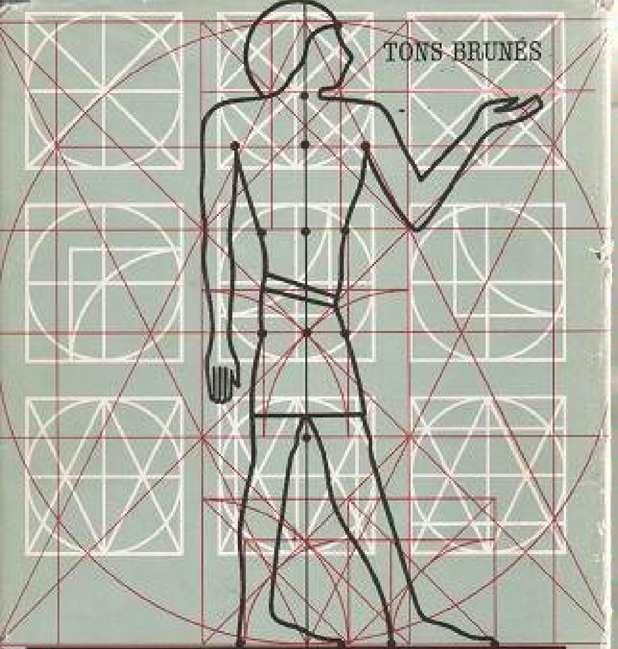
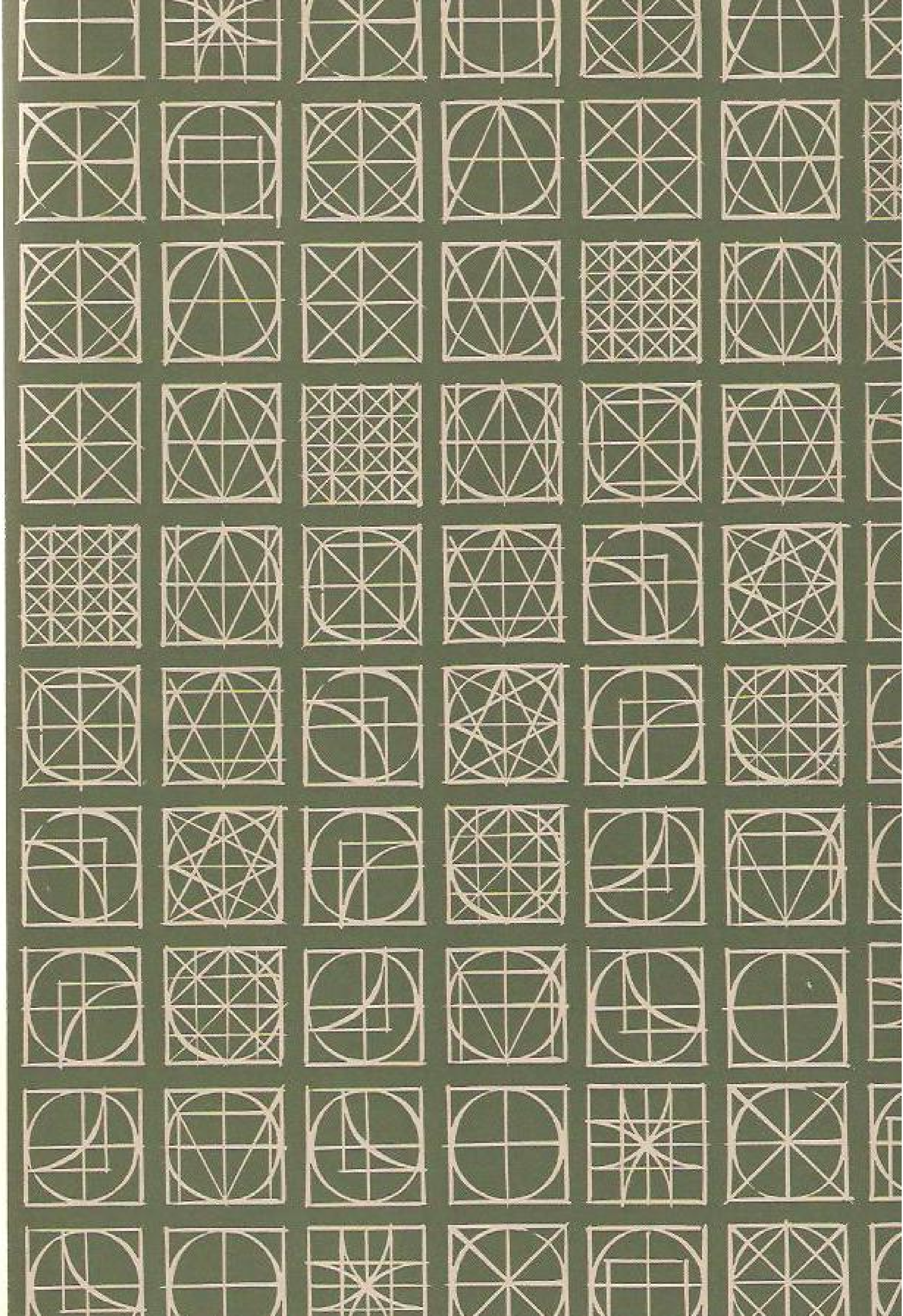
TONS BRUNÉS



The Secrets of ANCIENT GEOMETRY -and its use



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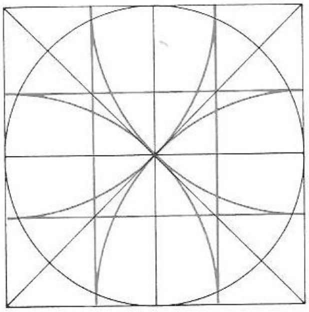
THE SECRETS OF Ancient Geometry - AND ITS USE



VOLUME II

RHODOS

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THE SECRETS OF ANCIENT GEOMETRY
was translated by *Charles M. Napier* from the original Danish manuscript
Den Hemmelige Oldtidsgeometri og dens Anvendelse.
All drawings and analyses have been made by the author.

★

The Greek quotation at the beginning of the book can be rendered thus:
"Only he who is familiar with geometry shall be admitted here".

★

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VOLUME II

CHAPTER FOURTEEN

Ancient Geometry reaches the Middle Ages

Passing from the era of Antiquity with its clean, almost severe, lines of temple design over to the colossal and much more intricate form of church and cathedral building of the Middle Ages, we must of course acknowledge the effect of centuries of development upon the methods and art of building.

Experience had taught builders that dimensions and proportions could be whittled down and given a lighter appearance with no detrimental effect on stability—within reason. Brand-new construction materials were produced, opening up fresh possibilities of design. The development of brick-making in particular, in conjunction with mortar, enabled buildings to be erected to heights unknown since the epoch of the ancient pyramid. Not to forget the fact that it was much easier in the Middle Ages to procure building materials in places where previously natural stone had to be brought in at great expense and inconvenience.

The development was understandably slow. The columned temples of ancient days were followed by buildings with slimmer columns, thinner lintels and increasingly smaller wall blocks. Eventually the brick as we know it today was evolved.

But one factor remained unchanged. Building, construction and design was the domain of the Temple, the Monastery and the Church.

Antiquity and the Middle Ages are generally accepted as merging (in Europe) towards the end of the 5th century A.D. The Middle Ages then went on until relative modernity took over about the middle of the 15th century. Whereas in ancient times the building brothers of the Temple had designed, organised and built the great edifices of their era, in the Middle Ages the monks took over this task. But the difference between the two groups was small.

The principle of complete secrecy was still all-powerful. It was the monks who taught the people whatever accomplishments the Church decided might be revealed to the ordinary man and woman. And together with this enlightenment the people were at the same time fitted with the all-embracing cloak of religion. A cloak that may have been heavy and inconvenient—but there was no choice. If he wanted education, a man could tread only one path: to the Church's school.

The numerous convent schools scattered throughout Europe are of course a memory of that period.

The power of the Church throughout the Middle Ages was virtually limitless, and it was materially rich, too, to an unbelievable degree. Witness the many magnificent cathedral buildings all over Europe dating from that period. To see anything approaching their ornate, ex-

pansive splendour we must search in the fairytale lands of the mysterious East. The Western world has nothing else to compare.

But contrast these huge vaulted monuments with their surroundings! The homes of the ordinary people were tiny, poor and frequently hovels.

Church design with each generation became bolder, more audacious—almost outrageous in its frankness. Builders rejected the traditional stone as a material for inside work. Timber, carved and moulded, took its place. There were timber rafters, wooden floors, and other structural details of wood.

This move towards "unknown" materials such as timber for work of these proportions naturally involved a degree of risk concerning the building's permanence. The durability and suitability of the new materials were largely unknown factors. Time alone could tell.

Structurally buildings were provided with rather thick walls at the base, the thickness of brickwork decreasing as the building rose. Only experience could tell where the limit of this taper effect lay. The wall or structure which withstood wind, weather and time really told the designer nothing. It was the structure that cracked and crashed to the ground that taught him a positive lesson!

With a geometric analysis in mind one must therefore be careful about choice of subject as far as the architecture of the Middle Ages is concerned. As often as not the buildings in question have been either rebuilt or restored. Extensions have been built, new spires and towers of a different form put up in place of the old—which had either crumbled with time or failed structurally. Or it may have been that the church had burned down and been rebuilt several times in its history—not always in the same manner and style.

It does not necessarily follow that these alterations and restorations deviated entirely from the generally accepted principles of geometric construction. It depended entirely on which group of builders were responsible for the work.

Free art, i.e. designers and builders who had dared to work outside the framework of the Church, began to exert a limited influence. Naturally an atmosphere of tension and jealousy existed between these and the men who stuck indomitably to traditional principles in their building design.

The "free" group nevertheless gained a gradually widening foothold within certain sections of the Church itself, and we see a typical example of the animosity between the groups in 1398 soon after the "free" builders had won the contract to erect Milan Cathedral.

The Italian master builders, soon after the job was started, found themselves in difficulty over a point of structure. They turned to Jean Mignot, the French architect, for assistance.

After a thorough examination of the structural plans Mignot sharply criticised the fact that they were not based on geometric principles. The Italian defence was: *"... quod scientia geometriae non debet in iis locum habere eo quia scientia est unum et ars est aliud."* (... that science of geometry has really nothing to do with the matter, since science is one subject and art quite another.)

This brought the crushing retort from Mignot: *"Ars sine scientia nihil est!"* (Art is nothing without science.) His reference was to the science of geometry.

This is a splendid illustration and proof of the existence of two building schools. One tried to blaze new trails, designing according to a personal concept of beauty and divorced from accepted geometric practice. The second school continued to derive inspiration from the ancient, established mode of construction.

The latter were without doubt the larger of the two groups. Cathedral design in the Middle Ages affirms this. But the greater number of buildings from this period have been restored or rebuilt several times in the intervening 500 years. And the design of restoration work may have followed one of two patterns.

If the builder subscribed to ancient geometric principles, he naturally applied these to the structure. It is quite possible in fact that the structure was not an exact replica of the original although based on ancient geometry. The restorer may simply have selected another point or line in the diagram on which to base his work. The building might turn out to be higher and wider than the original—and still fit the lines of the geometric diagram.

If on the other hand the building was converted or rebuilt by architects of the new school, it would prove difficult if not impossible to trace the plan of construction on which restoration was based. His choice was a personal affair, in no way governed by geometry. He simply decided for himself what he thought was beautiful and what was not.

In spite of a spirited and determined resistance the new school gained ground relentlessly. From the 15th century onwards the traditionalists were on the retreat. Ancient geometry was on the decline. So thoroughly was it thrust aside that today its principles have long since been forgotten.

The demand for buildings, houses, factories, etc., increased with each new age. New materials were continually being introduced. These new fields and materials attracted their own craftsmen's guilds which were not in the same way as before bound to the traditional past.

The powerful building sections of the various monastic orders irrevocably lost their grip on construction work. They were replaced by guilds of stone-masons, which in essence were a poor imitation of the once-powerful temple brethren whose handful of leaders had been the inspired builders of their day, the men who produced the plans for the brethren to follow. Always guided of course by ancient geometry.

We can still with a bit of care trace the application of these geometric principles in buildings erected far into the Middle Ages. If we confine ourselves to the period up to, say, 1200 A.D., we find the old guard still in command. Inaccurate restoration work on these old structures may be evident in the form of variations from the principles of geometry. But these are relatively easy to spot.

Cologne Cathedral: the Old

One of early Christendom's first footholds in the area of Europe now known as Germany was Cologne. By the 3rd century there was already a large Christian community there.

The town at that time belonged not to the North Europeans but to the South. It had been a Roman province since 50 A.D.

When the Roman Emperor, Constantine the Great, adopted the new faith in 312 A.D. the current Christian leader at Cologne, Bishop Maternus, saw the occasion as a favourable opportunity to have a cathedral built in the town.

Legend and history say this building was richly decorated with splendid (imported) granite columns, marble walls of

delightful colouring, and intricate mosaic floors.

We can only—alas—guess at the layout and appearance of this original building since nothing of it remains apart from the awed writings of men who had visited it. There is certainly no illustration suitable for the purpose of reconstruction.

Over the centuries Cologne attained considerable importance and influence, and in the year 814 it was decided to provide the growing city with a new cathedral to take the place (literally) of the old.

Instead of erecting the new cathedral on another site in the city the builder, Hildebold, Archbishop of Cologne, decided to put it on exactly the same spot as the old. Consequently the original building was bodily removed, and the new cathedral started on the site. It was completed around 873.

The new cathedral was added to in 1056. In that year Archbishop Anno built a new wing adjacent to the eastern entrance. He dedicated the new part to the Virgin Mary.

The cathedral now comprised the front church, complete with spires, the nave and the additional wing, in effect the new chancel. This latter part, too, was topped by spires.

This old church building was reconstructed from contemporary drawings and descriptions by Sulpiz Boisserée in his book, *Geschichte und Beschreibung des Doms von Köln,* Munich 1842. The church itself was destroyed by fire in the 13th century.

His illustrations consist of a ground-plan and side elevation. Boisserée's book does not, however, have a front elevation of the old cathedral.

In *Fig. 222* we have the ground-plan in which the original church and nave are on the left, the later extension on the right.

It is impossible of course to check whether the details as shown in Boisserée's reconstruction are accurate. We can only trust that they are. The material from which the reconstruction was compiled was obtained from various sources, but much of it stems according to the author from the church itself.

Any attempt to trace the principle behind the original cathedral plan must naturally start with the old building, i.e. setting aside for the moment the part dedicated to the Virgin Mary. We would normally look first at the facade but since, as mentioned, we have no facade illustration we must rely on the ground-plan for our analysis.

Our first analytical diagram is *Fig. 222.* We ignore meantime the structural addition on the right and take as our line of origin the total length of the old cathedral, represented by line 1-2. On this basis we construct our basic square 3-4-5-6. The whole area of the old cathedral lies within it.

The four sacred cuts, horizontally and vertically, are entered, and we see immediately that the two horizontal lines (in typical Greek temple fashion) indicate the width of the church proper. These are 7-8 and 8-10.

As we are well aware by now the union of the four sacred cuts produces a new central square. Here again we enter the sacred cut arrangement.

We see that horizontal cuts 18-19 and 20-21 mark the placing of the two rows of columns that run the length of the nave.

We note also that the columns contained by the inner square (resulting from the sacred cuts) number 7×2. A most natural choice, considering the importance of this number to ancient geometers. Number 7 (as we saw in Chapter Three) opened several doors to the mysteries of geometry.

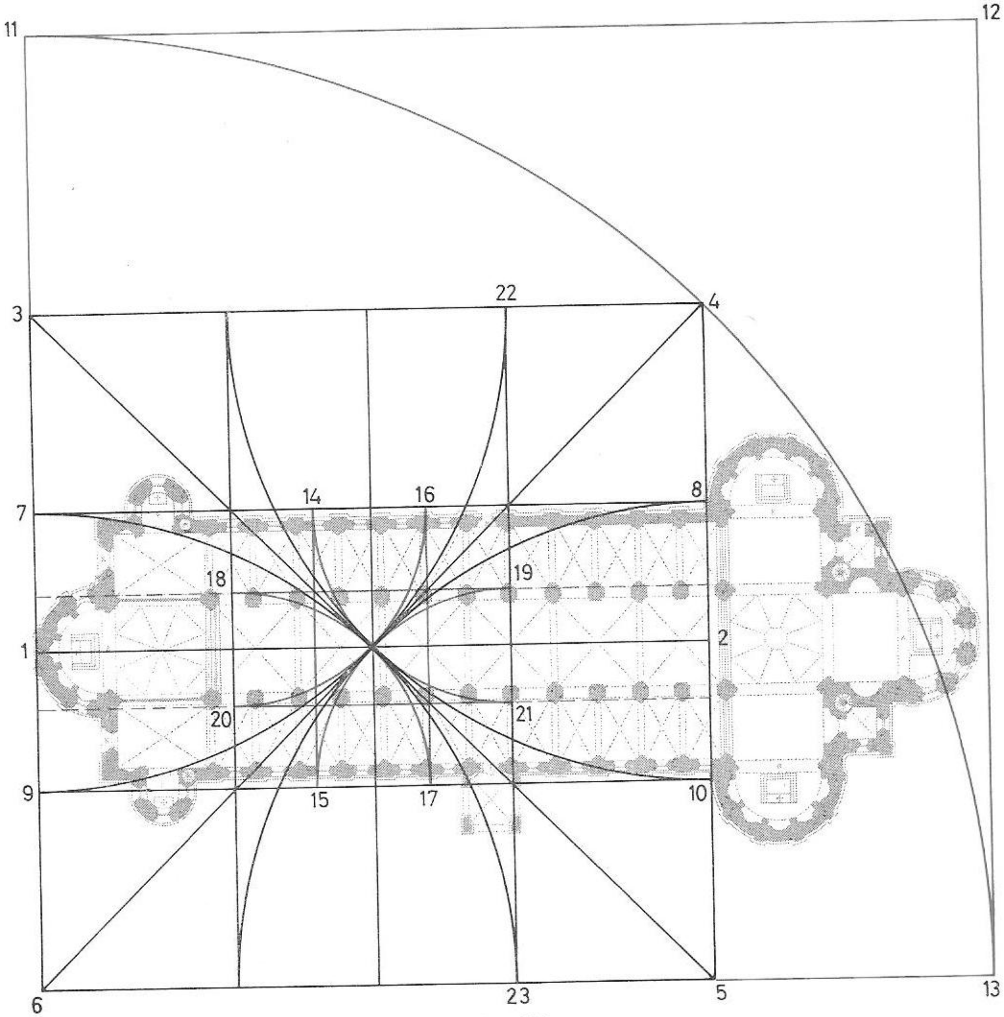

Fig. 222.

Thus we see in our basic square that, true to tradition, the sacred cut in two forms indicates (a) the total width of the church, and (b) the width of the inner columned hall.

We see, too, that the entrance doorway at the side of the nave appears to butt against the vertical sacred cut 22-23 in the main square, and that the inner horizontal cuts indicate the diameter of the circle on which the semi-circular structure on the facade is designed.

These few factors would seem to hint that we are on the right track and that our choice of basic square was correct.

If at a later date an architect wished to extend the church and yet retain the framework of ancient geometry, he could for example choose as the basis of his design the double-size version of the original basic square. This apparently is just what he did. Again we turn to *Fig. 222,* where we find this double square 6-11-12-13.

One of the vertical sacred cuts in the

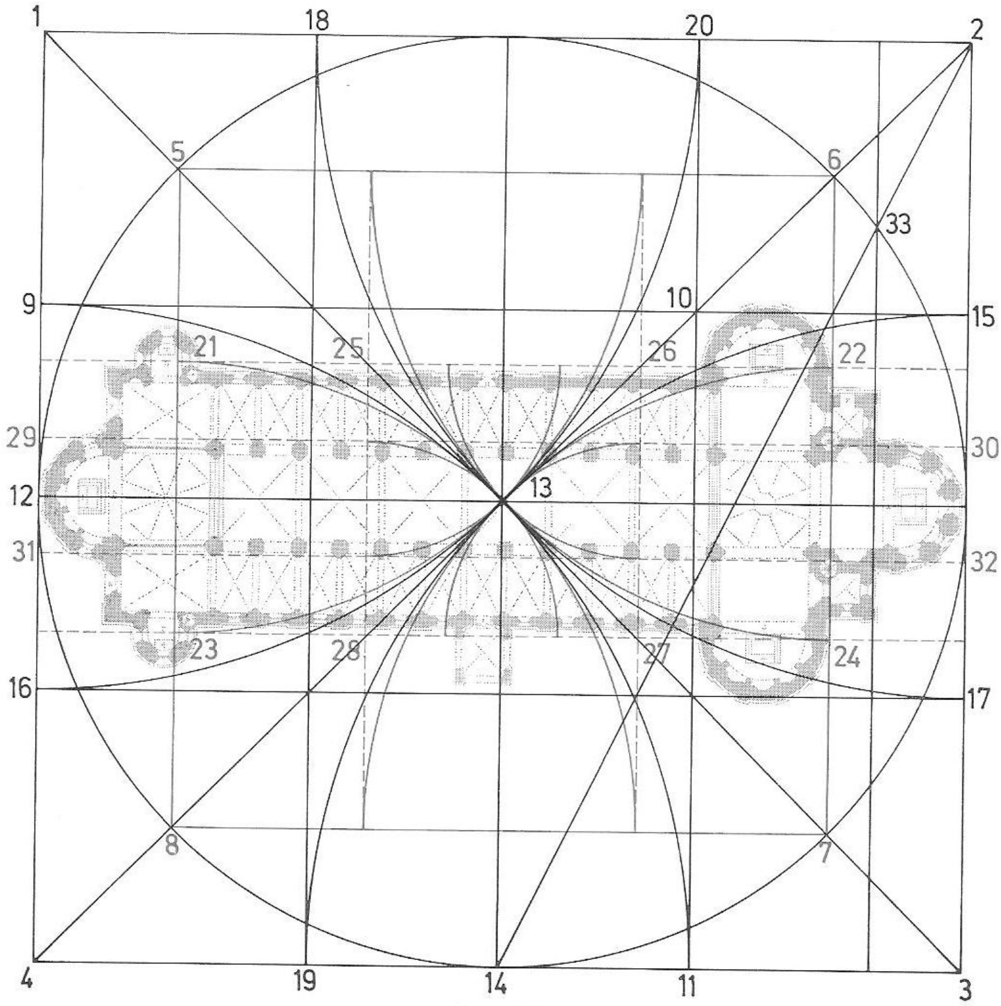

Fig. 223.

new square is line 5-4 which is of course the extremity of the old cathedral.

The additional building will now be placed in the area measured by line 5-13. The illustration shows that this was indeed what the architect intended.

Since the old part of the cathedral was enclosed in the half-size square of the new cathedral's basic square, the architect was able to place the whole building in its proper position in the centre of the diagram. We see this done in *Fig. 223.*

The complete ground-plan of the church has been placed centrally in the large square 1-2-3-4 (i.e. the square which in the previous diagram was named 6-11-12-13). None of the dimensions of the new building has been indicated so far, apart from the length.

We must imagine that our square at this stage contains only the old cathedral. Only the overall length (1-2) of the completed new cathedral is known. The old building is contained in square 4-9-10-11 which, as we know and can see, is half the size of the large basic square.

The sacred cut arrangement is executed in 1-2-3-4. The lines are 9-15 and 16-17 (horizontally) and 18-19 and 20-11 (vertically).

We observe first of all how the horizontal cuts mark the width of the new building. The two round towers are flush with the lines of the sacred cut.

As we have already noted that one sacred cut (20–11) marked the beginning of the new building, we can now state that the *length* of the addition (20-2) is determined by the sacred cut, and the *width* of the new building is also (9-15 and 16-17) indicated by the sacred cut.

When the basic square's circle is inscribed we are able to complete the square's half-size version (5-6-7-8).

This square is equal to the square originally used as the basis of design for the old cathedral.

We see how side 6-7 of this square indicates the diameter of the two new towers. This diameter is the distance between 20-11 and 6-7.

The sacred cut arrangement in 5-6-7-8 is shown with a broken line. This naturally reveals the external width of the old cathedral nave, as we saw in Fig. 222. The lines are 21-22 and 23-24. If we follow them into the new building we find that they give the location of the two square areas in the large towers.

The broken lines as usual create a new square (25-26-27-28). We examined the same square in the previous diagram, in which we found the placing of the rows of columns. The sacred cut in this inner square is produced to the outside of the diagram in lines 29-30 and 31-32. Immediately we see that the diameters of the towers at each end of the building take their dimensions from these lines.

Furthermore these horizontals, as we have seen, run flush with the outside of the columns and, in the new building, we see how they indicate the location of two rosettes on the corners of the square towers.

Diagonal 2-14 is entered to show the position of the circle's rectangle. Intersection point 33 is the one we require. A perpendicular through this point is in fact the same as one of the sides of the circle's rectangle. It indicates, as we see, the face of the square tower at each corner. These are square from ground-level to the level of the roof, after which they alter to an octagonal shape and are topped by an eight-sided spire.

Thus we see how finely the old and new structural plans have been dovetailed together. The geometric harmony of the whole project has been retained. And the ground-plan is ample proof of the geometric basis applied to its design.

I think coincidence can be set aside. It is surely impossible that so many details of the structure would fit the diagram if they had not been planned accordingly. The principal factor of design in laying out the ground-plan, both of the old and new buildings, was the sacred cut. It revealed all the main dimensions in the building.

As mentioned previously, Sulpiz Boisserée's book carried no illustration of the facade, neither of the original nor of the new building. And I have not been able to trace any plan or picture of the front elevation suitable for a geometric analysis.

Thanks, however, to the assistance of photography I believe I have come as close to a facade illustration as possible. And accurate enough for the purposes of analysis.

The new building, as opposed to the western original end, possesses the remarkable geometric property of displaying the same view to the side as it does to the front. Let me explain.

If we study the building from the side, we see ½ facade plus ½ of one of the circular towers (in fact rather more than

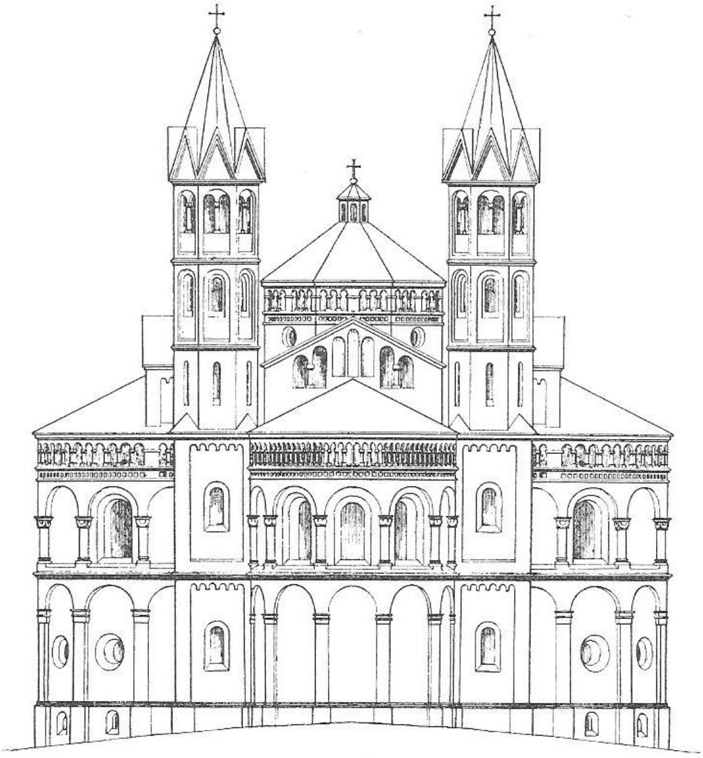

Fig. 224.

½ the latter). In other words, the nave plus a section identical to rather more than half the facade.

If we take two photographs of this side-view, printing one as it appears in reality and the other *back to front,* and placing them together at the point which would correspond to the middle of the facade, we find ourselves with an illustration of the cathedral's front elevation which must reasonably be assumed correct. This is what in fact was done.

It was only possible with the new building as its width and depth were in the ratio 1:1. It would not have been possible to execute the same "manoeuvre" at the other end of the cathedral. The proportions are not suitable.

We see the reconstructed front elevation in *Fig. 224* and our task now is to discover the location of the surrounding square in order to find the constructive basis of the facade.

This analysis is the reverse of our normal procedure. We have in the past first examined the facade, searching for a clue to the lay-out of the ground-plan. In this instance we are forced to examine the facade and ground-plan separately, and then to discover the link which we know must exist between them.

To construct the basic square of the facade we take as the length of its side the distance from ground-level to the uppermost tip of the twin spires. This square therefore equals the facade in height but in width is somewhat larger.

By the simple expedient of measuring we are delighted to discover the link between the basic square of the facade and

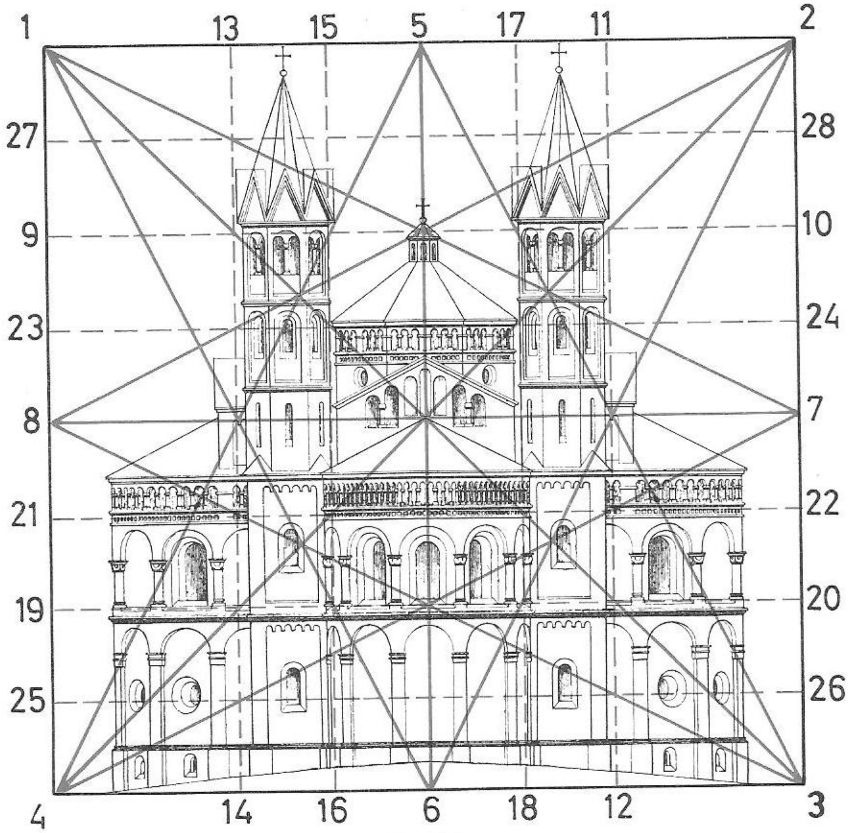

Fig. 225.

that of the ground-plan. The former is exactly ¼ of the latter. Equal therefore to 4-12-13-14 in Fig. 223. It is just conceivable that we are on the right track.

In *Fig. 225* we have the facade's basic square 1-2-3-4 and the lines of the vertical cross 5-6 and 7-8.

We see how the vertical axis (7-8) indicates the tip of the roof on each of the three circular apses. This is our first concrete clue that our square is correctly constructed.

We go on to enter the four acute-angled triangles 3-5-4, 4-7-1, 1-6-2 and 2-8-3. These make up the fundamental eight-pointed star which we discovered (Figs. 57 and 62) held the secret of the square's division into any number of strips from 4 to 10. Our plan here is to divide the square into 8 × 8 smaller squares.

In Fig. 225 we see the principal dividing lines as 9-10 and 11-12.

We observe how 9-10 marks the point at which the twin towers change shape from the vertical to a sloping spire. And 11-12 runs flush with the side of one of the towers.

When we complete the cross-lines of the 8 × 8 division we see immediately that this must indeed have been the basic factor of design in the facade. A wealth of information becomes evident.

We now see, for example, how line 17-18 marks the other side of the right-hand tower. Lines 13-14 and 15-16 indicate the width of the other tower.

The respective widths of the two towers are therefore ⅛ of the basic square, their location is 2/8 from the outside of the square and ⅛ from the vertical axis. They are 2/8 of the square apart.

This preoccupation by the architect with the number 8 is further underlined by the form of the towers themselves. As the diagram shows, these are not round but octahedral, and their eight faces each carry (at the top) a window and a small triangular crown.

We find a definite connection therefore between this 8-part division and the square's link with the ground-plan, since a square which is divided 8 × 8 is naturally also split 4 × 4.

But let us return to our inspection of the facade diagram. There are several interesting points yet to be recorded.

We noted earlier that line 9-10 marked the junction of tower and spire. The middle of the same line indicates the top of the small tower on the dome above the body of the church (between the twin towers). Line 23-24 is the level at which the dome's sloping roof begins.

We saw earlier that 7-8 marked the height of the three lower semi-circular buildings.

Line 21-22 marks the lower edge of the frieze at the top of the outer vertical wall of these buildings.

Line 19-20 indicates the position of a dark-coloured frieze which runs all the way round the new part of the cathedral, and line 25-26 illustrates the placing of the windows in the lower part of the building. They rest their sills on this line.

Thus we see how almost all of the 8-part dividing lines have their part to play in the facade. Most of them are of structural importance. One or two apply to decorative features.

The abundance of information derived from this first analysis of the facade must surely entitle us to record that, in his design of the facade, the architect certainly based his theme on the 8-part division of a square.

This was not, however, the sole theme. We still lack instructions on the pitch of the various roofs of the cathedral. The pitch of the spires on the twin towers seems obvious enough. It was obtained from the diagonal drawn from a point midway between 13 and 15 (17 and 11) and the horizontal 9-10. But what of the other roofs? The dome and the three semi-circular extensions?

In *Fig. 226* we see how the architect may have applied ancient geometry to determine these angles.

We start with the basic facade square 1-2-3-4 with its vertical and diagonal crosses and inscribed circle.

The half-size version 29-30-31-32 is constructed on the base-line of the diagram, and we see how the top of this square marks the top of the dome (excluding its small upper tower).

We saw that the facade's basic square was ¼ of the ground-plan's square. And we saw, too, that in the ground-plan the width of the facade in the new building was indicated by the sacred cut.

In Fig. 226 we have the outline of the ground-plan's basic square (bottom) and its relative proportion to the facade. We see the sacred cut (arrowed) produced as a pair of broken lines from the ground-plan to the facade, indicating again the width of the building.

Now we study square 1-2-3-4 and its half-size version 29-30-31-32 (at the base of the diagram). The horizontal axis of the former is 7-8, of the latter 39-40.

The diagonals of these squares intersect and create a small square immediately beneath the centre of the diagram, turned through 90° in relation to the larger squares.

The diagonal axis of this square indicates the point from which the roofs of

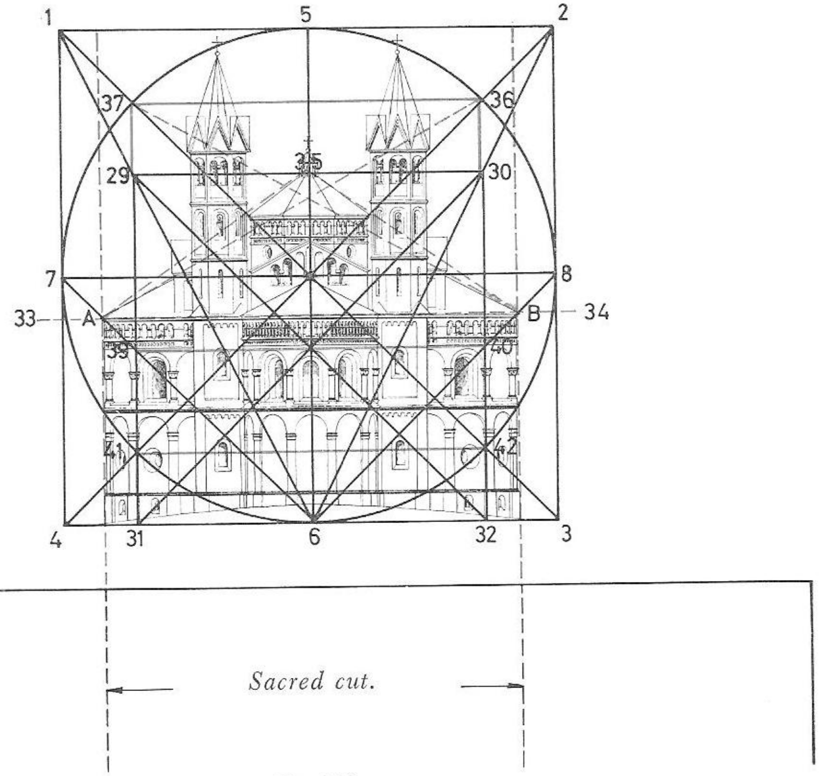

Fig. 226.

the three semi-circular apses begin their pitch. The line is 33-34. The intersection of this line with the width of the building is seen at A and B.

To discover the pitch of the three roofs, we require yet another construction; or rather, we must bring the half-size square up from the base-line to the centre of the diagram. We see this at 36-37-41-42.

Now we see that lines from A to 36 and from B to 37 provide the angle of pitch on the extension buildings. In other words, a combination of the line AB and the top of the half-size square placed centrally in the diagram.

We see a similar arrangement in fixing the pitch of the dome. But in this case the lines are from A and B to the middle of the top line (29-30) of the half-size square placed on the base. The lines are A-35 and B-35.

This leaves only few of the principal lines of structure unexplained. There may be one or two which we have not described but I have no doubt that their explanation, too, lies in ancient geometry and its principles. But the two analyses just completed of the facade should be sufficient to establish the association with geometry.

We thus applied an ancient procedure to the ground-plan and discovered the length and width intended by the architect of the extension. We went on to investigate how the facade was built up.

I believe that our course, i.e. taking the old ground-plan as a basis and establishing the length of the extension, was a

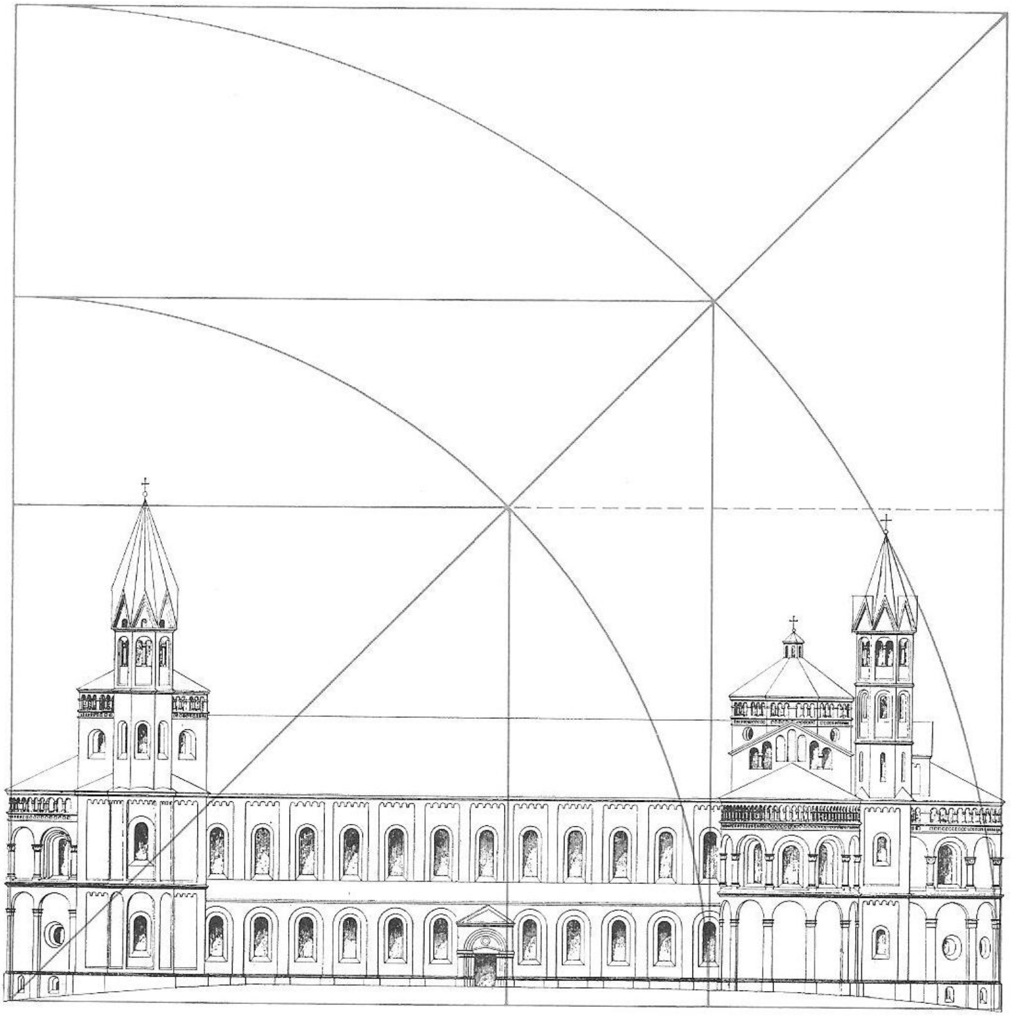

Fig. 227.

correct one. And it appears a natural step that the link between old and new should be the sacred cut.

We thus obtained a basic square for the ground-plan of the complete cathedral, old and new. This square was quartered to provide the basis of the facade plan. Its height was made to equal the height of the spires. In fact it is the height of these spires which decided whether the choice should be, for example, the centre square produced by the sacred cut or (as was the case) quarter of the ground-plan's basic square.

A comparison of the building's various dimensions reveals that the quarter-square decision was natural to allow the architect to build the towers of the extension building the same height as in the old building. This ensured a harmony at each end of the nave.

The reader may object that these towers could not have the same height since they were built according to two different dia-

grams. The towers and spires of the old building were erected nearly two hundred years before those of the new building.

But the explanation is that the height of the early church towers was determined by the sacred cut in the basic square applied to that old church. In other words, determined by the half-size square.

When we, in planning the new extension, also apply the rules of the sacred cut, we obtain three squares within each other: the three squares with which we are extremely familiar, where A is to B as 1:2, and B is to C as 1:2, and it follows that A is to C as 1:4.

We see the placing of the three squares in relation to the cathedral's side elevation in *Fig. 227*. We have no difficulty in recognising the trio. It would appear therefore that the plan of the cathedral is intended to symbolise the ratio of the three squares.

We can see quite readily that the architect of the old section of the cathedral chose to bring the tip of the spire up to the sacred cut but to place the cross above the line. The designer of the extension two hundred years later decided to keep both spire and cross within the basic square. But this virtually insignificant difference in personal taste becomes evident only in a geometric analysis. It is invisible to the casual eye.

The striking similarity we have encountered between the old reconstructed drawings and the principles of ancient geometry is in itself a reasonable guarantee that the restoration by Sulpiz Boisserée was essentially accurate. According to his book, much of the material on which the reconstructions were based come from original cathedral drawings and plans. A better source of material is perhaps difficult to quote.

Cologne Cathedral: the New

We have conducted an analysis of drawings of the old cathedral in Cologne, building of which was concluded around 900 A.D. But a visit to Cologne in the hope of inspecting this church or its remains would be pointless. It no longer exists.

The old Cologne Cathedral stood for only 350 years. In 1248 it was destroyed completely by fire. And it was on the site of the old church that its successor, the imposing structure of the present-day Cologne Cathedral, was erected.

According to history, few tears were shed when the old cathedral burned down. One might almost surmise that the reverse was the case. The building had been the subject of vehement criticism since the additional part was built on the instructions of Bishop Anno. It was always regarded—despite the effort to unify the structure—as two distinct buildings, each with its own characteristics. Over the years most cities around Cologne had erected impressive, attractive churches, and the religious leaders of Cologne were fairly agreed on one point: their cathedral was not a fitting representative for the city and the bishopric, particular since Cologne was considered one of the principal seats of religion in Germany.

Long before the cathedral's disintegration in a shower of sparks, therefore, plans were being prepared for the church

to end all churches. Certainly Cologne's neighbour-cities would never have seen anything like the new building.

Construction work in the case of churches and castles was, as we know, the responsibility of powerful religious brotherhoods, e.g. various monastic orders, who answered only to the hierarchy of the Temple or Church. One of the leading figures in such a brotherhood was a certain Meister Gerard who had been in charge of a number of church-building projects.

Whether Meister Gerard had been directly responsible for or had taken an active part in the selection of the constructive idea behind the future Cologne Cathedral is uncertain, but in any event he was fully conversant with and had an intimate understanding of the plans. It was therefore natural that he should be appointed in 1248 to manage the task of building the new cathedral.

The archbishop at that time was Konrad Graf von Hochsteden and he urged that construction should start as soon as the old site could be cleared. On 14th August, 1248, he had the pleasure of laying the corner-stone of the edifice that stands in Cologne today.

This building was to have one of the longest histories of construction of any in the Middle Ages.

Shortly after work began war and unrest broke out in the Rhine area. It not only delayed progress, it actually halted it altogether for long periods. And when the scene had settled and peace had returned there was a new setback: Meister Gerard, the man with the required experience and know-how to continue building, had died.

Gerard's death was a tremendous blow. He alone had been personally concerned with the cathedral's design, and for some (to us inexplicable) reason he had never trained up an assistant. What a situation! The proud city of Cologne with a partially built cathedral, and the builders with an armful of incomprehensible plans and drawings of the structure and its details. No one in the immediate vicinity had any idea how the plans should be followed. Not to mention something that no doubt existed but which is given no reference in history: a number of detailed geometric drawings. But who had the qualifications to carry on the work Gerard had begun?

Apparently only one man in the whole of Germany was sufficiently familiar with the respective aspects of geometry to bring order to the maze of lines that concealed the cathedral's lay-out. This expert was Albertus Magnus, a philosopher and theologian of the Dominican Order and a renowned man of science who boasted a particular knowledge of geometry and "the royal art", as church-building was termed.

According to Boisserée, Albertus Magnus was bishop of Regensburg about 1262: the year in which he was called in to help the builders of Cologne.

His efforts appear to have been entirely successful. After some study of the existing plans and drawings Bishop Albertus sorted out the threads, completed the architectural drawings, and handed them over to his brethren builders. They in turn, presented with concrete detail of the cathedral structure, were able to go ahead with construction under the charge of Albertus, or perhaps one of the other more trusted brethren to whom the bishop had revealed the vital factors on which the whole design depended.

This appeal by the cathedral builders to a high-ranking clergyman merely confirms that, even at this point in the Middle Ages, the real leaders of the building profession were officers of the Church. They were the architects and engineers who took charge of the work behind the scenes.

If the building trade, with its strict society of guilds, had been a purely professional affair (as is the case today), it is scarcely imaginable that the men who were faced with the task of building Cologne Cathedral would turn for help to a clergyman; that they would ask him to unravel the threads of a mysterious diagram spun by one of their one kind. One would hardly expect such a seemingly lofty churchman to have the necessary qualifications for solving the problem. This particular call to the Church illustrates therefore the solidarity that existed between the clergy and the building guilds. It was a perfectly natural step for the latter to seek the advice of their Church brethren. In the final analysis, in fact, they were actually turning to *one of their own superiors.*

Albertus Magnus and Meister Gerard were both fully aware of and familiar with the application of geometry as a constructive factor in building. Their knowledge and experience, although perhaps obtained in different places, stemmed certainly from the same source. It is not therefore greatly surprising that the bishop, by a study of the drawings and diagrams, was able to trace and define the factors which brought unity to the cathedral's planning—even though he had presumably never previously set eyes on the documents.

Once the plans had been sorted out and completed building was able to continue, and in 1322 (74 years after the laying of the foundation stone) the first part of the new church was finished and ready for consecration.

The church was far from complete but the people of Cologne at least had a building on which to concentrate their daily services. And building went on. The first difficult stages were over, although they had taken their time. Once again Cologne had a cathedral, and plans were available to allow the builders to go ahead with the rest of the structure. But more than 550 years were to slip past before the architect's original dream of Cologne Cathedral became reality.

Work went on laboriously until the early 16th century. Then it halted. The reason is commonly accepted to be that the day of the Gothic style had ended, in favour of the Renaissance. Whatever the reason, it was not until the mid-19th century that Boisserée came on the scene, flourished the recently rediscovered cathedral plans, and appealed to the German authorities to raise the capital to complete the unfinished building. This was done, according to the original plans, between 1842 and 1880.

Without being able to prove the point conclusively, it is not unreasonable to assume that the original plans—as completed and perhaps amended by Albertus Magnus—were adhered to throughout the cathedral's long period of building.

It would doubtless have been an extremely difficult job to alter significantly the plans of such a complicated giant. We may thus expect to find the building that existed in 1880 to be very close indeed to the one planned as far back as the 10th century or at any rate to that envisaged by Bishop Albertus.

The principles of ancient geometry reveal to us many of the finer details of the structure of Cologne Cathedral, and the presence of this ancient geometric system in the cathedral's plan is immediately apparent—as we shall shortly see.

Our analytical diagrams are in fact the same as those applied earlier to temples, arches, etc. But there are more of them, which means more lines.

Looking objectively at the intricate front elevation of Cologne Cathedral with its richly ornate facade, its towers and spires, windows on several levels, windows placed (vertically) irregularly of different

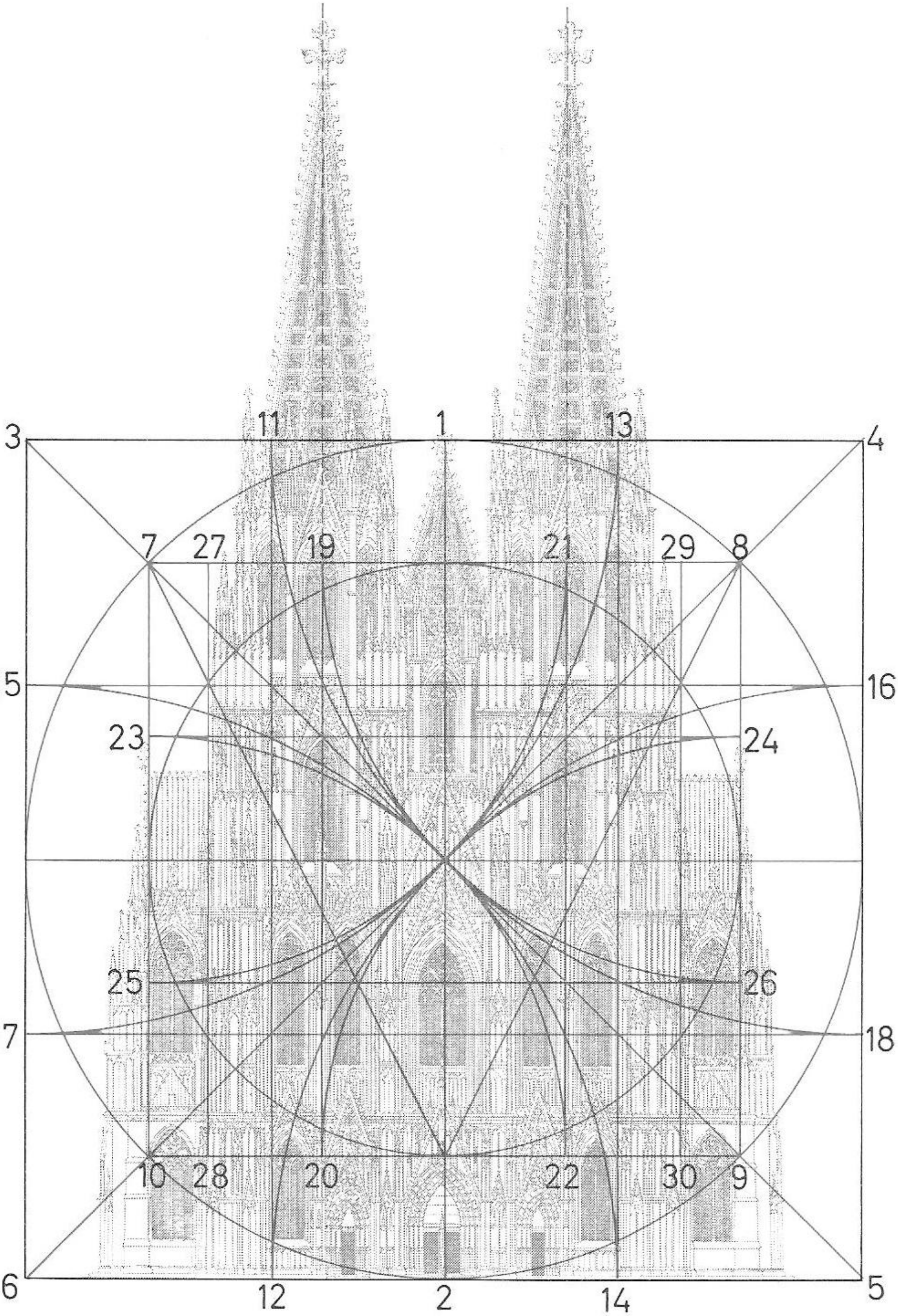
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Fig. 228.

heights and widths, its unending number of decorative friezes, the reader can readily appreciate the sheer massiveness of the task of analysing *completely* every line, every detail, every angle, of this whole building.

But it is not the function of this book to make such a detailed analysis of one building. Our diagrams, however, will reveal the principal dimensions of the facade and ground-plan and will pinpoint most of the main friezes and decorative features. Eventually we shall find ourselves able to reconstruct on paper the whole cathedral, without reference to a picture or photograph of the building itself. If ancient geometric principles can permit us to perform this feat, surely it is ample proof that this system did in fact form the basis of the cathedral's structural plan, a suspicion raised both by the building's history and its date of planning.

Our analysis starts with the view of the facade in *Fig. 228.*

This front elevation view of Cologne Cathedral is reproduced from Boisserée's *Geschichte und Beschreibung des Doms von Köln.* Despite extensive damage to the structure during World War II, the facade is largely the same today. Boisserée's drawings must, however, be considered nearer the orginal than any modern drawing or photograph could be.

The drawing in *Geschichte und Beschreibung* was photographically reduced somewhat in size. The subsequent geometric analysis was conducted directly upon the resulting photograph.

Before starting the analysis it would perhaps be advisable to satisfy ourselves on the accuracy and reliability of the material at hand. We know only too well that a proper analysis demands a great degree of accuracy from its subject.

Boisserée had at his disposal the actual drawings used in the construction of the cathedral. We may therefore take it for granted that the dimensions reproduced in his drawings are in the main accurate. Height, width, length, etc., have not been deduced or calculated from on-the-spot measurement, but have been obtained from the original source material.

We see in Fig. 228 the wealth of detail, decoration and ornamentation that covers the face of the cathedral. Friezes, towers, spires, etc., have been applied in a similar profusion to the ornamentation of, for example, the Arch of Constantine which we saw earlier.

All this has produced a mass of vertical and horizontal lines, each very close indeed to its parallel neighbour. The slightest discrepancy in reproduction of the original drawing would shift the geometric line of structure from one point to another.

Boisserée's drawings then are copies of the originals. They were executed about 1840 and reproduced in his book published in 1842.

The original drawings were undoubtedly of a fairly large format which Boisserée had been obliged to have copied by a graphics expert so that they could be used in his book. He was unable to share the present-day benefit of photography in order to reproduce, reduce and enlarge such diagrams at will.

The process of copying the drawings therefore presents an opportunity for slight inaccuracy to creep in.

The mechanics, too, of printing the book opens up a possibility of error. It is well-known that, in printing, paper can stretch and contract minutely depending on quality. Here, too, then another avenue for error—however slight.

These two points are mentioned not because the analysis is ridiculously ill-founded. It is not. It bears a remarkable degree of accuracy.

But one or two lines in the diagrams we are shortly to examine do not coincide

as precisely with their geometric counterparts as we might wish. The two possibilities of error in the original material are therefore mentioned as a conceivable cause of this discrepancy.

As always, the first problem in our analysis of the building's facade is to discover the size and position of the basic square, the square which in its division and multiplications will provide us with all the dimensions in the rest of the building.

The possible alternatives in this huge stone face of Cologne Cathedral are vast. Its high twin spires present a picture quite unlike any of the structures previously analysed in this book.

At the end of an exhaustive (and exhausting!) series of experiments I found the basic square. The key point in the entire front elevation is the centre of the 10-pointed star atop the central steeple.

If we take the distance from the centre of this star to the cathedral's base-line, and use it to construct a square on the base-line, we find this to be the basic square of the facade.

We do not, therefore, start—as with so many analyses—with a square which surrounds the whole facade. A square instead which extends outside the walls of the cathedral and which cuts across the two main steeples.

In Fig. 228 we see the vertical axis of the basic square as line 1-2, and the square itself as 3-4-5-6.

We enter the diagonals of the square and inscribe its circle. These permit us to construct the half-size version of the basic square, 7-8-9-10.

This is the familiar symbol "H", one square inside another. We go on to construct the sacred cut in both squares, i.e. in both squares we have entered symbol "M".

The two vertical cuts in the large (basic) square are 11-12 and 13-14, the two horizontals are 15-16 and 17-18. The corresponding lines in the half-size square are 19-20 and 21-22, and 23-24 and 25-26.

Furthermore, in the half-size square, we enter the symbol termed "Q", i.e. showing the circle's rectangle. It is shown by the lines 27-28 and 29-30. Thus our first diagram is complete and ready for inspection.

A building as complicated as Cologne Cathedral naturally has a larger number of marking lines than seen in previously executed analyses. I have therefore preferred to adopt a slightly different system from previous buildings in my study of the respective symbols.

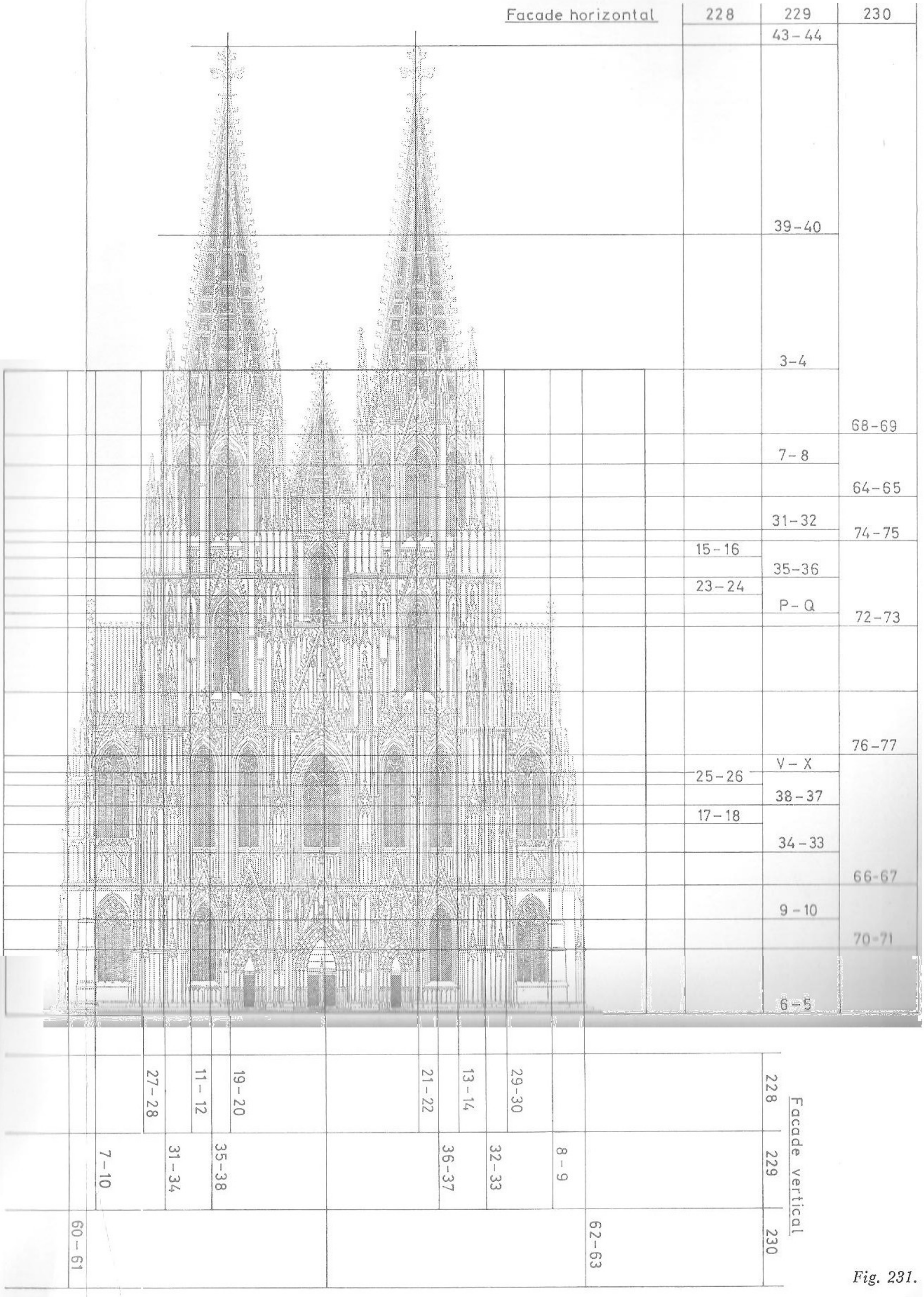
With, for example, the Greek temples we studied the horizontal and vertical lines of each symbol exclusively before moving on to examine the corresponding lines of the next symbol. With the building at present under the microscope, however, I think it will pay best dividends if —after constructing the complete symbol —we deal first with all the verticals and then with all the horizontals.

The cathedral's facade being symmetrical vertically it will suffice to mention only the vertical lines to one side of the axis.

We start with the vertical side 3-6 of the basic square. It has no contact with the building, and its function appears solely to be part of the square on which all the cathedral's dimensions will be based.

The next line in from the left-hand side is 7-10.

This is the first apparent observation we can record: the extreme vertical face of the transept is bounded by this line as can clearly be seen near the roof of the transept just above the horizontal axis of the basic square. Following line 7-10 down towards the ground, we see that it also marks the left-hand side of the two windows (one above the other) in the transept.


Facade horizontal
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43-44
39-40
3-4
68-69
7-8
64-65
31-32
74-75
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23-24
P-Q
72-73
76-77
V-X
25-26
38-37
17-18
34-33
66-67
9-10
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6-5
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29-30
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Facade vertical
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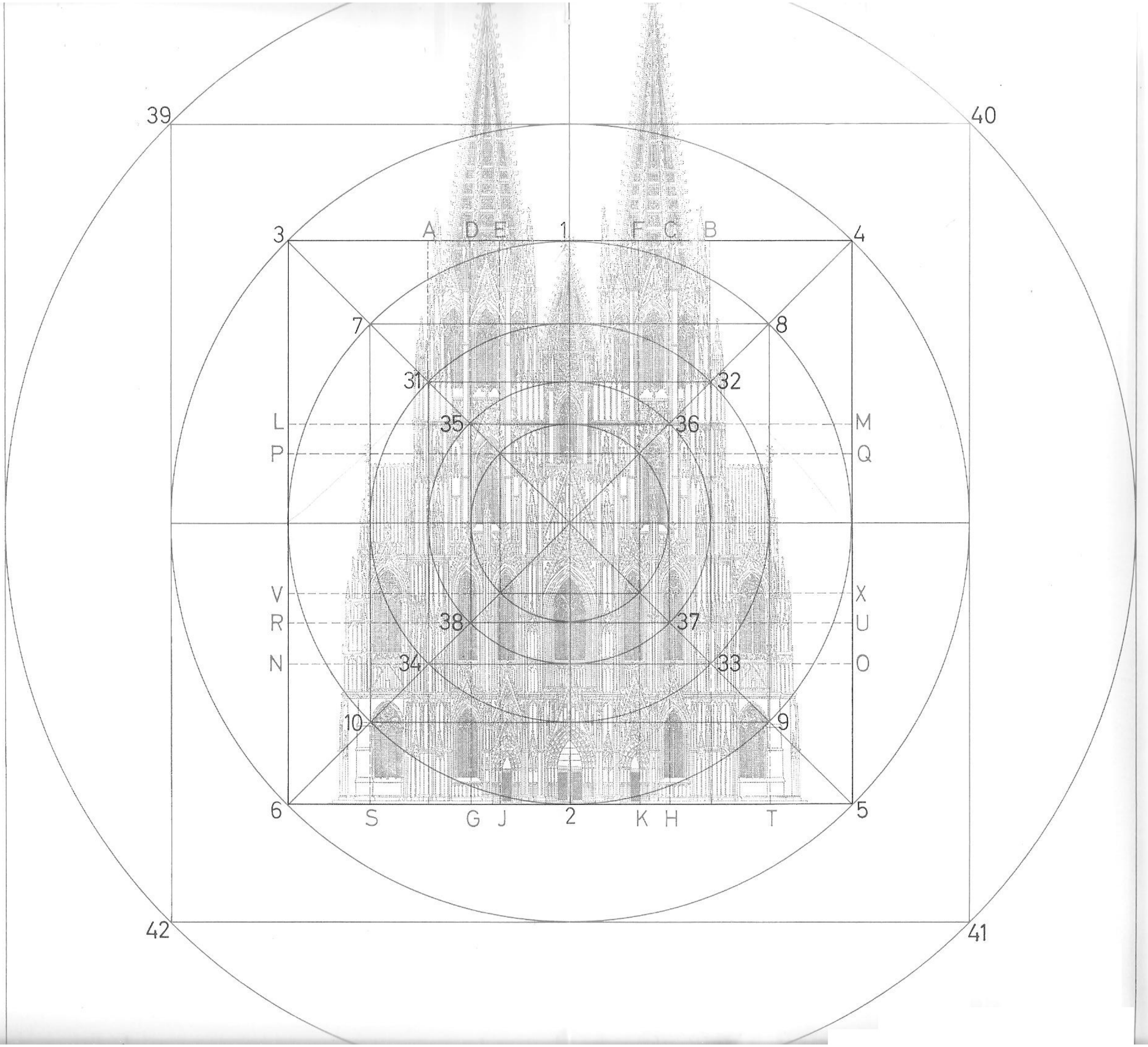
Fig. 231.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The next vertical line is 27-28, which is part of the circle's rectangle in square 7-8-9-10. This indicates the left-hand side of the facade, being the line from which the transept juts outward. The circle's rectangle therefore marks the width of the facade.

The next vertical is the sacred cut 11-12 in the basic square. At the top this line indicates the point from which the angled spire points skywards. Just under this mark the line cuts through the pointed decorative steeple to the left of the centre window in the main steeple.

As we move downwards we see that the line indicates part of the vertical ornamentation, and defines the left-hand side of the outermost of the five windows on the first floor of the building. It also marks the left-hand side of the window on the ground floor to the left of the entrance.

The next vertical we examine is 19-20, which is the sacred cut in the half-size square. This is one of the most striking lines of structure in the whole diagram. Produced upwards, it becomes the vertical axis of the cathedral's two principal steeples. Similarly it forms the centre-line of the pointed structure above the top steeple window, and is the axis for the window itself. It has the same function in the window underneath, and following it yet further downwards we see it pass between two windows on the first floor. Finally, on the base-line, it marks the left-hand extreme of the cathedral's arrangement of doors.

The next vertical, the diagram's original axis 1-2, naturally forms the axis of the stubby spire sticking up between the two main spires. As it makes its way groundwards it indicates the vertical centre of various ornamental faces and windows, until finally at ground-level it cuts through the main doorway. The door is a two-part affair, and the vertical axis therefore is represented at the very entrance to the cathedral.

As mentioned earlier, it will be unnecessary to repeat the vertical lines to the right of the axis since the building is symmetrical.

We turn instead to the horizontal lines of our diagram, and begin with the uppermost line 3-4.

It marks the centre of the star which proved to be the original starting point of the whole analysis. Moreover it indicates the uppermost limit of the pointed structure above the main spire's top three windows. At the tip of this superstructure there is a cross—which reaches precisely to line 3-4.

The next horizontal, 7-8, represents the level at which the tower windows change from the vertical to a pointed style.

Line 15-16 appears to have no particular marking on the facade apart from a piece of decorative frieze.

Line 23-24 indicates the pointed tip of the two windows on the third stage (i.e. second floor) of the facade. It also marks the height of the crosses on the ridge of the two parts of the transept, and the cross on the relief spire set across the vertical axis 1-2.

The horizontal axis marks the top of the seven windows of the first floor (five on the actual facade, two on the transepts). It will be noted how the crosses above these windows reach just to the height of this line.

Simultaneously the horizontal axis indicates the bottom of the two windows of the second floor. We have now obtained the vital statistics of these two windows: the top, the bottom and the vertical axis.

The next horizontal we come to is 25-26. It marks the top of the relief spires carved into the wall to the left and right of the five first-floor windows, and of similar spires between the individual windows.

The next horizontal, 17-18, has apparently no function to perform in the facade, whereas the final horizontal (apart from the base), line 10-9, holds a vital part in the structure. It marks the height of the huge arched doorway, and indicates the top of the ornamentation above the two smaller side doors. Finally, it marks the level on the two outer (transept) windows at which they alter from the vertical to a curved arch.

In the first analysis of the facade, then, we have studied nine vertical and nine horizontal lines. Apart from the two verticals that lay outside the facade altogether, only one single line had no function to perform in the structure of the cathedral.

Several features of our analysis seem to bring justification to our theory and to our choice of starting point. But one or two important parts of the facade structure are still missing—including the height of the building.

We shall find a few of these in our next analysis in *Fig. 229*.

We begin this part of our study with the same basic square as previously, 3-4-5-6, and with its half-size version 7-8-9-10.

Our procedure in this instance is to continue the half-size square construction by producing squares 31-32-33-34 and 35-36-37-38, and finally a small central square which we shall leave un-numbered. Thus from our basic square we have gone down in four stages, dividing each resulting area in half. This gives us five concentric squares.

Similarly, we reverse the procedure and build outwards from the basic square, two stages, constructing squares 39-40-41-42 and 43-44-45-46.

We now have seven concentric squares based on our original square 3-4-5-6.

The first thing we realise is that we have located the tips of the two main steeples with line 43-44. We can readily appreciate now why it proved so difficult to find the cathedral's basic square. It would have been natural to look for one that had as its side-length the total height of the two main spires. But no! Such a square and subsequent trials provided no information of value about the facade. The building lies not at the centre of our symbol with its base on the base-line, but at the top with its base on the third bottom line, i.e. on our basic square.

The next horizontal (after 43-44) is 39-40. This line has no real structural value apart from acting as an intermediate stage in building up the large outer square which governed the height of the twin spires.

The next two horizontals, 3-4 and 7-8, were discussed during the analysis of the previous diagram.

The fifth horizontal, 31-32, marks first of all the bottom of the uppermost tower windows. It also marks the upper edge of the ornamental strip of rectangles which stretches across the whole width of the facade. Each rectangle, at the point where 31-32 cuts across, alters from the vertical to a pointed arrangement rather like the tip of an obelisk.

The next line, 35-36, has been produced across the diagram as LM. It indicates the bottom of the ornamental strip of rectangles mentioned above. Its position is the ledge which juts out from the face of the building.

The top of the inner (small) square is produced as line PQ. It indicates the bottom of the uppermost middle window, and goes on to mark the stage at which the two windows of the second floor alter from their vertical form to an arch.

The bottom of this same square, produced as line VX, has a similar function to that of PQ. It marks on the first-floor windows of the transept the stage at which these alter to an arch. VX also indicates the base of the tiny, slim spires which rise from the face of the transepts.

Horizontal 37-38 is produced to RU. It marks, at the extreme right and left of the structure, i.e. the faces of the transepts, the junction of two ornamental layers.

Line 33-34, produced to NO, marks the bottom of the five windows on the first floor of the facade. It marks, too, the tip of the triangular decoration between the two windows in each of the transepts, and the crosses above the two secondary entrances as well as the crosses above the two main windows on the facade's ground floor.

The two lower lines 9-10 and 5-6 are discussed in the earlier analysis (Fig. 228).

The present analysis will not provide us with a great deal of new vertical information since three of the seven vertical lines (taking only one side of the symmetrical facade into consideration) are entirely outwith the cathedral's structure.

The first new line (bearing in mind that 3-6 and 7-10 were dealt with earlier) is 31-34. It has been produced upwards to meet the top of the basic square at point A. We see clearly that it marks the left-hand side of one of the main towers. We can also observe how the same line cuts the slim corner spire off from the tower proper.

The next vertical, 35-38, marks the left-hand side of the two upper windows of the main tower. The line has been produced to DG.

Line EJ is an extension of the side of the smallest inner square. It indicates the right-hand side of the same two windows, and at the same time the left-hand side of the left-hand doorway.

We are now in possession of all the required dimensions for the two windows on the second floor. The previous analysis gave us the height and vertical axis, and here we have been told the width and the point at which the arches begin.

The past analysis has revealed a number of salient points about the facade, not the least important of which has been the building's height. But we still lack an indication of the cathedral's width. For this we must go on to another diagram, *Fig. 230*.

Here again we begin with the basic square 3-4-5-6, in which we enter symbol "U", i.e. the symbol containing the circle's rectangle and the square on the latter. The circle's rectangle is seen as 60-62-63-61, and its square as 64-65-63-61, i.e. resting on the base-line of the diagram.

We further construct the same square centrally in the diagram, 68-69-71-70, and finally reproduce it suspended from the top of the basic square, seen as 60-62-67-66.

The diagonals of all three squares are entered, giving us the same version of symbol "U" as discussed at some length in the chapter on Plato's *Timaeus*.

The immediate factor of interest is that the circle's rectangle in the basic square was undoubtedly used to determine the cathedral's total width.

We now have the two main dimensions of the building's facade: its height and width.

Of the symbol's horizontal lines, the first new one is 68-69. The diagram illustrates clearly that it marks the tip of the ornamental arch above each of the top-floor windows.

The next new horizontal is 64-65. Its principal function is to indicate the level from which the central spire starts. The line has another, minor, purpose. In the two small spires situated on the outside of each of the two main steeples, the line cuts naturally between two sections.

The various intersections of the diagonals provide additional lines of design. One of these is shown by 74-75. It marks the bottom of the top-floor series of windows.

The diagonals also combine at the

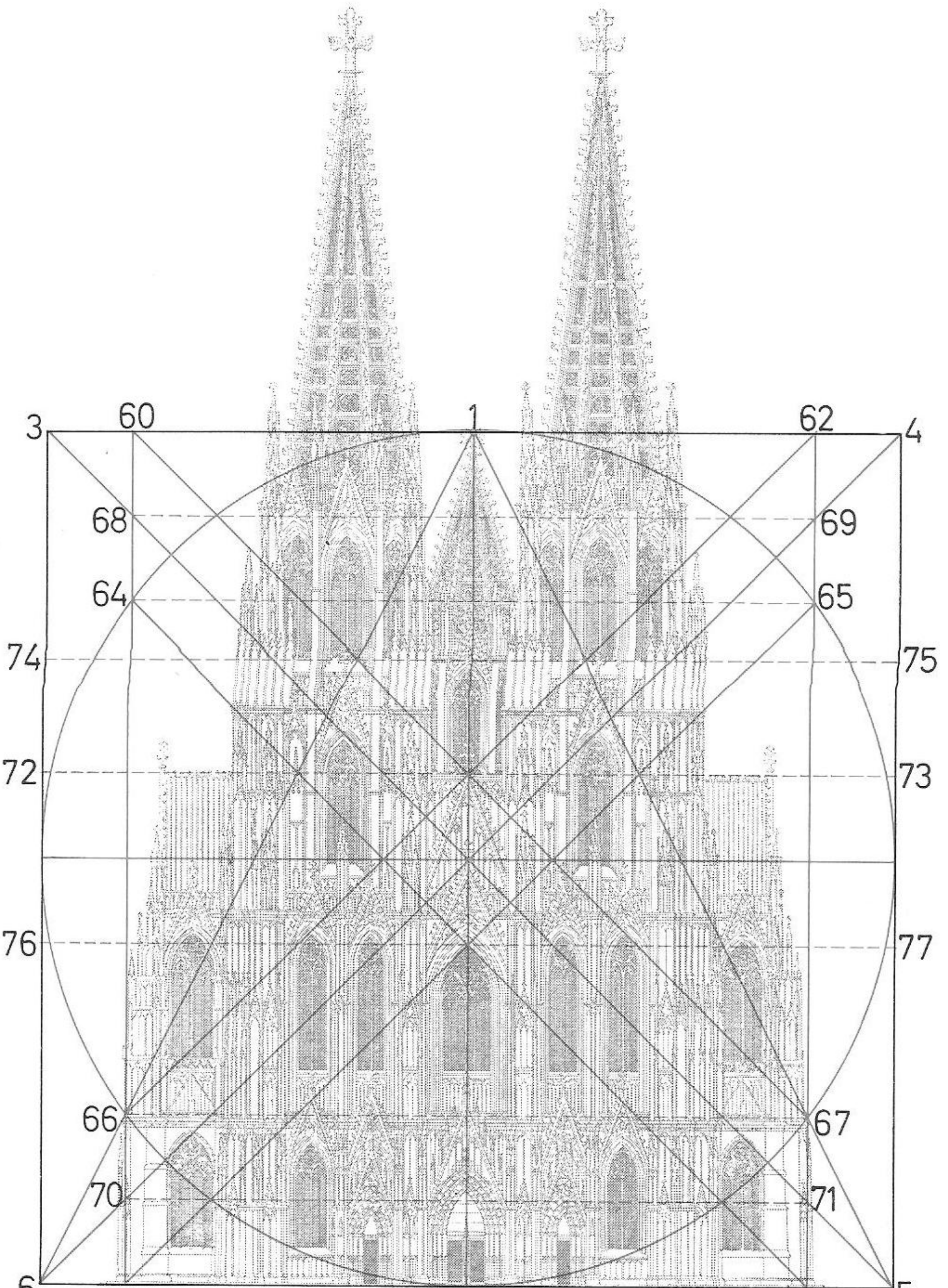


Fig. 230.

centre of the diagram to form a small quartered square, turned through 90° in relation to the basic square.

The upper corner of the small square is flush with line 72-73.

The latter marks first of all the height of the transept's ridge, a vital dimension. It also indicates the bottom of the window immediately under the middle spire.

The next new horizontal lies under the diagram's axis and is flush with the lower corner of the small central square, line 76-77. It marks the tipped arch of the largest window in the facade, placed on the first floor immediately above the main door. Line 76-77 also marks the tip of the two slim spires at the extreme right and left of the building.

Our next line is 66-67. It has a most obvious placing: running through the heavy frieze that stretches from side to side of the facade and which seems to "crown" the ground floor.

Our final new line, 70-71, also has a clear position in the facade, marking as it does a frieze that runs across the face of the cathedral and is repeated even between the entrance doorways.

All things considered, we have assembled a mass of precious information on the structure of the facade of Cologne Cathedral. Obviously the more complicated a facade is intended to be, the more symbols one requires. And the more symbols placed one on top of the other, the more difficult becomes an analysis.

Cologne Cathedral is surely one of the most inaccessible buildings to analyse on account of its maze of ornamentation. A complete book would be needed to do this particular job thoroughly. But I think the three analyses just completed have traced the *main* lines of structure of the facade, although naturally a myriad of other details would be required in order to complete the plan of the front elevation.

We see in *Fig. 231* as a form of résumé yet another picture of the cathedral's "face". Here we have all the horizontal and vertical lines discovered in our three analyses. We can see that, starting with any size of basic square, we know sufficient about the geometric structure of Cologne Cathedral to map out the whole front of the building.

The Ground-plan

After the analysis of the facade, we can go on to search for the basic unit of planning used for the cathedral's ground-plan. The facade, we saw, was designed mainly on the basis of seven concentric squares, each one double the area of the preceding.

We discovered that square no. 3 from the outside (see Fig. 229) was in fact the basic square, and from this we produced the other six.

The analyses on antique temples illustrated a definite geometric and numerical link between facade and ground-plan. We must look for such a link in the layout of Cologne Cathedral.

One of the first points that will register with the observer is that, contrary to earlier temple analyses, the ground-plan here occupies less area than the facade, the two long spires of which, if placed upon a ground-plan diagram, extend far beyond the church's length. We cannot therefore follow our usual pro-

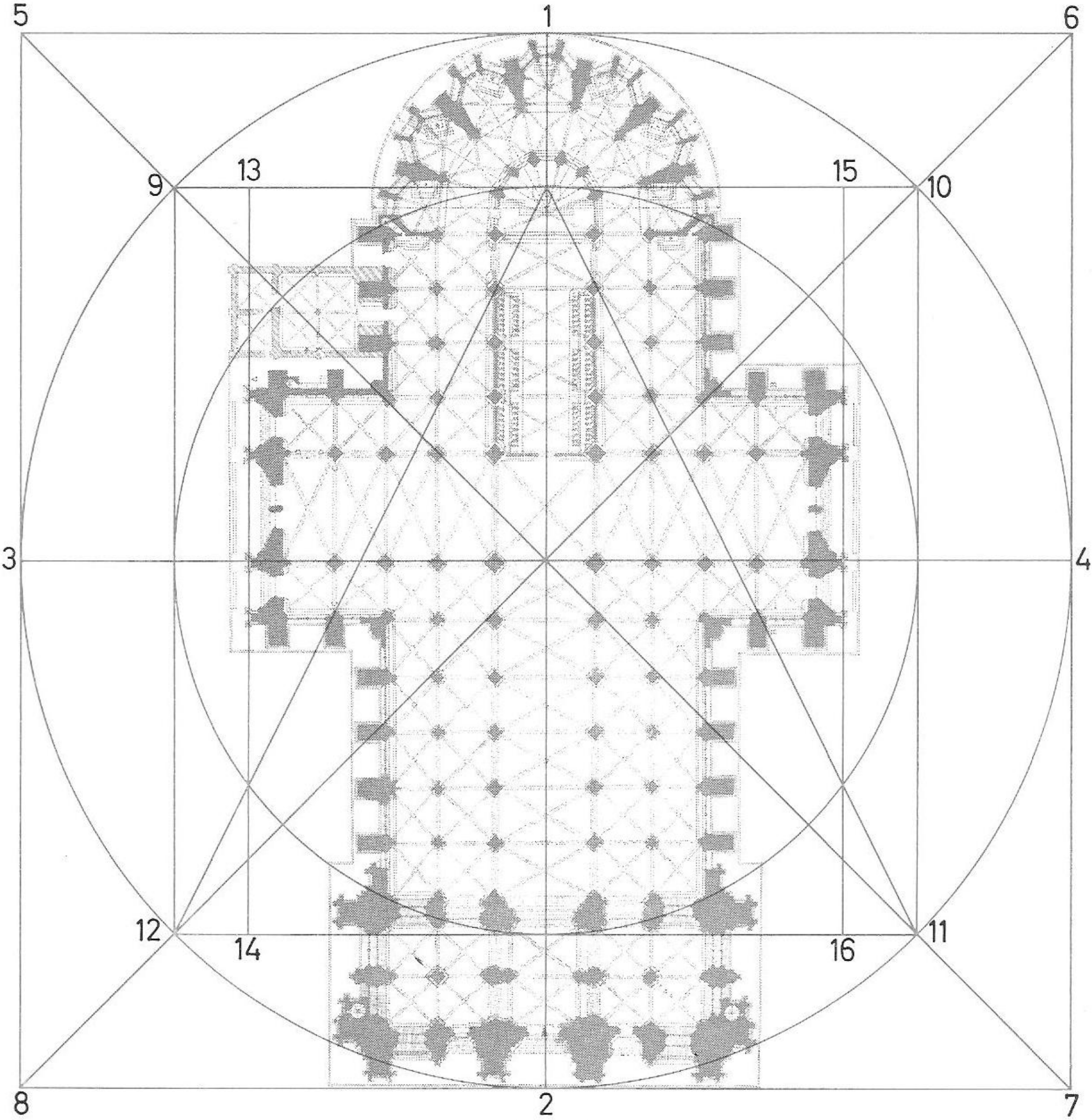


Fig. 232.

cedure of multiplying a particular square from the facade a certain number of times in order to obtain the ground-plan's basic square.

If we compare the ground-plan however with Fig. 229, we see that its length matches the dimensions of the second-largest square in that diagram, square 39-40-41-42.

This square is in area twice the size of the facade's basic square. Now we recognise the link between facade and

ground-plan. Instead as often the case previously of arithmetically multiplying the facade square 2 × 2 or 3 × 3 (providing a square four or nine times larger respectively), the planners of Cologne Cathedral took the facade's basic square and doubled it geometrically, the resulting square being used to lay out the ground-plan.

From a purely technical point of view it was not feasible to portray the facade and ground-plan in the same proportion when reproducing them for this book. But I can assure the reader that in my original analytical material the geometric link described above fits perfectly. We may thus establish, before moving on to the analysis proper, that the basic square of the ground-plan is identical in size to square 39-40-41-42 in the facade analysis.

In *Fig. 232* the longitudinal (vertical) axis is line 1-2. The horizontal axis is 3-4, and we add the circle in the normal way, completing its external square, 5-6-7-8, which is the ground-plan's basic square.

Entering both diagonals, we then construct the basic square's half-size version, 9-10-11-12, which is naturally the same size as the basic square in the facade. In Fig. 230 we discovered the total width of the cathedral by entering the circle's rectangle in this square, and we repeat the process here. The rectangle is 13-14-15-16.

The vertical sides of square 9-10-11-12 have no contact with the cathedral and are apparently merely guide-lines. The upper horizontal 9-10, too, seems to be without any particular placing in the cathedral, apart from marking the back of the altar.

At the other end of the building the square's horizontal side 11-12 marks the division between the cathedral's porch and the actual hall of worship, the line following the outside of the common wall.

The outer square 5-6-7-8 marks the total length of the building, being, of course, the basic square. But we ought to remember that the square was not decided according to the length of the cathedral. The length was discovered to fit perfectly into a square "borrowed" from the facade analysis, thus providing the link between the two.

The horizontal axis of the large square, line 3-4, is clearly placed in the diagram: it marks the position of eight columns situated transversely in the nave and transepts.

We pass on from that diagram to *Fig. 233* where we again have as our basic square 5-6-7-8. We enter the two acute-angled triangles 5-2-6 and 8-1-7, and the two diagonals.

These lines permit us to divide the basic square 3 × 3, shown vertically by 17-18 and 19-20, and horizontally by 21-22 and 23-24. The intersections of these lines are marked A, B, C and D.

We have thus executed on this ground-plan from the Middle Ages the same arrangement as proved to be one of the main themes in antique temples, and we note immediately that—just as with the temples—it plays an integral role in planning.

The vertical lines 17-18 and 19-20 mark the width of the nave + aisles to the outside of the wall but within the outside pillars. The cathedral is ringed by a large number of heavy pillars, connected with stone and brickwork, giving the effect from the outside of a number of niches. The above lines mark the inside of these niches, while in Fig. 228, we saw the outer edge of the niches indicated by lines 27-28 and 29-30. We are able to compare the two diagrams because of course it is in reality the same basic square with which we are working.

The upper of the horizontal lines of 3-part division, line 21-22, has the same

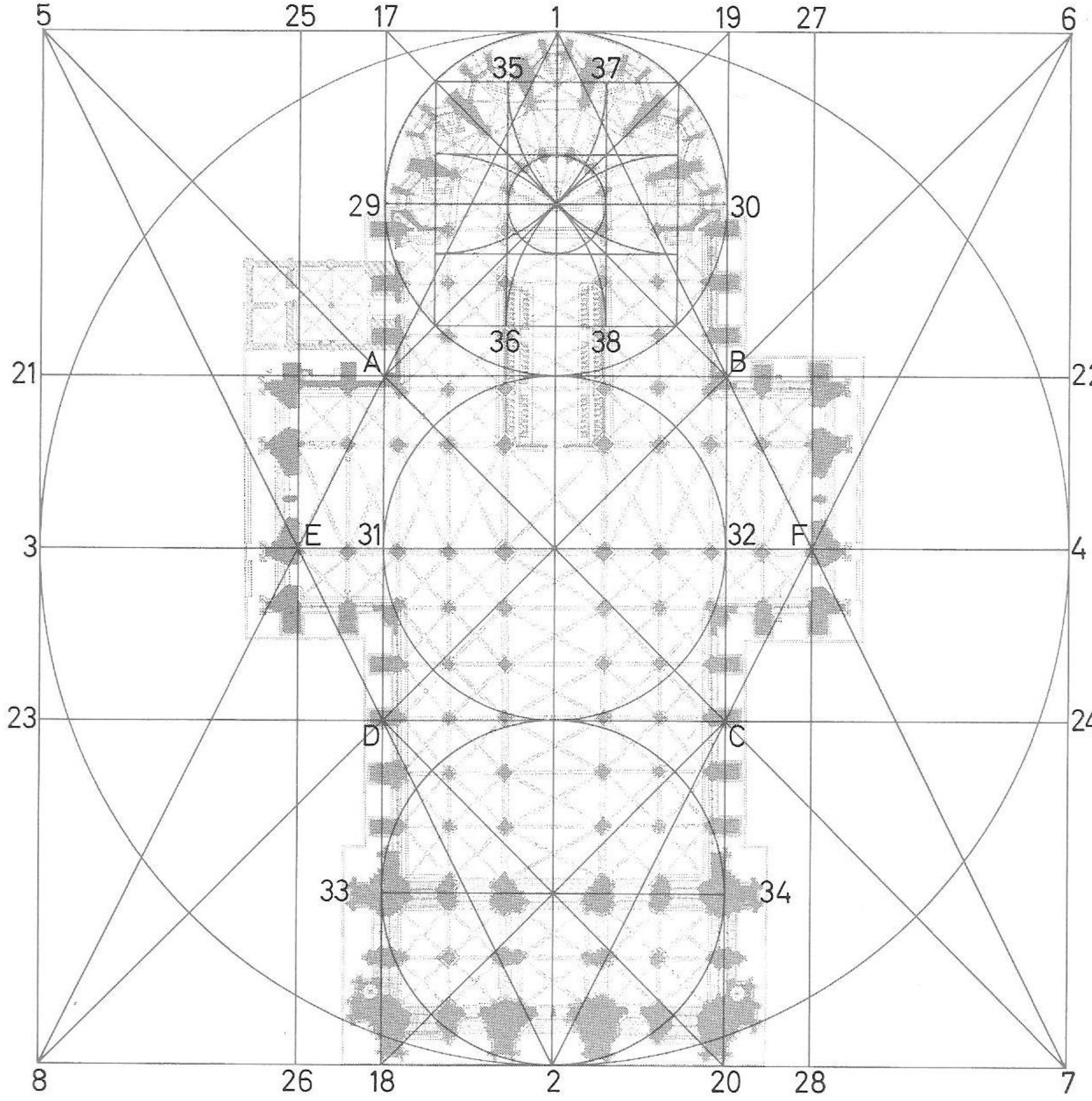


Fig. 233.

function as the two verticals, showing the same as that found in several samples of transepts.

When we examine the overall proportions of the ground-plan we discover another aspect of 3-part division. The cathedral's length:width ratio of 3:1 is the same outside niches on the sides of the classical temples. This cathedral, belonging as it does to an entirely different period, another style, has the addition of a transept but the main body of the

building comprises three simple adjacent squares.

The upper of these three squares, 17-19-B-A, when its circle is inscribed within it, indicates exactly the sweep of the semi-circular apse. The horizontal axis of the small square, line 29-30, bears the centre-point of the apse, immediately in front of the altar.

The semi-circular apse is also built of heavy pillars and brickwork, interspaced with windows. The square's diagonals A-19 and B-17 each cut through a pillar.

We can see from the direction they face that the pillars point to a common centre, and we can also see that their placing was determined by the diagonals. We may also recognise a 12-part division process as shown in Fig. 67 in which the problem of dividing up the circle's circumference was treated.

If we enter within the small circle the half-size version of square 17-19-B-A, and inside this new square execute the sacred cut (with two horizontals and two verticals as seen many times before), the latter forms a very small square in the centre which contains exactly the circular group of pillars that surround the altar. The vertical sacred cuts, 35-36 and 37-38, mark the width of the nave, cutting through both rows of columns that run the full length of the cathedral. These two lines actually end in the two massive pillars that flank the main entrance to the cathedral.

While our attention is directed at this end of the building, let us examine square D-C-20-18, which encloses part of the main church and the entrance to the porch.

The square's horizontal axis is 33-34, and we see how this line clearly indicates the wall between the porch and the church proper. This wall contains the entrance to the main church. As the same axis in the corresponding square at the opposite end of the building lands directly in front of the altar, it is not inconceivable that this symbolism (the middle of one square: entrance; the middle of another square: altar) was part of a wider plan in which the architects of the Middle Ages were experts.

There are three circles and three axes, and the mind is inevitably brought to bear on the Christian trinity. From the doorway (circle no. 1), through the church (circle no. 2), to the altar and kingdom of God (circle no. 3).

We add another two lines to our diagram. The acute-angled triangles at their intersections with the diagonals mark, of course, the square's 3-part division. But their mutual intersections also indicate the 4×4 division.

When we enter the vertical lines of division, 25-26 and 27-28 we see how they indicate precisely the internal length of the transept.

As we saw in the preceding analysis of the ground-plan the external length of the transept, we thus have an indication of the columnar thickness.

What in effect have we derived from the present analysis? The length of the cathedral is divided into six units by the horizontal axes of the three squares, $\frac{1}{6}$ is used as the porch, $\frac{4}{6}$ as the main body of the church, and $\frac{1}{6}$ as the rounded apse at the east end.

Lengthwise the cathedral is split by the basic square's 4×4 division: the two middle portions form the internal length of the transept, while the two outer parts fix the position and thickness of the outer walling and columns of the transept.

We are not quite finished with the 4×4 division of the basic square, although we shall not work further on the present diagram. For reasons of clarity we move to *Fig. 234* which illustrates various ways of utilising the 4-part division.

Fig. 234 A shows the normal cutting of

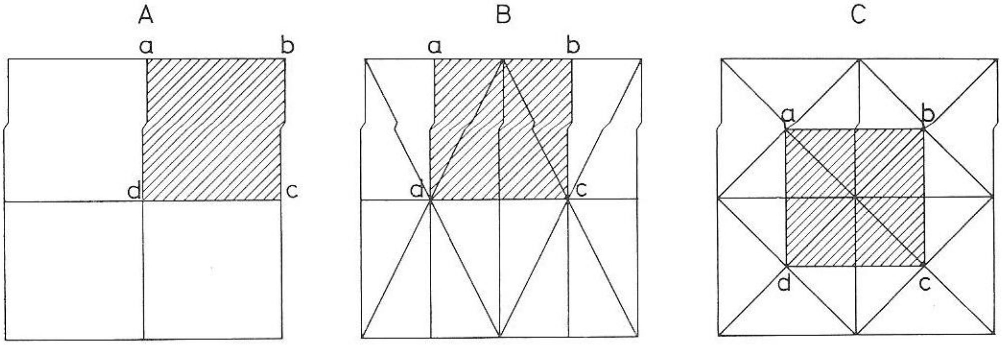


Fig. 234.

the square in four pieces by means of the vertical cross. The result is four equal squares, one shown as *abcd*.

Fig. 234 B shows the same division as applied in the ground-plan of Cologne Cathedral where, instead of placing $\frac{1}{4}$ of the square either to the right or left of the vertical axis, the $\frac{1}{4}$-size is placed top centre, with half a square on either flank.

Thus square *abcd* in diagram B is identical in area to the square of the same name in diagram A. The division is simple enough to appreciate but differs sufficiently to warrant individual examination.

Fig. 234 C illustrates another variation, with $\frac{1}{4}$ of the basic square lying in the centre of the diagram.

Division A splits the basic square into four complete squares, each in area $\frac{1}{4}$ of the main square.

Division B provides two complete $\frac{1}{4}$ squares straddling the vertical axis, flanked on either side by parts of the square which together amount to half the area of the basic square.

Division C allows only one complete $\frac{1}{4}$ square, the remaining parts of course totalling $\frac{3}{4}$ of the area of the basic square.

Division B is, as stated, the one applied to Cologne Cathedral, and we see it again in *Fig. 235* where we see the upper of the two squares at 25-27-F-E.

The two squares share a common boundary, i.e. side F-E. This line is part of the horizontal axis and, as we saw in earlier analyses, marks one of the rows of columns in the transept.

If we take the lower square E-F-28-26 and construct its double-size version 47-48-49-50, we see how line 47-48 indicates the second row of pillars in the transept.

We see therefore in the lines of these two squares how the distance between the two rows of columns was determined.

At the same time we may record that square 47-48-49-50 is the same area as the basic square in the facade, since each is half the area of the basic square in the ground-plan.

Having doubled E-F-28-26 in area, we now take the opposite course and inscribe the square's circle and half-size version, 39-40-41-42.

We see how the vertical sides of this square run very near to flush with the outside of the nave, and the two lower corners of the square rest almost in the centre of the two rosettes in each of the two heavy corner columns. Line 41-42 indicates the inside of the thick pillars by the entrance, and as line 26-28 marks the outer dimension of the pillars, the new square enables us to record the thickness of these supporting columns.

Yet another half-size square, 43-44-45-

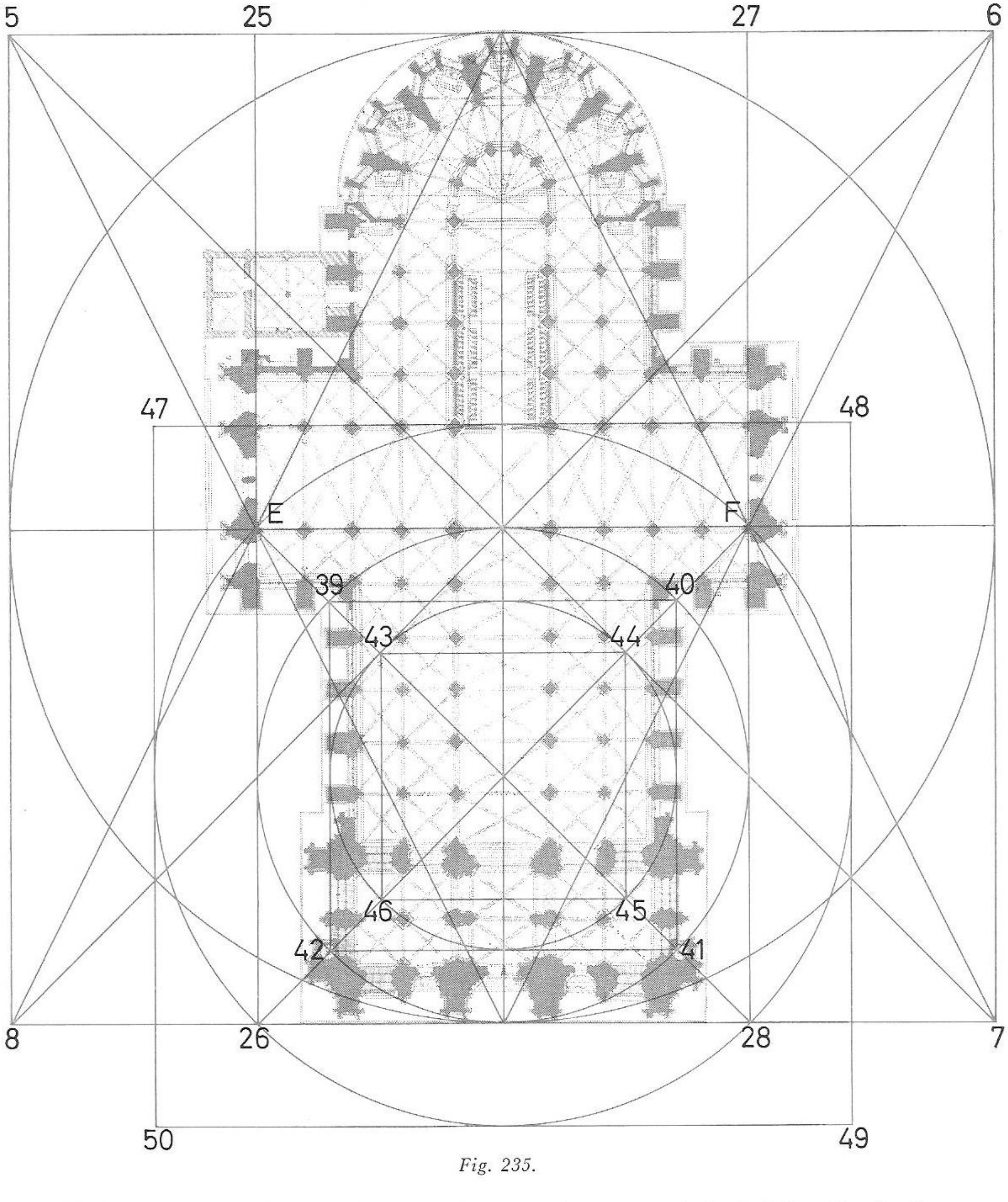


Fig. 235.

46, has been entered in the diagram. It appears at this stage to have no bearing on the plan of the cathedral since its lines touch no part of the building. But in the next part of the analysis, in which the sacred cut plays an important role, square

43-44-45-46 forms the basis of this construction and has therefore been entered in the present diagram to illustrate the natural conclusion of halving the area of the square.

We move now to our analysis of *Fig. 236* which has the same proportions as and similar construction to the preceding diagram. The ¼-size square E-F-28-26 was doubled and then halved and quartered in area.

In this analysis it is the sacred cut in these squares that we shall study. To simplify our examination the squares have been designated B (the ground-plan's basic square), C (its half-size, 47-48-49-50), D, E and F. Square A is taken to be the (largest) outside square in the facade plan.

We begin with the sacred cut in B, its vertical lines being 63-64 and 65-66. These two lines mark the total width of the nave, the widest point of which is the two great pillars at each end of the porch. In their course through the ground-plan the lines mark the position of the four external wall supports in the transept.

The sacred cut in square C is shown by lines 59-60 and 61-62, which mark the internal width of the nave and aisles. The lines have been produced to points 59 a and 61 a to illustrate their value at that end of the cathedral.

The sacred cut in square D is represented vertically by lines 55-56 and 57-58, which position the two outer rows of pillars running the full length of the nave and ending at the altar. The lines have been produced in the diagram to points 55 a and 57 a.

The sacred cut has not been entered in square E since it has apparently no purpose in the ground-plan. The square's vertical sides were discussed in connection with Fig. 235 where we saw that they marked the outer edge of the nave's supporting wall pillars.

Side 39-40 shows where the transept cuts across the nave. The square is thus firmly a part of the cathedral plan, but not by virtue of its sacred cut. In addition, it forms the intermediate link with square F.

It was the latter square that we saw in Fig. 235 as having no apparent part in the lay-out of the cathedral. We now see that its vertical sacred cut clearly indicates the two rows of pillars that march up through the centre of the nave from entrance to altar. The lines are 51-52 and 53-54 and end at the altar as 51 a and 53 a.

If we again construct, in our mind's eye, the half-size version of square F and take the vertical sacred cut of the new square upwards through the cathedral, we discover that it marks the net width of the central aisle—since we observe that before reaching the altar the aisle narrows slightly to allow for the choir.

Counting this latter square (which could be called G) and the large square (A) that surrounds the whole diagram and which we saw in the analysis of the facade, we have again our seven concentric squares, i.e. the same geometric diagram as applied to the planning of the facade. And it is the verticals, horizontals and sacred cuts of these squares that determine the ground-plan lay-out of Cologne Cathedral.

The presence and use of 3-part division also underlines the strong traditional link between Middle Ages and Classical times. It was applied in nearly every ancient temple, for example.

If we continued in this vein, it is unlikely that we could ever complete in each fine detail the analysis of Cologne Cathedral. The towering spires and lacy Gothic stonework contain thousands of applications of ancient geometric principles and diagrams. It is a question merely of time, patience and (in our instance) space to

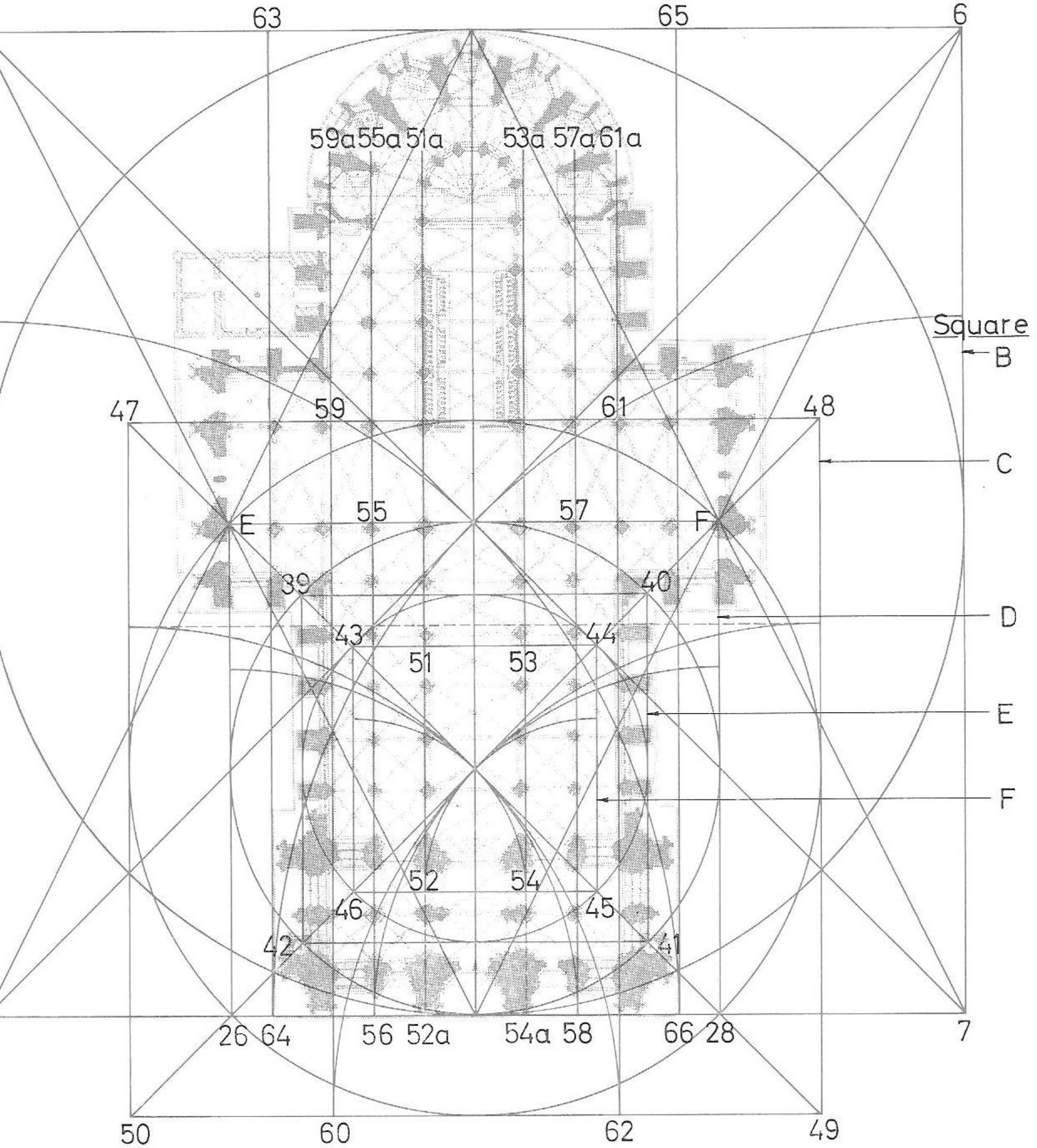


Fig. 236.

draw back still more of the mystic veil that conceals the ingenious and traditional designs.

One example is the 12-part division of the circle used to lay out the rounded apse at the east (top) end of the build-

ing. The same division was applied to the half circle that surrounds the altar.

But the main aim of our search has (I trust, convincingly) been fulfilled: to illustrate that the design of Cologne Cathedral was built up according to geometric principles that were already hoary with age and use hundreds of years before the cathedral was built.

The analysis has picked out in the main the dimensions and proportions that give the cathedral its distinctive appearance and character. It pointed out the framework round which the details were built, and it brought clearly to notice the definite link between the design of the facade and that of the ground-plan, familiar from previous analyses.

The Pantheon

Walk through the older quarter of Rome, capital of the ancient world, and you are bound sooner or later to come face to face with one of the most impressive buildings from antiquity: the Pantheon.

The building rests like a giant animal amid a conglomeration of tightly packed houses and apartments, which seem loath to yield room for the great structure. From floor to its beautiful, domed ceiling the Pantheon measures almost 45 m. (approx. 140 ft), and from side to side almost 56 m.

The main entrance to the building faces a small *plaza,* and from this opening in the dense housing mass one can stand back and see the grand approach and entrance to the Pantheon in its entirety.

The complete frontage, as it stands today, is quite obviously inspired by early Greek temple structures, having eight frontal pillars and richly decorated capitals.

Above the columns the sloping roof rises at a somewhat steeper angle than one is accustomed to seeing in a Greek temple, but the style is the same.

But comparison with a Greek temple goes no further than the door of the Pantheon. The frontage appears almost to be stuck on as an afterthought to a rounded temple that has an all-enclosing wall instead of a series of columns. We would look in vain therefore for the typical cloister effect found in Greek buildings.

As in the case of most monumental structures from early Christian times, the Pantheon has had a dappled history of reconstruction and no longer bears its original appearance.

In *Fig. 237* we have a picture of the Pantheon, showing its present face.

The name Pantheon denotes that it was a temple dedicated to all gods in the community, and history says it was built originally by Agrippa in 27 B.C. as a ten-column temple, but nothing remains now of the original structure apart from one or two pieces of foundation. These show that the original building was 3.7 m. lower at its base than the present version.

The old, original temple burned to the ground about 81 A.D., and Domilian built a completely new style of temple in the ruins of the old.

The apparent reason for the new building being placed so much higher than the older seems to be that the burned-out shell

Fig. 237.

of the former temple was simply flattened, rolled and used as the foundation.

The new building, too, has since disappeared almost without trace. Historical records indicate that the building had a large circular courtyard at its centre. It would appear to have covered a larger area than the present Pantheon.

Its life was short. In 110 A.D. the building, like its predecessor, was razed by fire. The job of planning and rebuilding a new temple was given to Hadrian.

Hadrian was not content to produce a carbon copy of the previous temple, but planned a new structure from foundation to roof, taking as his starting point the dimensions of the old inner courtyard. This he made into a circular temple which he topped with a fine domed roof, the ceiling of which he decorated with a peculiar stucco design.

The ornamental ceiling does not however bear the same design as the temple capitals or later decorative panels. Its lines and circle look more like a geometric diagram than an architectural decoration.

A large hole was left in the centre of the domed ceiling, and through this the light of day streams into the church producing a most unusual shadow effect on the plaster ceiling. We see this in *Fig. 238.*

Hadrian's reason for erecting a temple which in height exceeded its predecessors but which occupied less area of ground was perhaps that Rome even then was densely built up, and crowded so close to the site of the Pantheon that he was forced to restrict the ground area in order to create a sense of space.

There is little doubt that the new structure is smaller than the old in ground area. In the previous building the round courtyard was a part of the building, but only part; the same area is not a part of

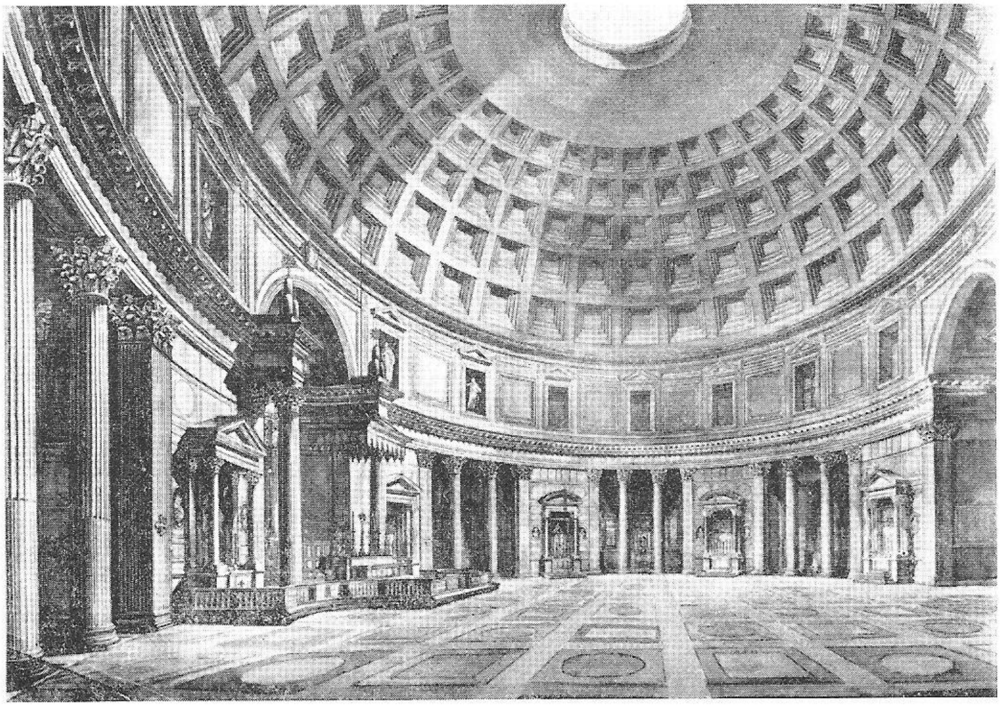


Fig. 238.

the new building—it *is* the new building.

Historians are not too sure who built the present entrance. It may have been Hadrian, but may possibly have been a later builder, Antonius Pius. Or even a third party.

But the builder of the entrance is not of great interest to a geometric study such as ours, since the original facade probably exists no longer. As late as 1606, for example, Pope Barberini had the Pantheon's bronze porch beams removed and melted down to cannon. The building suffered another "face-lift" in 1747 when, among other things, the windows were altered in size.

In spite of these attacks on its originality, the church has retained a composed beauty. It strikes one immediately and inevitably. It is an impressive experience passing from the darkened hall through a narrow opening in the beaten-copper door into the dome-shaped church hall. Light pours down from the gaping roof, reflecting on the stucco ceiling, and providing considerably more natural lighting than usual for a Catholic church.

The floor is laid in variously shaded marble, shown at its best in the strong daylight. The design resembles a giant chess-board, the squares in turn being laid in mosaic form.

There are two motifs: a square with a circle inscribed, and a square with a smaller square inside. The patterns alternate as do the black and white squares on the chess-board (see Fig. 238). Already one senses the presence of ancient symbolism and geometry.

In our study of the Pantheon through the eyes of the ancient geometer, we must ignore the two previous buildings that stood on this site. Of them there is virtually no trace. We may consider the pre-

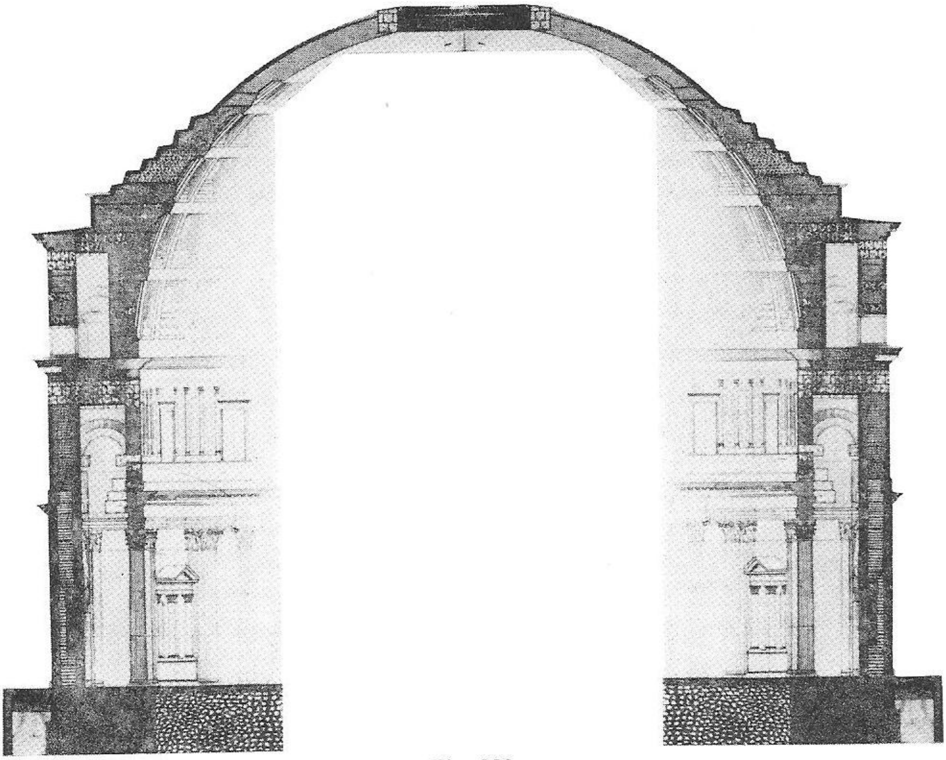


Fig. 239.

sent building to be a brand-new erection by Hadrian around 117—138 A.D.

Hadrian, whatever else he may have been, was an ingenious planner and designer. In addition to a wonderful gift of unfettered imagination, he had at his fingertips "the royal art" of geometry. We ought thus to expect to find this monument to his expertise laid out in accordance with ancient geometric principles.

The rather severe alterations to which the building was subjected in 1606 and later in 1747 leave some doubt about the original design and dimensions of the entrance and windows. These will therefore be omitted from the survey, and we shall stick as close as possible to the building thought attributable to Hadrian alone.

It was a rather difficult task tracing suitable material for an analysis, since most available drawings and plans are in freehand with no indication of source or authority, and cannot therefore be used in a critical geometric study.

However, I came across the best material on site, i.e. in the front hall of the Pantheon in a booklet describing the building.

Called quite simply *The Pantheon,* the booklet is the work of Roberto Vichi. Its illustrations are apparently based on accurate measurement of the building, and the sources are quoted of both drawings and photographs.

One sectional drawing shows the interior of the building and the structure of the outer walls. It also shows the passages that lie between the outer wall and the temple hall.

The drawing gives only one (the left) side of the building, as far as the vertical axis. But as we require the whole cross-

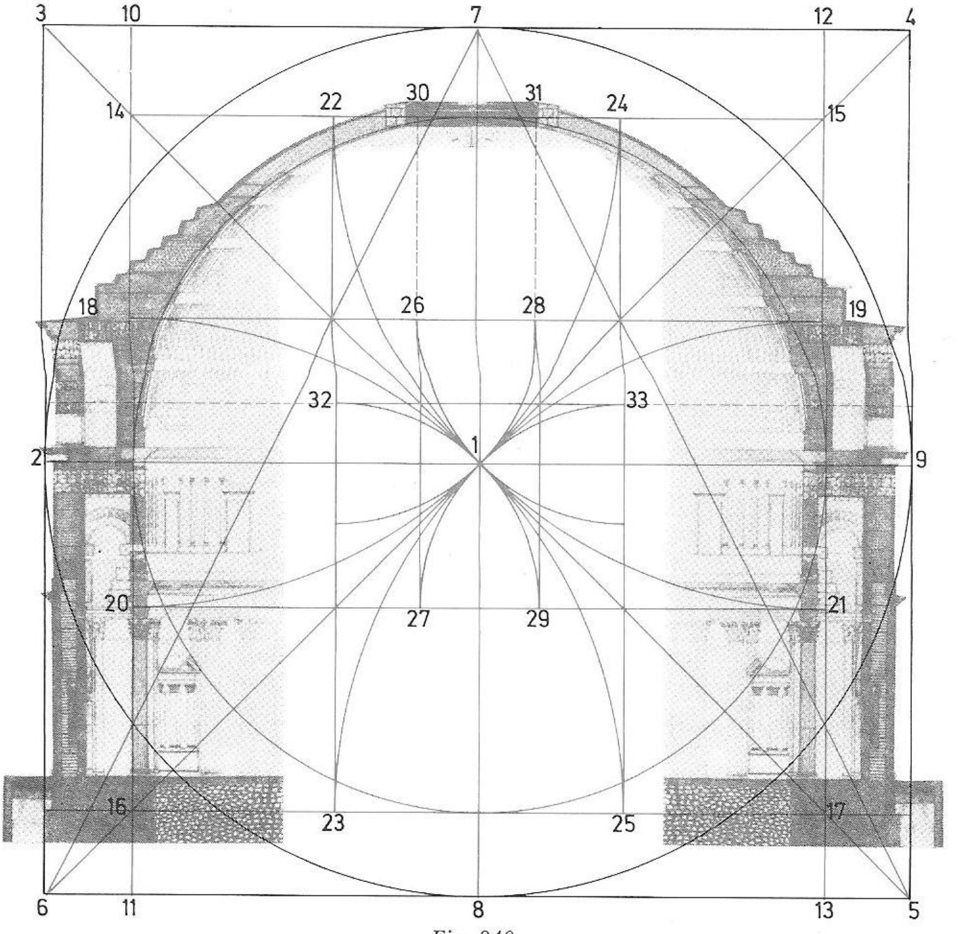


Fig. 240.

section, the illustration was photographed and repeated on the right of the axis in order to produce a complete picture of a slice of the Pantheon. Apart from the double width, the illustration in the present book, *Fig. 239* is almost identical in size to the original.

As usual, the first step is to find the constructive basis of the building's plan. As with most of the other buildings we have examined, there are a number of possibilities in the Pantheon.

Its width exceeds its height, and the natural (and correct) assumption is that the basic square's vertical sides run flush with the building's outer wall. We saw this applied to samples of Grecian temples.

In *Fig. 240* the two verticals were entered flush with the walls, and the problem then remained to find the Pantheon's horizontal axis. I found this to be at line 2-9, where it is marked both outside and inside the building by a frieze. The inside frieze marks the level at which the inner hall wall changes from the vertical to the curved dome.

Having fixed the horizontal axis, we can go ahead and construct our basic square: 3-4-5-6.

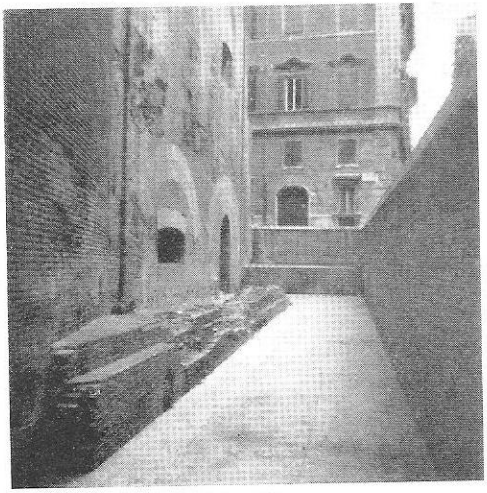


Fig. 241.

Fig. 242.

We notice immediately that neither the upper side nor the base of this square has any contact with the building in our diagram. As regards the upper horizontal (3-4), we are accustomed to seeing it placed well above the roof of the building or temple under discussion. But the square's base-line usually has its place at the foot or base of the building.

This is in fact also true of the Pantheon. But whereas we normally analyse a *facade* drawing or photograph of a particular building, with the Pantheon we have a *sectional* drawing, a slice down through the middle. And therein lies the explanation.

Facade plans normally show the outside steps leading up to the actual temple entrance, but these steps do not show in a sectional drawing.

We may therefore assume that base-line 5-6 marks the bottom of these steps that lead up to the temple.

But there are no steps leading up to the front of the Pantheon! We saw this clearly in the photograph in *Fig. 237* taken from the square in front of the building.

This apparent absence of approach steps is caused by the fact that the square has been raised and surfaced so often in the past that it is now level with the floor of the Pantheon's porch. A further inspection reveals however that there were steps round the building at one time.

The Pantheon is built in a fairly undulating area of the city, there are steeply sloped streets all around. The street, for example, running behind the Pantheon is about 10 m. higher than the church's base.

To protect the building against encroachment from surrounding houses, the authorities built a wall. Outside it lies bustling Rome.

In *Fig. 241* we see a view af the right-hand side of the Pantheon towards the rear of the building, illustrating clearly the "moat" between Pantheon and the adjacent neighbourhood.

Fig. 242 is a photograph of the building's left side, showing a similar variation in ground level.

In *Fig. 243* the camera has caught a view along the right side of the building towards the front, and we see here that recent authorities have placed steps from ground level up to the temple's porch—proving that the base of the building lies

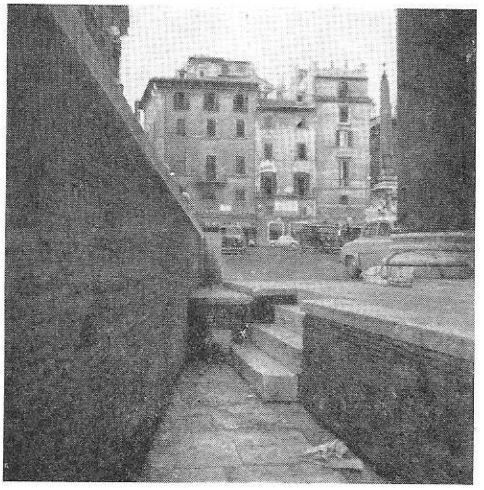


Fig. 243.

in effect much lower than the level of the front *plaza.*

This is evidence that our base-line 5-6 in Fig. 240 is properly positioned, and proof, too, that a geometric analysis can uncover or rediscover factors that only an intimate search on site can establish. Factors which may be totally absent from the drawing we analyse.

We now add to Fig. 240 the acute-angled triangle 6-7-5 in order to mark off the circle's rectangle. The latter is seen as 10-11-12-13.

The basic square's proportions were selected so as to run flush with the extreme outside of the building, i.e. the projecting sills that encircle the Pantheon in the form of a frieze.

Now that we have entered the circle's rectangle we can establish another part of the plan: the rectangle's vertical sides form the basis of the positioning of the ring of pillars that run around the outside edge of the main hall.

If concrete indication were required that the Pantheon was planned according to the principles of ancient geometric diagrams, the positioning of these pillars certainly whets the appetite. Hadrian was obviously a planner of the ancient geometric school.

Our survey continues with the entry of the square on the circle's rectangle, which we place centrally in the diagram. Even more lucidly we see the building's plan open before our eyes.

The square is marked by lines 14-15 and 16-17. The former, we see, indicates the internal height of the main hall, passing along the uppermost curve of the dome.

When we inscribe a circle in square 14-15-16-17 we see that it follows exactly the sweep of the dome and takes in part of a colonnade halfway up the inside of the hall.

Other drawings of the Pantheon exist, showing a circle drawn in a similar position, but that circle lies entirely within the hall and does not touch the surrounding columns. But if the arc of a circle is to follow precisely the curve of the dome, it must be drawn as indicated here. A circle with either a shorter radius or a different centre will not match the curve of the ceiling exactly.

The other drawings are not in detail, and give the distinct impression that the hall has been sketched round a circle rather than a circle placed within the building.

Later analysis will prove that the circle I have arrived at is more likely than any other to be correct, since it marks a number of factors both in the sectional view of the building and in its ground-plan.

We enter the sacred cut in the square on the circle's rectangle. The respective lines are 18-19, 20-21, 22-23 and 24-25.

Line 18-19 plays an important part in the building's structure, representing the height of the outside vertical walling. This is the level from which the domed roof starts.

The lower horizontal sacred cut, 20-21, is also marked both inside and outside the

building. Inside, we see it as the height of the niches placed between the columns bordering the main hall. The actual marking is represented by a frieze and can perhaps best be seen in Fig. 238.

The external marking is the edge of the sill that encircles the building.

Fig. 238 also illustrates clearly the stucco ceiling in the main hall. The design is composed of five concentric rings of geometric figures, the rings reducing in size towards the centre. The smallest of the five stops short some distance from the hole in the centre of the roof.

As near as I can ascertain, the point at which the fifth and inner ring is met by a smooth area of plaster is marked by the vertical sacred cut, lines 22-23 and 24-25. The junction is indicated at the intersection of the curved dome and the two sacred cut lines.

The combination of the sacred cut in square 14-15-16-17 creates another smaller square. If we again enter the sacred cut in this latter square, we find that the two vertical lines, produced upwards to the ceiling, mark the diameter of the large "sky-light" in the centre of the roof.

Line 27-26 is produced to point 30, and 29-28 is produced to point 31 showing the diameter.

The upper sacred cut (horizontally) is 32-33 and indicates the top of the upper row of windows in the building.

The analysis of Fig. 240 has provided us with details of a number of important features in the Pantheon. Most of the diagram's lines were used by the architect to place some factor or other in the building. We may thus assume that our choice of basic square was correct.

Our next diagram, *Fig. 244,* is constructed in the same manner as Fig. 240. The basic square is 3-4-5-6, in which we inscribe the circle and acute-angled triangle 6-7-5, following this with the circle's rectangle 10-11-12-13. The square on the latter is again 14-15-16-17. Thus far the diagram is identical with Fig. 240.

The next step is to construct the basic square's half-size version in the centre of the diagram. This is done by joining the intersections of the basic square's inscribed circle and diagonals. The required square is 34-35-36-37.

We execute the sacred cut in this square and observe first the placing of the upper horizontal cut. It is produced across the building as line 38-39.

In the previous analytical diagram we found that line 18-19 marked the upper edge of the frieze or sill that tops the outside walling. Here we find that 38-39 indicates the underside of the same sill. The thickness of this projection is thus determined by the distance between the two upper horizontal sacred cuts in squares 14-17 and 34-36 respectively.

The lower horizontal sacred cut is also produced across the building as 40-41. It indicates the top of a lower projecting frieze. The underside of the same frieze was marked in the previous analytical diagram by line 20-21, i.e. the sacred cut in square 14-17. Thus the thickness of this sill, too, is determined by the distance between the two (lower) horizontal sacred cuts in squares 14-17 and 34-36.

The base of the latter square, line 36-37, forms the floor of the main hall.

The vertical sides of the half-size square, lines 34-37 and 35-36, appear to have been the determining factors in fixing the net width of the main hall just as the sides of the basic square marked the gross width of the whole building. We notice that the first-mentioned set of lines touch the projecting sills of the inside columns in the same way as lines 3-6 and 4-5 run flush with the friezes of the outer columns.

In the horizontal lines of the sacred cut in the inner square, lines 38-39 and 40-41, we can find the relationship with the ba-

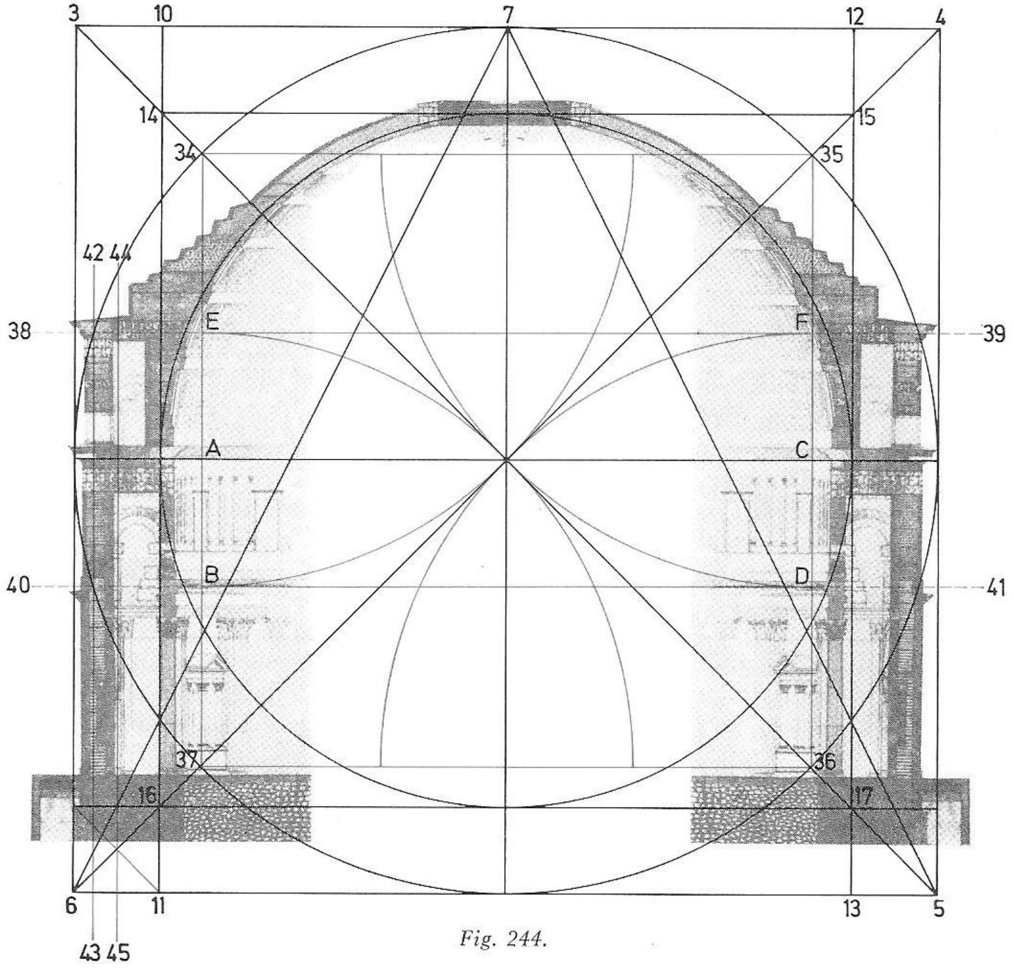


Fig. 244.

sic square's inside circle that decided the dimensions of the temple's outer walls. And the factor that determined the width of the upper "shelf" or sill from which the domed roof rises.

The sides of the basic square coincide with the extreme tip of this ledge and, as we have seen, indicate the temple's total width.

To decide on a thickness for the outer wall Hadrian then drew a vertical line through the intersection of sacred cut 38-39 in the half-size square and the basic square's inside circle. The vertical is seen as line 42-43 and marks the outside of the main wall.

That determines the outside of the wall, but how did he fix the inside? He took the area between the basic square and the circle's rectangle (area 3-10-11-6) and split it in two pieces down the centre. The dividing line is seen as 44-45. The actual process of division was carried out in square 16-11-6, by means of the simple diagonal cross.

Thus we have the five outer vertical lines of the Pantheon: line 3-6 is the temple's total width; line 42-43 the outside

of the wall; 44-45 is the inside of the wall; 14-16 the outside dimensions of the colonnade in the main hall; and 34-37 marks the effective width of the main hall.

This is a typical example of geometric diagrams being applied as a form of static curve, in the same way as a present-day engineer uses graphs to make rapid calculations of height/weight/strength ratios, etc.

The ratio within the diagram remains constant, of course, irrespective of the size of the finished building. The deciding factor is the length of the basic square's baseline. The smaller the basic square, the slimmer the wall; the larger the square, the thicker the wall will be.

The ability to appreciate these ratios depended to a great extent on the experience and wisdom of the master builder, and it was also a matter of routine in applying these experiences. But I have no doubt at all that builders in ancient times followed certain rules and ratios laid down by ancient geometry when they had to determine bearing thickness, strength, proportions, etc., in a building. It was a matter of knowing the properties of the basic square.

It was naturally a stage that he and his predecessors had approached very slowly over the centuries.

Whereas in the dawn of geometric thinking and draughtsmanship the actual lines and diagrams were a sacred subject, revealing wisdom of occult geometry and numbers, it gradually over thousands of years became a subject applied to more practical spheres—such as the building site.

For thousands of years Temple brethren had gone about the business of fixing the dimensions of a building by the principles of ancient geometric symbols. But in the beginning they knew nothing of (and gave little regard to) the most economic form of structure, wall thickness, arch curve, etc. They went ahead with geometric symbols, diagrams, plans and finished structures that could support ten, twenty times the required weight—and more.

The Great Pyramid of Egypt, for example, is more a mammoth memorial to ancient geometry than a building planned for the sake of economics. For its effective interior is nothing compared with the tremendous mass of material used in the building's construction.

As the religious builders gained more experience, studied their finished efforts, returned for a second look at their geometric plans, and speculated on whether such massive walls, roofs, etc., were strictly necessary, so the style of building altered. More space was allowed within the structure, dimensions were slimmed down, columns became tapered, new materials were made available—and geometric diagram became even more than before a tool of the builder. Once he had experimented with a particular height/thickness ratio and had found a suitable set of lines in his favourite geometric symbol, the builder stuck to it. He had found a successful shape from which to advance. He could afford to be bold. But always he came back for a second look at the geometric diagram.

Hadrian was certainly a brilliant and experienced builder. A less experienced, more timorous planner would have been tempted to select line 3-6 as the outside of the main wall and not, as we have seen in the analysis, line 42-43. But the difference between the two lines meant, for example, a considerable saving in building materials. And cost was presumably also a factor worth remembering in those days, too.

In *Fig. 245* the basic square is subdivided 10×10 in the same manner as we saw in Chapter Ten. Plato, we remember, split the square on the circle's rectangle 8×8, and by producing his

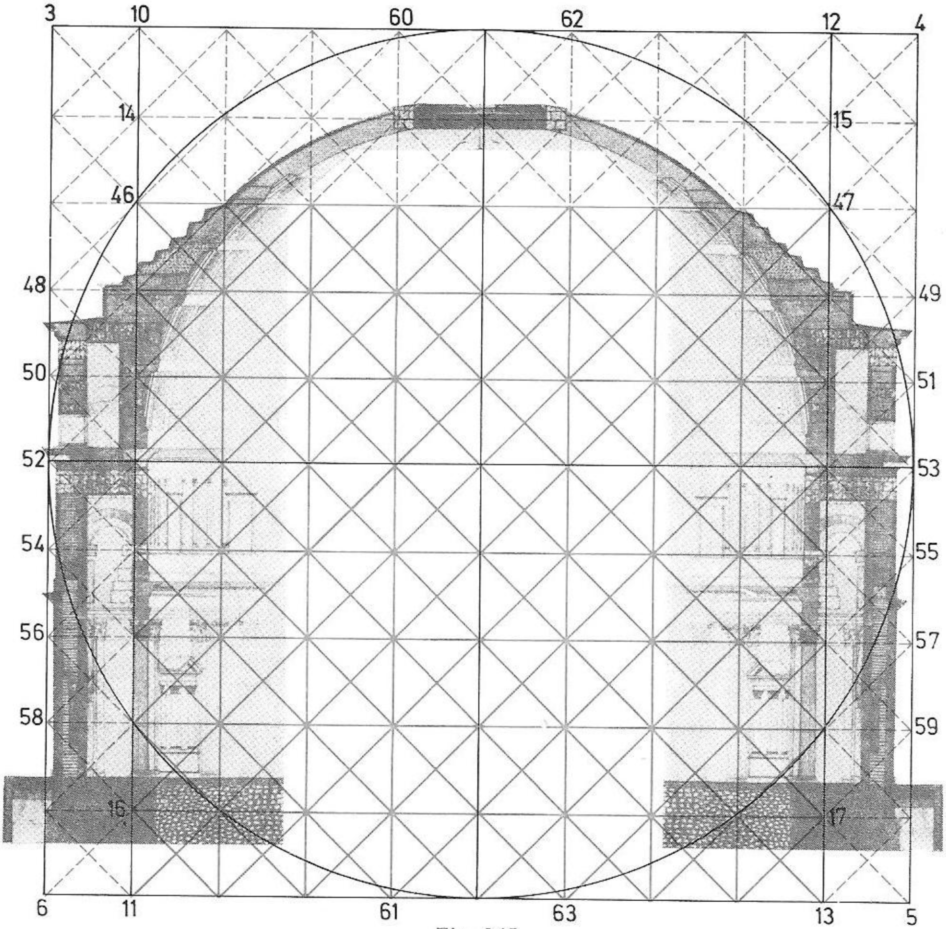


Fig. 245.

lines of division to the basic square, divided the latter 10 × 10.

In the present diagram the circle's rectangle is 10-11-12-13, and its square lies at the bottom of the figure: 46-47-13-11.

We divide the latter square with verticals, horizontals and diagonals as shown previously, and produce these to meet the sides of the basic square. The extensions are shown as broken lines.

We shall now examine the horizontal lines of 10-part division—and the result is surprising. Almost all of them are responsible for marking some dimension or other in the building.

The top line, 3-4, is the constructive starting point. It lies outside the building and therefore has no place in the actual structure.

The next line, 14-15, runs flush with the inside ceiling of the dome, and is in fact the same line (in Figs. 240 and 244) as the top of the centrally placed square on the circle's rectangle.

We examine next line 46-47, which runs through the upper half of the dome. From this level down to the next line, 48-49, we note that the outside of the dome is broken in a series of seven steps.

Line 50-51 has apparently no special

place or value in the diagram, running merely through a frieze on the outside.

We are already familiar with line 52-53 as the horizontal axis of the basic square. In Fig. 240 it was numbered 2-9 and its value was illustrated in that analysis.

Line 54-55 indicates the lower sill of the windows and niches backing into the gallery above the main hall. These are more clearly seen in Fig. 238. As far as can be ascertained, the distance between points 52 and 54 is equal to the height of the windows.

Line 56-57 marks, as we can see, the lower edge of the corinthian-style capital in the inner colonnade.

The representative marking of line 58-59 is most interesting. The line lies so low in the building's plan that it plays no constructive part in the structure, but nevertheless, to point out the existence of the line, a mark has been made in the waist of the pillars that surround the main church hall.

Line 16-17 is the same as seen in Figs. 240 and 244. It is the base of the square on the circle's rectangle when this is placed centrally in the diagram.

Finally we come to line 6-5, the basic square's base-line, which is part of the original construction. Since it lies somewhat lower than the building itself, it has no real marking there, but perhaps acted as the level of the foundation or the steps which originally led up to the temple.

These three surveys of the building's sectional view ought to be sufficient to demonstrate the guidance given by ancient geometry in planning the structure of the Pantheon.

Pantheon's ground-plan

THE CHOICE of starting point for an analysis of the Pantheon's ground-plan must, as we know from past experience, be geometrically linked with the building's facade (or in this case its cross-section).

Since the Pantheon is a circular building and since the building's complete elevation was contained by the basic square in the earlier analysis, it would be natural to transfer the dimensions of this square to the ground-plan.

In *Fig. 246* we see this taken as the basic square: 1-2-3-4. It embraces the entire circular part of the building. Additional structures at the front and rear of the main building are excluded from the basic square. In order to include these in a geometric study, we construct the basic square's double-size version: 5-6-7-8.

We notice first that lines 1-2 and 3-4 were used as the determinating factor in fixing the depth of the triangular rooms at the front and rear of the building on either side of the vertical axis.

We see, too, that line 5-6 completely encloses the ground-plan at the rear of the building, while the lower line of the outside square, 7-8, appears to run aimlessly through the porch. Later analysis, however, will prove that the depth of the porch was determined by other factors than the basic square's double-size version.

The sacred cut is executed in the large outer square as 9-10 and 11-12 (vertically) and 13-14 and 15-16 (horizontally).

Immediately we notice that the verti-

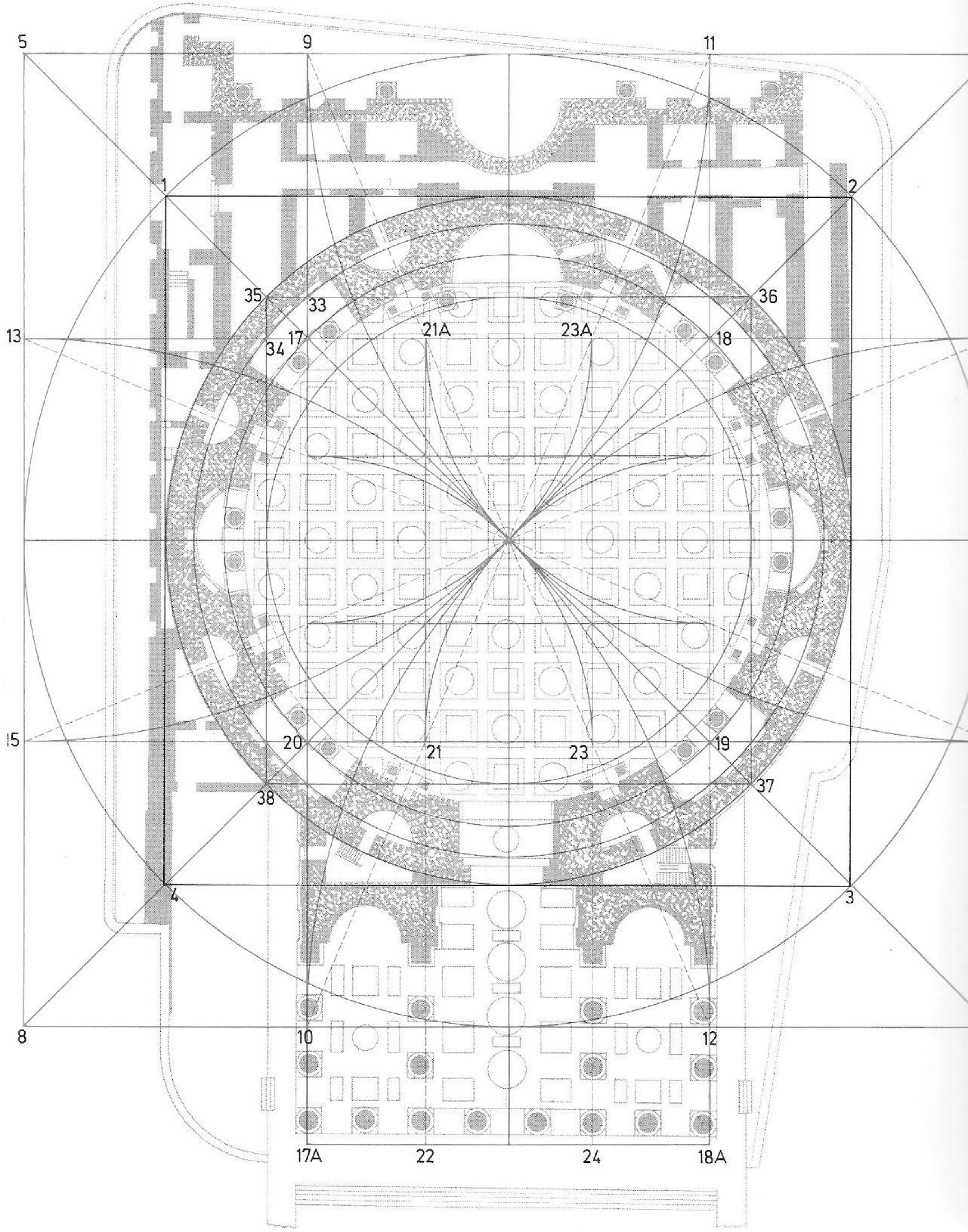

5
9
11
1
2
35
33
36
13
17
21A
23A
18
34
20
21
23
19
15
38
37
4
3
8
10
12
17A
22
24
18A

Fig. 246.

cals indicate the width of the porch. They run precisely through the centre of the two outer rows of columns.

The combination of the four sacred cuts creates as usual a smaller square in the centre of the diagram: 17-18-19-20.

If we can imagine this square flipped over in the direction of the Pantheon's entrance, with line 19-20 as the "hinge" or axis, we achieve a new square: 19-20-17A-18A. And we see that the new square takes in exactly the complete entrance arrangement, including the part that lay outside square 5-6-7-8.

Thus we have covered the area of the whole ground-plan. Line 5-6 contains the rear, the vertical sides of the basic square (1-4 and 2-3) contain the long sides of the building, and line 17A-18A contains the entrance.

In square 17-18-19-20 we execute the sacred cut. We are particularly interested in the two vertical lines. These are 21A-21 and 23A-23. Produced to the entrance of the building, they mark the total width of the approach—passing through the middle of the two rows of pillars that line the entrance and through the square butt-end pillars on either side of the doorway.

The extensions are to points 22 and 24 respectively.

We have concentrated in the Pantheon's ground-plan so far on the horizontal and vertical lines of the diagram and the interplay of the various squares.

But a circular building of this type is naturally influenced considerably in dimensions by the circles within and outwith the respective squares. A square, we recall, has two constructive circles: one described around the outside, the other inscribed within the square.

Before examining the applications of the circle, however, we require to add one more square to our diagram. It is the basic square's half-size version (seen as 35-36-37-38), and its associate circles have an important role to play in the building's lay-out.

We now have four squares placed concentrically within each other, and we begin our study of their circles with that of the outside square.

Square 5-6-7-8's inner circle was a deciding factor in planning the rear of the ground-plan, being the constructive link between the basic square and its double-size version.

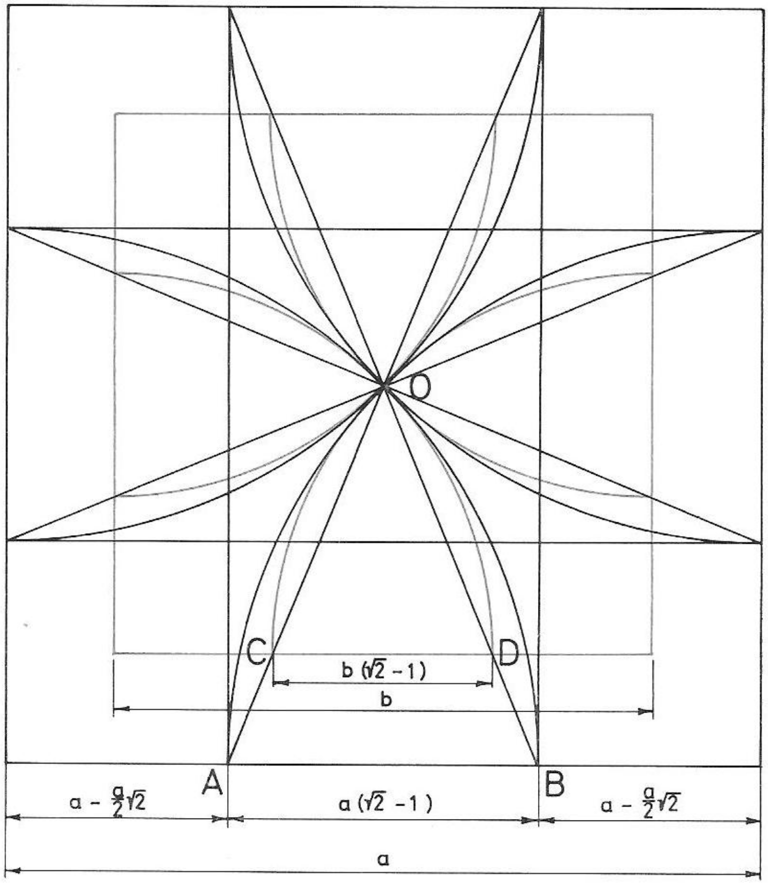
Being the inside circle of the large, outside square, it is the outside circle of the next (basic) square: 1-2-3-4. This same square's *inside* circle, we see, follows the curve of the outer wall of the Pantheon and was obviously the factor that determined the outer dimensions of that wall.

This circle is simultaneously the outside circle of the basic square's half-size version: 35-36-37-38. This square's *inner* circle provides the absolute net floor area in the large main hall. The circle touches the outside of the two large columns on either side of the altar (on the left of Fig. 238). The actual perimeter of the inner hall is broken by a series of recesses and projecting walls.

Our attention turns now to the inner square created by the combination of sacred cuts in the large outer square. The small square is 17-18-19-20. We observe how its outside circle forms the back of the small niches cut into the projecting wall all the way round the main hall. The arc of the same circle positions the pillars at the mouth of each of the six main recesses in the hall.

As we saw with earlier Greek temples, the intersections of existing lines and squares often provide opportunities for entering new lines.

One such intersection in the Pantheon's ground-plan is the meeting of the square created by the sacred-cut combination (17-18-19-20) and the basic square's half-



$$\triangle AOB \sim \triangle COD \implies a(\sqrt{2}-1)\,\frac{\frac{b}{2}}{\frac{a}{2}} = b(\sqrt{2}-1)$$

Fig. 247.

size version (35-36-37-38). Their junction produces four tiny squares, one of which is 33-17-34-35.

Taking the diagonal (33-34) of this square as the basis of a new circle with the same centre as the previous, we see how the new circle follows the inside sweep of the main wall of the circular hall. And the distance between this circle and that drawn within the basic square gives the total thickness of the wall.

The various concentric circles we have examined are so much a part of the structural plan of the Pantheon that they represent a reality that cannot be ignored. There can be no doubt that they were part of the original ground-plan sketched out by Hadrian and his fellow-builders. Not only was Hadrian well aware of the structural value of geometric symbols, he also made full use of them in this sample of his work.

At the centre of the large square the sacred cut forms a rectangle. Vertically

this is 9-11-12-10, and horizontally 13-14-16-15. When we enter the diagonals in these two rectangles (i.e. 9-12, 11-10, etc.) we observe that they pass exactly through the middle of the eight windows in the outer wall of the church.

This is a construction we have not seen previously. What are its characteristics? It is not usual for lines to possess a certain geometric property?

A close examination will, however, reveal that these lines do indeed have a purpose, a geometric birthright. Hadrian did not include them in his plan accidentally.

The diagonal lines under discussion have the following property: their intersections coincide with the sacred cut in any square sharing the same centre and axis as the basic square.

Whether this discovery was made by Hadrian or whether by some earlier geometer is difficult to state categorically. Only the existence of another, earlier building showing the same construction would prove the latter.

That the discovery is correct is illustrated in *Fig. 247* which shows the geometric construction and corresponding arithmetical calculation.

We move on in the analysis of the ground-plan to a new diagram, having obtained the majority of the principal dimensions in the church from the preceding diagram.

The object of the next two analyses is to demonstrate how a similar analysis of the same squares provides further information on the recesses and column spacing within the main hall.

In *Fig. 248* we start with the basic square 1-2-3-4, and construct its half-size version 35-36-37-38. This is a repeat of the previous analytical diagram, but in this case we go down one stage further, constructing yet another half-size square: 39-40-41-42.

This new square is divided 3×3 in the same manner as executed in the Greek temple analyses. The simplest method of doing this is to enter the diagonal cross and the acute-angled triangle. The intersections of these two figures indicate the 3-part dividing lines. Vertically these lines are 47-48 and 49-50. We see immediately that they indicate in the rear wall the width of the semi-circular niche in which the altar is situated. The lines run through the centre of the two columns flanking the entrance.

At the opposite side of the temple the corresponding lines mark, as I believe was intended, the net width of the hall entrance.

The same lines horizontally (43-44 and 45-46) show the same thing: they indicate the width of the two niches at right and left of the main hall.

The 3-part division just executed contructs a new small square at the centre of the diagram, and in this square, too, we enter the 3-part dividing lines. They are (horizontally) 51-52 and 53-54. The vertical lines are not required.

We recall that 3-part division was the most common mode of spacing pillars in earlier Greek temples.

If we produce lines 51-52 and 53-54 across the diagram, we see that they were apparently responsible for positioning the columns in the niches on the right and left of the hall.

Fig. 249 shows in effect the same diagram, the difference being that the inner square has been swung through 45°. In other words, square 39-40-41-42 corresponds in the new diagram to square 55-56-57-58.

We see how the 3-part division of this square was used to mark the maximum width (at the rear) of the four rectangular recesses in the main hall. When this width is linked to the centre of the diagram in the form of radii we see that the

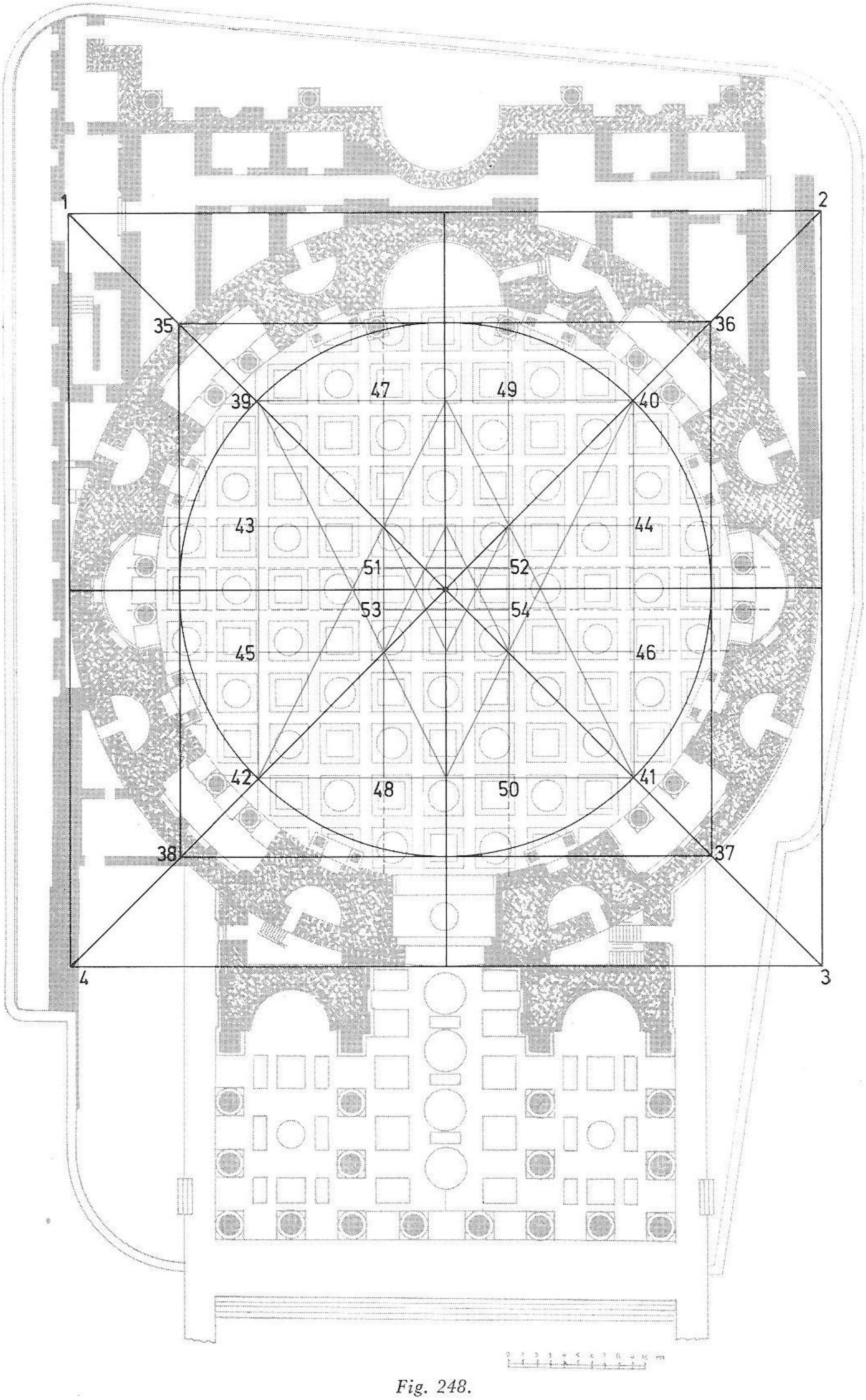


Fig. 248.

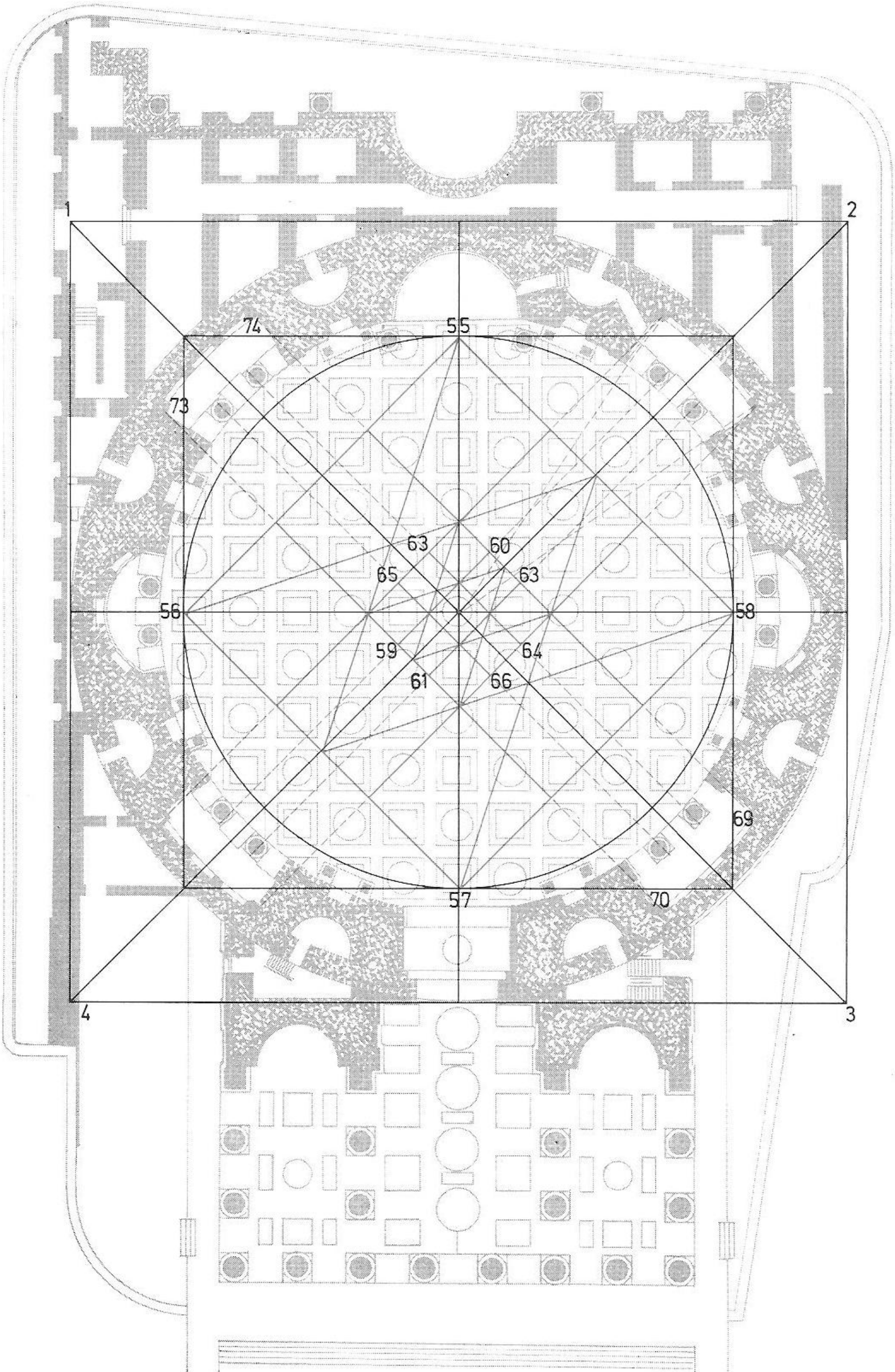


Fig. 249.

angle matches perfectly the angle of the recess wall.

The small inner square created at the centre of the diagram by the lines of 3-part division is also divided 3×3, and we see how the resultant lines appear to have been the factor that positioned the two columns placed before each of the four rectangular recesses.

We see this carried out, for example at recess 69-70. The dividing lines are extensions of 63-64 and 65-66.

As in previous analysis, a mass of detail is still hidden geometrically in the diagram, but we have nevertheless traced as many of the main lines as permit us—as long as we remember the order and manner in which Hadrian applied the symbols—to reconstruct the chief structural features of the Pantheon. We require neither measurements nor drawings. Only a full appreciation of the circles and squares of ancient geometry.

A continued analysis would certainly provide us with a wealth of additional information. The diagrams have not been exhausted, possibilities are many. But as emphasised previously, the aim and intention of this book is not to explain every line of any specific building. It is to demonstrate the presence of ancient geometry in planning a particular building.

CHAPTER FIFTEEN

The Golden Section versus the Sacred Cut

WE HAVE made free and frequent use throughout this book of a completely new term in geometry: the (by now) familiar *Sacred Cut,* a label applied by the author.

This geometric newcomer distinguishes itself from other similar terms in that it is not simply a geometric or mathematical phenomenon; it was a concrete factor in the sphere of building right from the earliest days of religious constructions through time as far as the Middle Ages.

Notwithstanding that the *sacred cut* existed as one of the chief factors in apportioning dimensions to almost all monumental structures of the past and can be traced and revealed in those examples of such structures as (from the point of view of preservation) lend themselves to study, the Temple and later the Church were completely successful until now in keeping secret from uninitiated the mathematical knowledge that the term conceals. Indeed the *sacred cut* and ancient geometry, still hidden from the casual observer, died an unnoticed death the day that building, design and construction work were wrested from the hands of the religious orders and passed instead to professional builders ignorant of the training and tradition of the cloister and temple.

We have studied the practical application of the *sacred cut* in the planning of the Great Pyramid of Egypt, we have picked it out from among the building instructions for Moses' tabernacle in the Midian desert, and we have seen it employed as undoubtedly one of the principal motives of design in building temples of antiquity and religious structures of the Middle Ages.

In fact the *sacred cut* has proved itself the corner-stone of the entire building industry of ancient times in precisely the same way as it represents the main factor in ancient geometry itself, the subject to which this book is devoted. The ancient system of geometry can almost be regarded as a direct development of meditation on the *sacred cut* and the sacred number seven, and in the same way as these latter it has remained part of the early Church's occult teaching.

The phenomenon arose, as we saw, at an immeasurably early stage in Man's mathematical speculation, and assisted the developing geometer to calculate the circumference of the circle to within an error of less than 1 %. The *sacred cut* has the property of being explicable to any intelligent observer devoid of mathematical knowledge and experience as we understand it today.

The concept of the *sacred cut* is so devastatingly simple, requiring only a primitive system of numbers and a reasonably intelligent operative to apply it successfully, that it is scarcely surprising that it was observed and recorded so early in

Man's mathematical speculation. What *is* perhaps astounding is the fact that it has not subsequently been revealed and expounded to the world at large.

The term "sacred cut" is a name I have personally given to this particular observation, and it no doubt (not unintentionally) calls to mind a similar established term associated with ancient mathematics: the *Golden Section.* The latter has, by some scholars, been attributed the place in classical structural design that I have described for the *sacred cut.*

But I consider there is a major difference between the two conceptions: whereas the *sacred cut* can be traced to and proved to be present in countless examples of ancient buildings, design, etc., the same cannot be said for the *golden section.* The second term is much more of a speculative nature, never having stood the rigours of practical application although attempts (which I regard as groundless) to prove otherwise have been made from time to time. The *golden section* is moreover out on a limb, so to speak. All on its own. It belongs to no "family" or system of geometric thinking. It apparently has never (as it must, if it is to make good the boasts made on its behalf) been the cause, intermediate stage or final result of a series of observations based on geometric knowledge.

Exactly what is this *golden section?* What is its history? And origin? These are probably the first queries one registers on encountering this established feature of geometry. And no doubt one is interested in its purpose.

Finally, the question arises: How can such a phenomenon as the *golden section* go down in history as a wonderful mathematical reality, if in fact there was nothing special about it?

It would be altogether a mammoth assignment (and too great for this present work) to go in detail into the multitude of postulates and theories to which the renowned *golden section* has given birth. My course therefore has been to concentrate on the studies of one researcher and to allow his assessment of the subject to act as a representative for the work of others.

I have chosen to examine the work of the Danish engineer Vilhelm Marstrand who, in his book *Arsenalet i Piræus og Oldtidens Byggeregler,* included a chapter on the wisdom contained in the *golden section.* He also mentions the phenomenon in his chapter on rules of structure.

Marstrand's deliberations were chosen because, with the thoroughness of the trained mathematician, he probed the legend and story of the *golden section* from numerous angles, and he attempted to trace its origin both in the sphere of numbers and in practical application.

As we shall presently see, the results of his effort are sparse.

Marstrand admits, for example, that the *golden section* has apparently never in its pure form been employed in building design, and I believe that his attempt to fit the concept into the context of Plato's *Timaeus* also fails.

Faced with these two "failures" one would expect Marstrand to say that the *golden section* had in fact nothing to do either with Plato or with building. But instead we have the odd experience of Marstrand suggesting that (a) Plato must have made some kind of error, and (b) a rough approximation must be deemed satisfactory in design.

The uncertain nature of his conclusions is typical of the theories and explanations that surround the *golden section.*

It is widely believed that the mystical *section* was a term of proportional beauty administered in classical architecture and other design. But confirmation (let alone genuine proof) has never been forthcoming to substantiate the belief.

Sure enough, by his very thoroughness, Marstrand brings out several discrepancies and illustrates that something somewhere must be wrong. Yet he concludes by acknowledging the existence of the phenomenon while stating one or two qualifications.

Marstrand's book is available only in Danish. Rather lengthy (translated) extracts are therefore included in this volume in order to provide as much of the background to his examination of the *golden section* as possible.

*

What is the *golden section?* How did it originate? With which age was it associated?

These are questions one might well direct at engineers or architects. They, of any complete classification of worker, should know the answers. But one normally discovers that although the label "*golden section*" is recognised, perhaps even bandied about in conversation, only a minority of people know its history. To the majority it is simply a means of achieving proportional harmony used by the ancient Greeks.

Occasionally someone will say that it is a continued fraction, expressed as $\frac{\sqrt{5}-1}{2}$.

And it is rare that one comes across the expert who can state, correctly, that the term *golden section* implies that a given straight line be cut in extreme and mean ratio, i.e. that the line is divided at such a point that the whole line should have the same proportion to the larger part, as the larger to the smaller.

It has never, however, been my pleasure to meet any scholar who could demonstrate or even refer to its existence in ancient building structure. The term is merely accepted as an established postulate, requiring (or having?) no proof.

Marstrand's book *Arsenalet i Piræus og Oldtidens Byggeregler* (The Arsenal at Piraeus and Rules of Ancient Architecture) was published by H. Achehoug and Co., Copenhagen, 1922. He touches several times on the *golden section.*

In the references scattered throughout the book's 286 pages Marstrand confines himself entirely to a mathematical and speculative search. As the volume is intended as a revelation of the general system of rules governing ancient building practice (with particular attention paid to the arsenal at Piraeus) one might expect an example or two of the use of the *golden section.* But none is offered.

The absence of any such example is doubtless due to Marstrand's total inability to trace one. But let us have a look at his own explanation of the *golden section:*

"Concerning the rules of proportion, it is immediately apparent to the observer that quadratic sub-division and simple numerical ratios were frequently employed in design. Moreover, researchers such as Hultsch, Dörpfeld and Durm have written lengthy reports of their results in this field. In more recent times Wolf and Thever have demonstrated that simple ratio and whole numbers of inches, handbreadths, feet and cubits are to be found in the majority of classical dimensions.

"On this basis, there is no doubt whatever that the aim of the ancient architect and designer was to achieve dimensions in whole and unbroken numbers. Behind this inherent goal must lie a purely practical reason. And this is the case.

"Just as in present times craftsmen (although extremely accurate rulers, gauges and measuring sticks are readily available at little cost) dislike marking off long distances, so the craftsman of ancient times actually found the process *impossible.* If a distance to be measured was greatly in excess of their ruler or gauge, they were never twice able to execute the measure-

ment and obtain precisely the same result on both occasions. Their measuring tools were simply not accurate enough for the task. It was thus essential to work with whole numbers or complete units.

"A series of fine observations in this field has already been conducted. Call to mind, for example, Thiersch's report on the Temple of Poseidon at Paestum, and his discovery that the sub-division of the triglyphs in the temple frieze corresponded in height and width to the temple's main proportions; and the illustration by Thever that certain numerical ratios are repeated again and again within the same temple group: for example, 2:5 in the Temple of Apollo, 3:8 in the Temple of Hera, 5:11 in the Temple of Zeus, and so on.

"Thever's further theory that a connection existed between these temple dimensions and the Pythagorean speculation on numbers was never taken to a satisfactory conclusion, but that such a relationship in fact prevailed seems obvious.

"The belief of the Pythagorean student, based as it was on the mystery of numbers, that everything had its origin in numbers, that god was numbers, was nothing new. It did not evolve with Pythagoras. Thousands of years earlier it had been preached by the Babylonians and Egyptians.

"The real work of Pythagoras in this particular field was to assimilate such numerical speculation with the so-called Egyptian rules of construction, and to present the result to his fellow-Greeks as something of practical value in architectural design.

"He perhaps co-ordinated the system and its many aspects, and almost certainly spotted a linking thread which appears to have been foreign to the religious leaders of Babylon and Egypt. As the symbol of this "link" he selected the five-pointed star, and contemporary accounts indicate that it was forbidden on pain of death for a Pythagorean to reveal the said star's secret.

"Consequently one would expect to find all aspects of Pythagorean speculation incorporated in temple buildings, including particularly the ratios associated with quinary division.

"And this is actually the case. Quite independent of Thever's study, Macody Lund has enjoyed considerable success in proving that the most sacred of these ratios in the religious structures of antiquity and the middle ages was the so-called *golden section;* in other words, the division of a given straight line such that the smaller of the parts is to the larger as the larger is to the whole line.

"Few subjects have been disputed more fiercely than the *golden section*. As a rule of architectural design, it was first acknowledged during the period of Renaissance. Previously, its very existence had been challenged. It was denied that antiquity knew anything about it. But this was quite wrong. Plato lauded its qualities, and a large portion of Euclid's famous *Stoicheia* was devoted to its construction and background. In general, however, classical literature has little to say on the matter. No direct mention is made by Vitruvius in his 10 works *De Architectura*. Not really surprising if in fact an aura of mystery and secrecy surrounded the *golden section*.

"But naturally the secret must have been more profound than the simple five-pointed star, which everyone can sketch and the mathematical properties of which must be (and must have been) known to all arithmetical scholars.

"The secret must surely have been the rules for applying quinary divisions in building. By 'rules' we mean in the first instance simple practical guide-lines.

"I shall attempt to show later that it was by developing to a fine pitch their knowledge and experience of comparison

and similarity, coupled with the development of the *golden section,* that builders and architects of antiquity and the middle ages ensured that their work would not collapse, that its design and structure were sound. But such rules demanded a much greater degree of experience and practice than ours today. Naturally enough, therefore, those within the profession did their best to exclude novices and possible pirates, preferring to hand on their knowledge and secrets to people who proved themselves wise and able enough for the responsibility.

"In reality, the only difference between ancient and modern practice in this respect is the exclusion nowadays of the word 'secret'.

"Today all the rules, every guide-line, every scrap of experience is readily available to anyone interested in the subject, but the proper application of all this knowledge necessitates on the other hand such an enveloping mass of previously-obtained qualifications and mental ability that relatively few people make the grade. Success in a series of searching examinations is a *sine qua non* to any prospective architect/engineer (as indeed is the case with other professions).

"It is a matter of some surprise that certain scholars persist in denying that the ancients applied such rules of structure. What denial can withstand the proof set out by Macody Lund? The only aspect on which the doubters can direct a derisive finger is perhaps the mysticism that Lund lays forth as the background to these structural laws. But his reasoning alone can be questioned. There can be no suspicion whatever of the manner in which he demonstrates the use of the ancient rules.

"Further proof of the authenticity of the rules is forthcoming in the fact that they allowed the user to produce an infinite variety of structural designs typical of building in classical antiquity and the middle ages.

"It is scarcely probable that two buildings exist which are identical in every detail, and the objection that measurement has later disproved the 'rules' is not in fact valid. Kroman has illustrated this. Considering the large number of instances in which the rules have been proved by Lund to have been applied to a particular building, even a rough approximation is more than mere coincidence. The law of averages excludes coincidence.

"Division of a line according to the *golden section* (or, as it is also termed, to cut it in extreme and mean ratio) implies that a given line is divided in two pieces, a and b, such that the shorter piece (a minor) is to the larger (b major) as the longer is to the whole line:

$$\frac{a}{b} = \frac{b}{a+b}$$

If this ratio is x, then:

$$x = \frac{1}{x+1}, \text{ therefore } x =$$

$$\frac{\sqrt{5}-1}{2} = 0.61803.$$

"As can be seen from its square root, the proportions of the *golden section* form an irrational fraction. In other words, it cannot be expressed precisely in complete numbers in the numerator and the denominator. But it can be accomplished to any desirable degree of accuracy by developing the expression 'in series', which can best be done with the aid of the infinite fraction we found to be identical to the above expression:

$$x = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots\ldots}}}}$$

"The series corresponding to the continued fraction is termed the fraction's convergents, and is in turn the part of the

series which can at any point be broken off from the remainder, since it is true of these infinite fractions that their real value lies always between two fractions and nearer the latter of the two.

"It is also a quality of convergents that they make it possible with smaller numbers than any other fraction to state an approximate expression for the continued fraction's precise or real value. In the case of our particular infinite continued fraction the convergents are:

	$1/1$	$1/2$	$2/3$	$3/5$	$5/8$	$8/13$	$13/21$
	1.0	0.5	0.666	0.6	0.625	0.615	0.619
an error of	+ 60 %	− 19 %	+ 7.9 %	− 2.9 %	+ 1.1 %	− 0.5 %	+ 0.2 %

"The simple formation of this series (each step is formed by the addition of the two preceding numerators and denominators respectively) was well-known and used by ancient planners, and its proportions are those most often discovered in classical buildings.

"The lower row in the series is taken as a simple numerical ratio, beginning 3:5, and the latter as a result is often referred to as the ratio of the *golden section*. As a rule, however, all the numbers of the lower row are normally to be found represented in the building's proportions.

"The foregoing applies in particular to the vertical lines of division and to the ratio of height to width.

"When it is required to divide up a horizontal line in accordance with the *golden section,* the process is altered slightly and becomes a matter of symmetry (taking the form minor-major-minor):

$$\frac{1:1:1}{3} \quad \frac{1:2:1}{4} \quad \frac{2:3:2}{7} \quad \frac{3:5:3}{11} \quad \frac{5:8:5}{18} \quad \frac{8:13:8}{29}$$

Here again the first three fractions are converted to simple ratios, starting with 3:5:3, and again this is referred to as the *golden section*.

"Drawing I (*Fig. 250*) shows the normally accepted geometric construction of the *golden section,* and the associated construction of a regular pentagon and decagon.

"Radius AC has been cut in extreme and mean ratio at point A, and A_1B is divided similarly at point C, the whole diameter thus being split up as follows:
$R_q{}^2 + R_q + R = 2R$

"There is of course nothing to prevent a continuation of this process of division, upwards or downwards, as shown in drawing II (*Fig. 251*) which also illustrates the simple connection that exists between the *golden section* and two concentrically placed squares."

The quotation continues with several examples of how, with arcs within a circle, one leg of the compasses being on the circumference and the other inside the circle, one can construct stars within each other, the intersections of the curved lines representing approximate values for the *golden section*. But since these examples add nothing to the present discussion they have been omitted here.

Later Marstrand continues:

"It is characteristic of the use of the *golden section* in building that it is not scattered here and there, but that it is employed in a definite series of steps $a \times aq \times aq^2 \times aq^3$, etc., but in answer to any query on whether it has been applied in any specific building one should bear in mind that instead of the precise ratio, the ancient builder used one of the approximate values which—as we have seen—can vary as much as 3 % from the correct value. And further, a fact that I have emphasised on several previous oc-

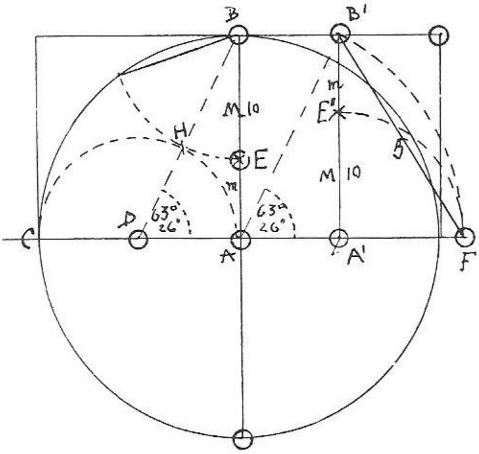


Fig. 250.

casions, the executed measurements only to a certain degree can be regarded as identical to the originally planned dimensions. It is absolutely an exaggeration to state that Greek temples are built precisely according to plan to within a fraction of a millimeter. The ancient measuring equipment simply was not accurate enough to do this."

Here Marstrand concludes his initial comments on the term, *golden section,* and proceeds to explain in text and figures

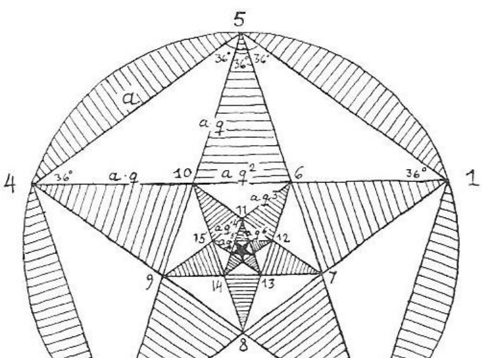


Fig. 251.

several inaccuracies about the Temple of Hera.

We can see from that extract that Marstrand was plainly inspired by Lund's *Ad Quadratum;* also from the fact that he refers to Plato and Euclid as the original sources in which the *golden section* is mentioned; Lund mentions them in this connection, too.

This of course is not particularly amazing if the Plato-Euclid source was correct. Naturally both Lund and Marstrand would come to the same conclusion.

Whether the conclusion can be attributed to Macody Lund or whether it has its origin elsewhere is perhaps difficult to discover, but when we examine the passage in question a little later in Euclid's *Elements,* we shall find that Euclid himself never once mentions the *golden section.* He does on the other hand prove the qualities of a geometric term which he calls the *section,* without in any way emphasising its value in preference to other geometric terms, or indicating that it should be regarded as something outstanding and deserving of the mystical name, *golden section.*

Marstrand begins his account by establishing (or stating) that one of the factors that immediately makes itself obvious in ancient building work and its aesthetic value is the quadratic division and simple number-ratio of the planning.

His further statement (with no accompanying geometric proof) that the properties of the *golden section* were the most important and sacred in religious building, and his indication that these properties can be illustrated only by a complicated chain fraction whose final result is an approximation, or by an irrational fraction called $\frac{\sqrt{5}-1}{2}$, must be regarded as clashing somewhat. When he continues that, "in answer to any query on whether it has been applied in any

specific building one should bear in mind that instead of the precise ratio, the ancient building used one of the approximate values", then I am afraid confusion reigns completely.

It seems to me to ring rather hollow that a particular factor is accepted as a mainstay of building design and is never applied independently but always with approximate values; perhaps even grossly inaccurate values, for Marstrand underlines further discrepancies in Greek construction work, commenting: "And further, a fact that I have emphasised on several previous occasions, the executed measurements only to a certain degree can be regarded as identical to the originally planned dimensions."

Solution of the fraction $\frac{\sqrt{5}-1}{2}$ produces a decimal fraction which, taken to five places, is 0.61803. This is the desired ideal, but if it is possible to come only to within a rough degree of accuracy, why bother with so many decimal places?

It would have been interesting to examine in connection with the same extract a single example of the *golden section* applied in a building, but none is given.

It appears therefore that the first mention of this geometric phenomenon fails to provide a complete answer to the query: Where was it applied? And particularly: Why, if in fact it was used in planning, was it something special?

The only concrete piece of information we have drained from the passage is a geometric description of the *golden section:* "Division of a line according to the *golden section* (or, as it is also termed, to cut it in extreme and mean ratio) implies that a given line is divided in two pieces, a and b, such that the shorter piece (a minor) is to the longer (b minor) as the longer is to the whole line."

This is then expressed in the fraction: $\frac{\sqrt{5}-1}{2}$, which equals 0.61803.

The object is therefore to split a line in two pieces so that BC is to AB as AB is to AC.

Fig. 252.

Before we look into the basis of the problem's solution, we shall examine the result 0.61803.

We assume that the given line has a total length of 1, and the point of division is as indicated by the above decimal.

Thus line AB equals 0.61803, and line BC equals 0.38197, since these total 1.0000.

We are told that

$$\frac{AB}{BC} = \frac{AC}{AB}$$

thus,

$$\frac{AB}{BC} = \frac{0.61803}{0.38197} = 1.618,$$

since $0.38197 \times 1.618 = 0.61802746$

$$\frac{AC}{AB} = \frac{1.0000}{0.61803} = 1.618,$$

since $0.61803 \times 1.618 = 0.99997254$

We find therefore that the postulate in fact fits the calculation, there being an infinitely small error of one or two $^1/_{100,000}$. This error can be traced arithmetically but never in any circumstances geometrically since we may take it for granted that the ancients certainly did not work with mathematical quantities and dimensions obtainable only by arithmetic. To bear any relevance at all to the building art, it must have possessed a geometric solution. And moreover we should bear in mind that the development of decimal calculation took place about the year 1000 A.D. If therefore Euclid wrote about and discussed the *golden section* with such decimal accuracy, it was nearly 1500 years before the decimal system came into use.

If we wish to fit the formula $\frac{\sqrt{5}-1}{2}$ into a geometric diagram, we must first find a familiar factor in order to begin the construction. This we find in $\sqrt{5}$. A line which is $\sqrt{5}$ in length can be found in a right-angled triangle whose short perpendicular is 1 and long perpendicular is 2.

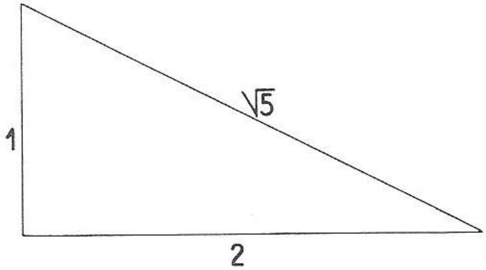


Fig. 253.

Thus $a^2 + b^2 = c^2 = 1 + 4 = 5$. Since $c^2 = 5$, then $c = \sqrt{5}$.

We find this geometrically in *Fig. 254* in which the short perpendicular is AB, the long is BC and the hypotenuse CA.

The only line in the diagram that is 1 unit in length is AB. We shall now proceed therefore to mark off along its length a distance equal to $\frac{\sqrt{5}-1}{2}$.

The first step is to execute $\sqrt{5} - 1$. This is done by measuring the length of AB (= 1) along CA, producing point D.

This is the first part of the formula, and we are told next that the result (DA) shall be cut in two. In other words, we cut line DA at E and mark this distance out along AB at point F, which should thus be the point of the *golden section* of Line AB.

To test our geometric construction we say:

$AB = 1$
$BC = 2$
$CA = \sqrt{5} = 2.236068$
$CD = 1$
$DA = \sqrt{5} - 1 = 2.236068 - 1 = 1.236068$

$$DE = EA = AF = \frac{1.236068}{2} = 0.618034$$

and we have now succeeded geometrically in constructing the same arrangement as before when we simply measured off the length on a horizontal line, namely: AF major = 0.618034, and FB minor = 0.381966.

Thus line AB has been divided in extreme and mean ratio, according to the requirements of the *golden section,* and

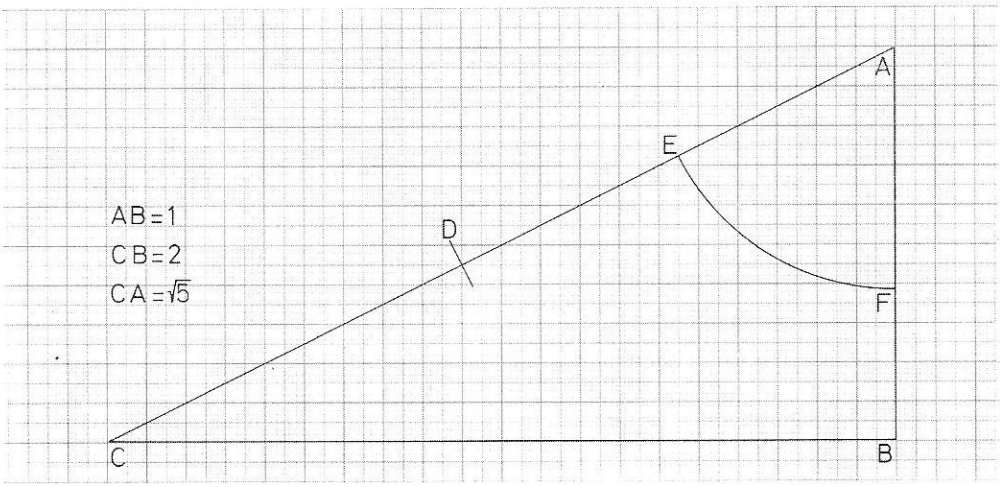


Fig. 254.

we have examined the term both geometrically and arithmetically without deriving much information.

We learn nothing of a harmonious relationship with other numbers or sizes, nor of any special geometric factors apart from the fact that the term arises from the acute-angled triangle we have studied so much in previous analyses.

We have reason to suspect that Marstrand's main source of information and inspiration was Macody Lund's book *Ad Quadratum,* and like Lund Marstrand spends some time trying to show how Plato is discussing the *golden section* in his dialogue *Timaeus.*

That particular portion of *Timaeus* was admittedly dissected fairly thoroughly in Chapter Ten, when we saw in detail that it involved the *sacred cut,* not the *golden section.* But as we did not at that stage have any information on the *golden section,* it would be interesting to reappraise that same passage now.

Marstrand writes (page 218):

"Euclid uses the word *analogas* in precisely the same context as we say *proportional,* but to provide Vitruvius's statement (that the Greeks employed *analogia* to give their temples a comparative identity) with a mantle of fact, the word must have had a more restricted meaning. That this in fact was the case is apparent from the well-known passage in Plato's *Timaeus,* in which it is said:"

(The following is an English version of what appears to be Marstrand's own translation into Danish since the Latin text is also given):

"'It is impossible to combine two things without the aid of a third because there must be a bond between the two to form a whole. And the most beautiful bond is one that acts on its own. *Analogia* is doubtless the most beautiful way to achieve this.

"'For when three numbers whether 3- or 2-dimensional (author's suggestion: three cubes or squares) are such that the first is to the middle as the middle is to the last, and vice versa, when the last is to the middle as the middle is to the first, then the middle can be the first, and the last and first can both become the middle. It necessarily follows that all will be the same, and when all are mutually related, then unity is complete.'"

Marstrand's own comments to the above are as follows:

"Appreciation of this passage has naturally given rise to great difficulties and been the subject of many written works over the centuries, not only as the the explanation of the term *analogia,* which merely states that when

$$\frac{a}{b} = \frac{b}{c} \quad \text{or} \quad \frac{c}{b} = \frac{b}{a}, \text{ then}$$

$$\frac{b}{a} = \frac{c}{b} \quad \text{or} \quad \frac{b}{c} = \frac{a}{b}$$

and the three quantities, as long as they are bound by b, will constantly provide two equal ratios and are so constituted that they form a unity. But immediately afterwards Plato says that it is only between two plane (2-dimensional) numbers that there can be a bond of the kind he mentions. Between 3-dimensional numbers there must be two bonds, and this was why god placed air and water between fire and earth in order to bind the two together.

"In this case therefore the terms 3-dimensional and 2-dimensional numbers must be regarded in their strictest sense to mean cube and square numbers, since between square numbers a^2 and c^2 there is the bond ac, while on the other hand between cube numbers a^3 and c^3 there are only the terms a^2c and ac^2.

"Thus

$$\frac{a^3}{a^2c} = \frac{ac^2}{c^3}$$

and even then A and C must be prime

numbers, otherwise Plato's assertion is unfounded, because between the square numbers a^2b and bc^2 there is the single bond abc, and the cube numbers a^3 and $(n^2a)^3$ are linked by the bond na^3.

"But if this strict interpretation is placed on the passage, 'For when three numbers .... etc.', which has hitherto been translated literally, 'or masses or forces' but which ought undoubtedly to be translated in accordance with the latest English versions as here, 'whether 3- or 2-dimensional', then one arrives at the conclusion no matter how the passage is translated that no such bond or link exists since the entire development requires whole numbers.

"Apparently Plato, whose mathematical calculations generally are not renowned for their clarity, has not given this much thought. This was probably because he confused two ideas. One, the simple double ratio $6/2 = 18/6$ (*analogia* in the sense in which Euclid used the word), and two, the idea of the bond that best links itself with the two factors it must unite (*analogia* in the sense that Pythagoreans and Vitruvius used the word) namely, the irrational ratio of the *golden section* $\frac{a}{b} = \frac{b}{a+b}$.

"It is quite normal in the above passage from *Timaeus* to include praise for the beauty of the latter ratio, a Pythagorean throwback. For despite the efforts of Plato and subsequent mathematicians to oppose the Pythagorean number mysticism on purely philosophical grounds, it was inevitable that many of Pythagoras's ideas and expressions should reappear often unconsciously in the work of those who followed.

"Even in the work of Euclid, whose ideas are perfectly accurate and shorn of all mysticism, the *golden section* (or as he terms it, the division of a line in extreme and mean ratio) occupies a very important place. He solves the problem as early as Book II, Prop. 11."

It should be mentioned that in his version of Plato's writing, Marstrand uses in one vital part a different translation from the generally recognised one. The latter states:

> "For whenever you have three cube or square numbers with a middle term such that . . . ."

Marstrand translates this:

> "For when three numbers whether 3- or 2-dimensional are such that . . . ."

The difference is distinct. The first excerpt relates quite clearly to cubes or squares, while Marstrand takes a wider view: that it should be 3-dimensional and 2-dimensional factors. Theoretically the translations are identical, since one might hold that "3-dimensional" and "cube" carry the same significance, while "2-dimensional" (or plane) might be taken to mean "square".

But I must admit to trusting more to the first and more recognised translation on the grounds that Plato—who is exceptionally concise and exact in his statements—would never refer to numbers as 3- and 2-dimensional. To Plato numbers were, as for us today, an accurate gauge by which to measure something, but that gauge could never be 3- or 2-dimensional. I feel the Marstrand version lends an unnecessary vagueness to the meaning of the passage.

And I further believe—as stated earlier—that both translations are incorrect in as much as the passage ought in fact to read:

> *"For whenever you have three cubes or squares with a middle term such that . . . ."*

This almost negligible amendment throws the meaning of the passage into sharp relief: Plato is discussing the re-

lation between three cubes or three squares.

After his attempt to translate the passage Marstrand goes on in a bid to fit it to the background of the *golden section*—but fails. He drifts further and further from the real heart of the matter, and finally confesses:

"One arrives at the conclusion no matter how the passage is translated that no such bond or link exists since the entire development requires whole numbers. Apparently Plato, whose mathematical calculations generally are not renowned for their clarity, has not given this much thought. This was probably because he confused two ideas . . . ."

On one point there can be no confusion: Wilh. Marstrand's abilities as a mathematician are beyond question. But his failure to fit Plato's *Timaeus* into the framework of the *golden section* is due quite simply to the fact that Plato was not discussing this. Marstrand therefore could never have succeeded.

He began his attempt on apparently certain ground, and the reader expected him to be able to illustrate the connection between Plato's writing and the *golden section*—but after a strong assault on the text, he drifts into choppy water and reaches the conclusion that Plato has forgotten to take certain factors into consideration and has confused two issues.

Marstrand in fact proves very well that Plato's *Timaeus* had nothing to do with the *golden section,* and his final comment is thus inconsequential:

"It is quite normal in the above passage from *Timaeus* to include praise for the beauty of . . . . the *golden section's* . . . . ratio, a Pythagorean throwback."

Here he persists in saying that Plato was discoursing on the *golden section*—despite the fact that he himself has just proved otherwise.

Marstrand has applied his logic and mathematical knowledge to the task and has assumed from the outset that a solution was possible and proof at hand.

Naturally he had combed through every available page of relevant literature for proof without luck, and—unable to uncover anything that satisfied his mathematical mind—he had confidently started on the job of solving the puzzle.

As he worked he began to realise that he could not match the text to the properties of the *golden section,* and as he laboured further and further from the point so the chances of success decreased.

In the end he was certain that it must have been Plato who made a mistake.

He has in other words identified the *golden section* so intimately in his mind with Plato's writings that when the two fail to unite it must be Plato's fault. Marstrand ignores the possibility that popular theory may be wrong, and that Plato may be speaking of something else entirely.

To complete our study—although the circumstances were related in full in Chapter Ten—I shall repeat the theory which Plato discussed. We shall be referring to it again in our analysis of Euclid.

In *Fig. 255* we see three squares within each other, sharing the same base. Square A is the smallest, B the intermediate, and C the largest.

Square A has a side-length of 5, and square B was constructed with as its side-length the diagonal from square A, i.e. a purely geometric construction. The side-length of square B measures therefore $5 \times \sqrt{2} = 7.07$.

The side-length of square C is constructed from the diagonal of square B, $7.07 \times \sqrt{2} = 10$.

Thus in area

square A is $5 \times 5 = 25$
square B is $7.07 \times 7.07 = 50$
square C is $10 \times 10 = 100$

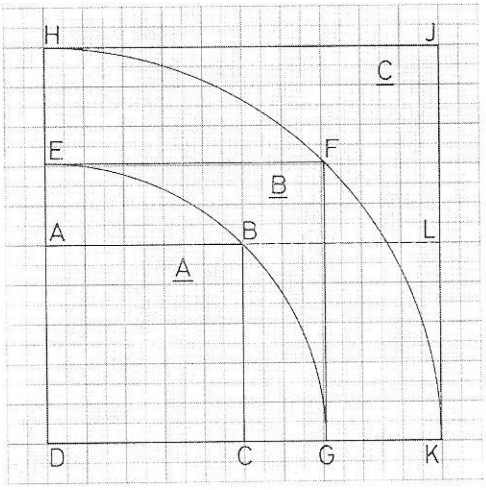


Fig. 255.

Therefore from A to B and from B to C, the squares double in area.

Plato says of this diagram:

"For whenever you have three cubes or squares (numbers) with a middle term such that the first term is to it as it is to the third term, and conversely what the third term is to the mean the mean is to the first term . . . ."

The first term is square A, which is to the mean term (square B) as 1:2. The mean term square B is to the third term (square C) as 1:2. And conversely, the third term is to the mean as 2:1, and the mean is to the first term as 2:1.

So on this score Plato's wording is illustrated plainly and simply. The sentence is no longer a mystery when it is linked with the factors discussed by the writer.

The *golden section* is concerned with the division of a given line "in extreme and mean ratio", but Plato has never mentioned any line. He states quite openly that his subject matter is squares or cubes.

Plato's text therefore deals with the doubling in area of the square, and as this construction contains and involves the *sacred cut,* it is this (and not the *golden section*) he refers to.

It is a fundamental error to identify *Timaeus* with the *golden section.* The latter belongs to a later period.

The following extract is from *The Thirteen Books of Euclid's Elements,* 2nd edition, translated from the text of Heiberg, by Sir Thomas L. Heath, Dover Publications, New York, 1956. It is Euclid's Book II, Prop. 11:

"To cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

"Let AB (in *Fig. 256*) be the given straight line; thus it is required to cut AB so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

"For let the square ABDC be described on AB; let AC be bisected at the point E, and let BE be joined; let CA be drawn through to F, and let EF be made equal to BE; let the square FH be described on AF, and let GH be drawn through to J.

"I say that AB has been cut at H so as to make the rectangle contained by AB,BH equal to the square on AH.

"For since the straight line AC has been bisected at E, and FA is added to it, the rectangle contained by CF,FA together with the square on AE is equal to the square on EB. But the squares on BA,AE are equal to the square on EB, for the angle at A is right; therefore the rectangle CF,FA together with the square on AE is equal to the squares on BA,AE.

"Let the square on AE be subtracted from each; therefore the rectangle CF,FA which remains is equal to the square on AB. Now the rectangle CF,FA is FJ, for AF is equal to FG; and the square on AB is AD; therefore FJ is equal to AD. Let AJ be subtracted from each, therefore FH which remains is equal to HD; and FH is the square on AH; therefore the rect-

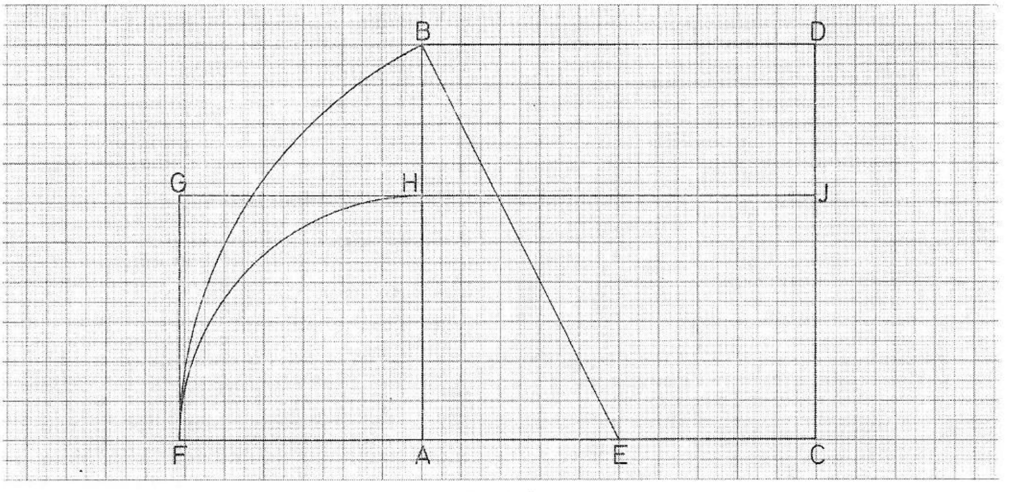


Fig. 256.

angle contained by AB,BH is equal to the square on HA.

"Therefore the given straight line AB has been cut at H so as to make the rectangle contained by AB,BH equal to the square on HA."

The extract from Euclid is accompanied by the geometric construction which illustrates the text.

The vertical line AB is the point of origin, the line we must divide in extreme and mean ratio.

Guided by the text we construct a square on AB, bisect AC at E, join EB and produce EA to F, then construct a square AF, etc.

After completing the construction, Euclid states:

> "I say that AB has been cut at H so as to make the rectangle contained by AB,BH equal to the square on AH."

He follows this assertion with a proof as to its veracity. We shall not go over the proof again as we can take a more advanced path.

Let us first see what rectangle he means by AB,BH. For there is no such construction in the diagram.

Line AB is the side of a square and BH is part of that side. The intention must be that line BH should be imagined as a continuation of DB, forming a rectangle with BH as the short side and AB as the long side. But as AB and DB are equal, we already have such a rectangle in the diagram and need not construct another.

Thus Euclid's assertion is that rectangle BDJH equals square AFGH, and we can examine this with the aid of arithmetic.

Since line EB is a part of the construction and forms the hypotenuse in a right-angled triangle whose perpendiculars (AB and AE) are in the ratio 2:1, we can make AE equal to 1.

$$
\begin{aligned}
\text{Thus } AB &= 2 \\
EB &= \sqrt{5} \\
EF &= \sqrt{5} \\
AF &= \sqrt{5} - 1
\end{aligned}
$$

As AF is one side of a square, it follows that the square's other sides are also $\sqrt{5} - 1$, which we can therefore say of AH.

Line BH $= 2-(\sqrt{5}-1)$, since AB $= 2$. Finally BD $=$ AB $= 2$.

Thus rectangle BDJH is HB $\times$ BD $=$

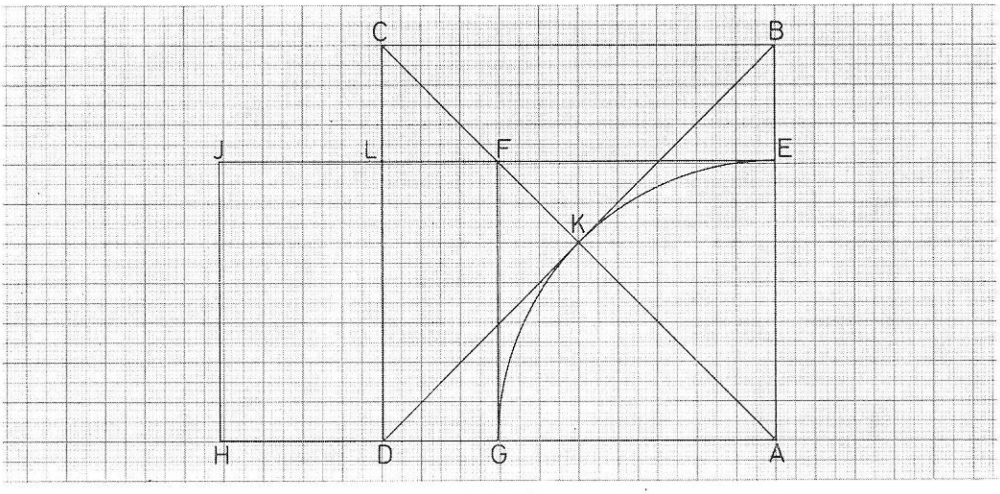


Fig. 257.

$2 - (\sqrt{5} - 1) \times 2 = (2 - 1.236) \times 2 = 1.528.$

And square AFGH $= AF^2 = (\sqrt{5} - 1)^2 = (1.236)^2 = 1.528.$

We have proved numerically that Euclid's assertion is correct, and we can accept his conclusion:

> "Therefore the given straight line AB has been cut at H so as to make the rectangle contained by AB,BH equal to the square on HA."

The object was obviously to divide the line in a particular ratio, and we can assume this to be the *golden section*.

We saw in our study of Marstrand that the formula for finding the *golden section* was $\frac{\sqrt{5} - 1}{2}$ provided we call the line we are to divide "1 unit".

In this case the line in question is 2, i.e. line AB, and the formula must therefore be $2 \times \frac{\sqrt{5} - 1}{2} = 1.236$. Line AH major $= 1.236$, line HB minor $= 0.764$. Here we have the ratio of the *golden section,* BH:HA as HA:BA

Whereas a search through Plato's *Timaeus* proved fruitless, we find here in the writings of Euclid the ratio that has been called the *golden section.*

But Euclid does not offer the ratio as a work of art or a thing of special importance. He gives it no ringing title. He says simply: "To cut a given straight line . . . ."

There is however a very interesting facet of Euclid's geometric construction which he fails completely to mention: rectangle FGJC is equal in area to square ABDC, since

the rectangle's long side CF $= 1 + \sqrt{5}$
since CE $= 1$, and EF $= \sqrt{5}$
the rectangle's short side FG $= \sqrt{5} - 1$
Thus: $(\sqrt{5} - 1) \times (1 + \sqrt{5}) = 3.9966$
And the square is $DB^2 = (AE \times 2)^2 = (1 \times 2)^2 = 4$

Euclid has thus succeeded in constructing a square and a rectangle of the same area.

We find the same thing in effect with the term I have called the *sacred cut.*

We see in *Fig. 257* a construction recognised from an earlier chapter. It contains a square and a rectangle with the same area.

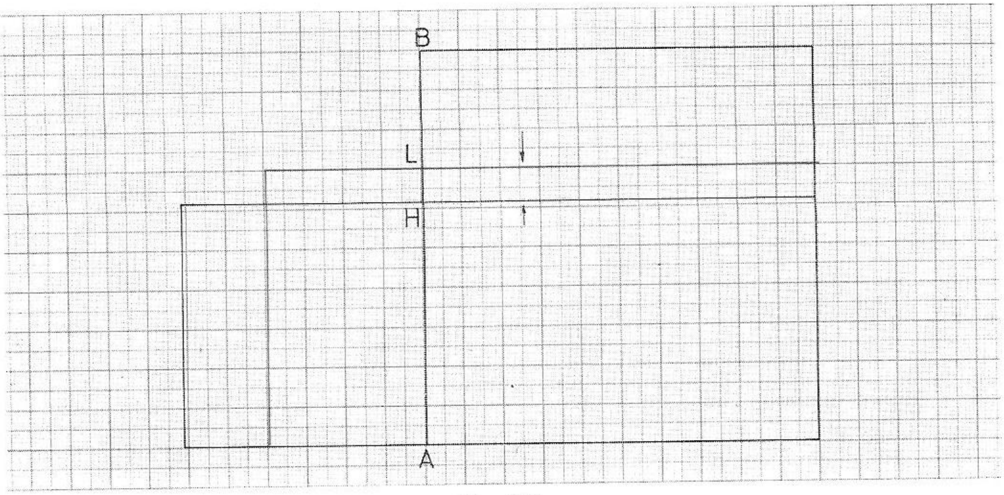


Fig. 258.

The square is ABCD. Half the length (AK) of its diagonal is taken as the side of the half-size square, which is seen as AEFG. G is the *sacred cut* in line AD and E the *sacred cut* in AB.

If we double the area of the half-size square, we naturally arrive at an area equal to the large square.

We do this by producing AG to H so that AG equals GH. EF is produced to J in the same way. This provides us with a rectangle AEJH which equals in area square ABCD, and the horizontal *sacred cut* in the square is seen as the rectangle side EFJ (cutting CD at L).

Comparing this diagram with the preceding, we can immediately appreciate an undeniable similarity: the principal features of both are a large square and a large rectangle.

In each the square and rectangle are equal in area but in Euclid's diagram the rectangle is somewhat longer and lower.

This slight difference in proportions gives the respective rectangles separate intersections with the side of the square.

In *Fig. 258* we find both constructions in the same diagram.

According to Euclid's statement and diagram, line AB is divided at point H, this being the *golden section*.

By my theory AB is divided at L, which is the *sacred cut*. As stated, all three figures, two rectangles and one square, are identical in area.

There is obviously a similarity in the manner in which the two diagrams are constructed but because of the different shapes of the two rectangles, their intersections with the square do not coincide.

The next time Euclid turns to the problem of dividing a straight line is in one of the definitions to Book VI:

> "A straight line is said to have been *cut in extreme and mean ratio* when, as the whole line is to the greater, so is the greater to the less."

In Proposition 30 of the same book he goes more thoroughly into the matter:

> "To cut a given finite straight line in extreme and mean ratio."

The problem he discusses here is closely related to that in Book II, Prop. 11, but the proof is not nearly so complicated in execution. Moreover, it is assumed that the reader is familiar with the con-

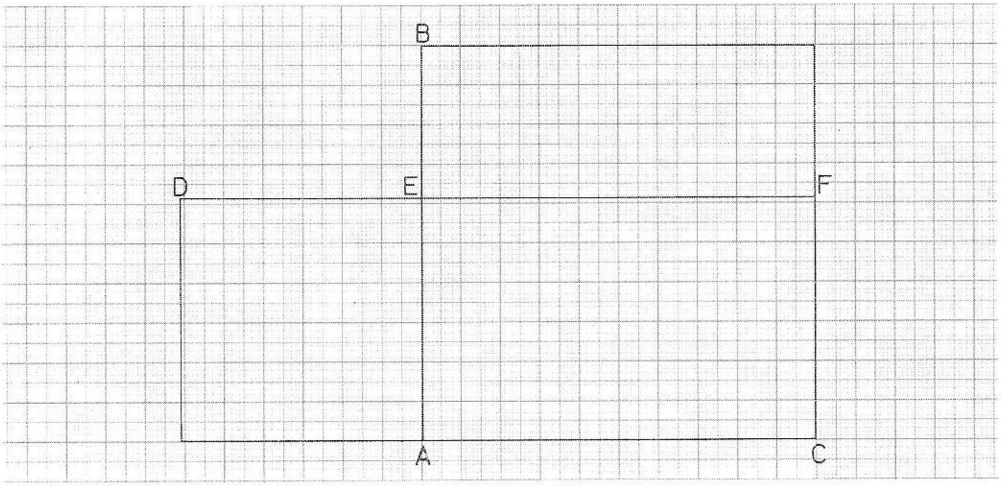


Fig. 259.

struction of the rectangle. But at the same time it is peculiar to record that Euclid mentions as an accepted fact that the square and the rectangle are equal in area:

> "Let AB be the given finite straight line; thus it is required to cut AB in extreme and mean ratio. On AB let the square BC be described; and let there be applied to AC the parallelogram CD equal to BC and exceeding by the figure AD similar to BC."

We note that the entire directive for constructing the rectangle (which in this case is called a parallelogram) has been omitted, when he states simply that parallelogram CD is equal in area to square BC, *Fig. 259*.

Again Euclid fails to emphasise any special feature of this problem, and gives it no particular name, concluding:

> "Therefore the straight line AB has been cut in extreme and mean ratio at E, and the greater segment of it is AE."

His third attack on the same problem is delayed until Book XIII, Prop. 5.

He constructs the same diagram as before but adds slightly to it by first having a straight line cut in extreme and mean ratio, and then reproducing the same ratio on a larger scale by adding another line to the original, which then becomes part of a longer line:

> "If a straight line be cut in extreme and mean ratio, and there be added to it a straight line equal to the greater segment, the whole straight line has been cut in extreme and mean ratio, and the original straight line is the greater segment."

Euclid—almost at the close of his long series of books—finally examines the repetitive element in the problem, showing how to cut the original line, add an extra length, and still retain the same ratio.

This brings one to think of the doubling in area of the square, where one square always produces the *sacred cut* in the next.

Euclid's construction is shown in *Fig. 260* in which the diagram is basically the same as before although employed slightly differently.

We recognise line AX as that which in the earlier diagram was cut in extreme and mean ratio (AB). By joining AE and entering CY, he succeeds in dividing BA

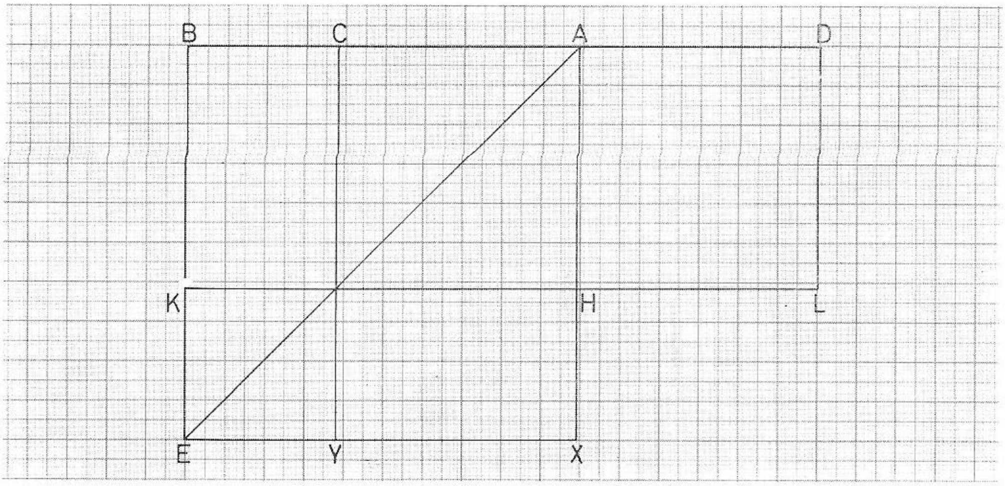


Fig. 260.

in extreme and mean ratio at point C, since BC equals HX and BA equals AX.

Squares CH and AL are equal in area, and Euclid states that BD is divided in extreme and mean ratio at A.

We can test his statement arithmetically, since we know the values for the various lines from earlier reckoning. Line BD was $\sqrt{5} + 1 = 3.236$, BA $= 2$, and AD $= \sqrt{5} - 1 = 1.236$.

Thus:

$$\frac{BA}{AD} = \frac{BD}{BA} = \frac{2}{1.236} = \frac{3.236}{2} = 1.618$$

Thus the ratios are identical, and Euclid's assertion and expanded construction are correct.

In this proposition Euclid demonstrates that if we take a straight line and, by the method he has shown, cut it in extreme and mean ratio, we have then the basis of a continuous series of divisions on the same principle.

Line BD is cut major and minor at point A. Major BA is again cut major and minor at point C. BD is thus cut in extreme and mean ratio at point A, and the larger of the two segments is cut similarly at C.

We could go on reducing the lengths of the segments in this way, always in the same ratio, and similarly we could multiply upwards taking each time the whole line as the major segment.

We have precisely the same repetitive picture with the *sacred cut,* where the mutual relationship of the squares produces a constant ratio.

If we turn again to *Fig. 255* and examine the well-known set-up of squares, A, B and C, we see that A is to B in the ratio 1:2, and B is to C in the ratio 1:2.

Square A is marked ABCD, with base-line DC. Square B is marked by the corners DEFG and base-line DG.

We may observe an interesting similarity between this diagram and Euclid's proposition, if we ignore for a moment one or two lines in square C. If we study square B (DEFG) we observe that its rectangle is ALKD.

We saw these two figures, too, in Euclid's various diagrams. Although his rectangle was longer and slimmer, the ratio of square:rectangle was 1:1.

The same progression is observed clearly in the diagram with the *sacred cut.* The base-line of square B, DG, is divided by

the *sacred cut* at point C by the base-line of square A, DC.

Thus DC is major, CG minor and DG major + minor.

If we move forward to square C, then line DG becomes major, GK minor and DK is major + minor.

By means of the *sacred cut* we can move on to larger and larger squares, a particular segment starting off as major + minor, and becoming major only. And we can work our way down through smaller squares as far as numbers permit.

This is a similar progression/retrogression process to Euclid's geometric construction, and lends support to the link between Euclid's postulate concerning extreme and mean ratio and the ratio of the *sacred cut*.

The results are not the same in bare numbers, but the basis of construction of the square/rectangle combination is the same, and so is the progressive series.

The two problems/theories appear very much to be of the same type, so close indeed that it ought to be possible to discover their association.

All the way through these two volumes we have seen the *sacred cut* employed on many occasions and in countless ways, not only as a speculative factor in solving the problem of the circle's circumference, and not only as a theoretical factor in speculation on geometry's origin. We have seen more. We saw it in practice from the Great Pyramid to Moses' Tabernacle, from classical temples to buildings of the Middle Ages. Yes, it was the thread that united the designs of all those buildings we examined—and not to a degree of "rough approximation", but to an amazing degree of accuracy.

There is therefore no need here to prove or illustrate the existence of the concept, or its application and significance both to esoteric geometry and to practical use.

But the *golden section?* It has a glittering title and mystic reputation—but what else?

Marstrand wrote a fine work on ancient building practice and, as we have seen, gave a detailed account of the *golden section* at the outset of his book—after which its presence or application were not demonstrated or revealed in one single instance. In the same way, Macody Lund touches on the problem of the *golden section* in the introductory remarks to his book *Ad Quadratum*. But his analyses omit any reference to the idea.

He works at one point with divisions and properties of the pentagon and decagon, the symbolic significance being that the former contains the properties of the *golden section*. But a direct proof or sample of its existence is not offered.

He is perfectly correct in suggesting that a square should be drawn about the facade in order to plan its design, but his squares are drawn by chance and have no specific link with the rest of the building's dimensions.

My analyses on the other hand show that the choice of facade square is paramount to the temple's design. It is tied up distinctly and irrevocably with every other dimension in the building.

I do not intend to dig deeply into the analyses contained in Macody Lund's otherwise excellent work, but shall quote from a set of notes on drawings held by the Academy of Art (Kunstakademiet) in Copenhagen. The drawings were made by Buus Jensen, the Danish architect, in 1922 when he was dispatched by the Academy to test Lund's theories on the Temple of Poseidon in Paestum, southern Italy.

On his return he wrote, among other things:

"The red constructions are an attempt to assimilate with actual examples the theories contained in Macody Lund's book *Ad Quadratum* on the general planning of Doric temples, but the basic outlines

differed so greatly that further investigation was pointless.

"The blue constructions show an attempt to test the theories on the Parthenon.

"The deviations are so pronounced that further study would probably be worthless."

Of the facade drawings Buus Jensen wrote:

"The theories held by Macody Lund and detailed in his book were tested on an accurately measured survey of the Temple of Poseidon in Paestum.

"The result was as follows:

Point 1 differs by 70 cm
Point 2 differs by 40 cm
Point 3 differs by 70 cm
Point 4 differs by 20 cm
Point 5 differs by 15 cm
Point 6 differs by 0 cm
Point 7 differs by 0 cm."

We see that Buus Jensen experienced some difficulty in matching theory with structure—doubtless because the theories were wrongly founded.

In every other context in which the *golden section* is mentioned we are served only with the statement that it existed within ancient building practice. The writers on the other hand, switch smartly across the table to the *section's* mathematical properties, with infinite fractions, etc. But nowhere—search as much as we wish—can we trace full geometric proof of its practical application.

If I can prove the use by ancient planners of the *sacred cut* and can illustrate its colossal weight as a factor in building, why does Euclid fail to mention it once in his thirteen books, why does he persist in discussing quantities and sizes that are a "rough approximation" of the real thing?

The postulates he offers are certainly accurate from a geometric point of view, but numerically they do not tally with the properties of the *sacred cut*. The procedure adopted for constructing a square and a rectangle with the same area is in effect the same in both cases, but Euclid's rectangle has a different shape than demanded by the *sacred cut*, and as such a more contrived figure whereas that of the *sacred cut* has a natural form.

The same to a certain extent is true of his postulate concerning the series in which major becomes minor in the next ratio. We saw the process earlier in the doubling in area of the square, and yet there is a basic difference. Why is his work just that little bit off target?

The answer perhaps lies in the surroundings in which Euclid worked. His background is interesting.

Roughly 300 years prior to Euclid's era, the man who first brought "popular" geometry from Egypt to Greece was Pythagoras. He set up his school of speculative geometry and initiated a number of young thinking Greeks in its secrets.

The learning was passed directly from teacher to pupil without the aid of books or documents. Education at Pythagoras's school was not free in the sense that his pupils were banned from telling friends or acquaintances of their new-found knowledge. Their learning was something religious, something sacred, something to be defended, protected. It was not the personal property of the individual. The pupil could speak openly on the subject among equal brethren—but in the presence of uninitiated he was silent. If, in the period of study, it was discovered that one or other of the pupils was "leaking" information outside the circle of brethren, the mildest punishment he could expect was expulsion from all further classes.

Study lasted several years, of course, and it became evident relatively early if any of the pupils broke his oath of silence.

Students in the upper and senior classes kept an eye on new pupils, and any irregularities observed or overheard were immediately reported to the school's master, who took appropriate action.

The system of secrecy worked well, therefore, as long as enforcement of law and penalty was strict.

Quite apart from the Pythagoras school of learning there were the usual, older established Temple schools at which initiation was considerably more involved and difficult, and the punishment for breaking the oath of silence much harder —leading often in fact to the risk to one's life.

The Norwegian writer, Scharabæus, who published *Mysteriesamfund,* Oslo, 1948, wrote:

"In 515 B.C. Alchibiades was accused of imitating the actions of the hierophant during the secret ceremonies, while he (Alchibiades) was under the influence of drink at a party. Neither his eloquence nor popularity could save him. As dictated by tradition, envoys from the Temple visited his home that evening and—with faces towards the setting sun—pronounced the death sentence on Alchibiades for blasphemy. He escaped with his life by fleeing the country during the night, but several of his friends were condemned to death and had their property confiscated.

"And Aristophanes tells of a price placed on the head of Diagoras of Metas for joking about the Mysteries.

"There are similar accounts of punishment meted out to offenders against the Greek Mystery temples."

If that was the fate of men who joked about the Temple, how much more serious would be the penalty for actually revealing its secrets?

One of her secrets was geometry. It was learned at the same source as Pythagoras visited, i.e. Egypt, but the Greek Temple had obtained the learning infinitely earlier than Pythagoras's era. And the information was held within a very tightly-knit religious brotherhood. Not even "outside" temple brethren were initiated in the occult mysteries of numbers.

The priests applied their geometric knowledge in planning and building temples, or as a part of their religious ritual, but few could hope to gain entry to the inner circle of mathematical priesthood. The select few had over a long period of years illustrated their reliability to keep lesser secrets. Their silence and oath were unbreakable.

Thanks to his popularity in certain parts of government Pythagoras made many enemies among prominent citizens of Greece—and equally ill-intending enemies within the Temple. The latter regarded his school as the thin end of the wedge, the beginning of public ownership of a subject that had remained the domain of the Temple for thousands of years.

Certainly, a pledge of silence was demanded of pupils to the Pythagorean school, but it could hardly be otherwise that selection was scarcely as strict or careful as for Temple training. Being subsidised partly by court and state, the school was well patronised and membership was sought after by the upper classes of citizenry.

Whereas the ancient Temple admitted to her ranks primarily men of mature years, Pythagoras opened the doors of his school to large groups of young students. One of his principal subjects was arithmetic combined with geometry. He considered this the best form of training in logical thought.

As numbers and geometric constructions were essential in later training as an explanatory factor in illustrating, for example, the then accepted picture of the world's origin, it is logical that Pythagoras should include mathematics. But natural,

too, that the Temple regarded his teaching as profanity.

If it had been the case that Pythagoras had received his training from the Greek Temple, the solution would have been simple: he would by some means or other simply have been forbidden to teach.

But this was an occasion on which the Temple was powerless.

According to ancient tradition, initiation in the higher degrees of learning was obtained—on recommendation—from the temples of Egypt. Anyone who had been admitted to and trained by the Egyptian Temple was supreme anywhere else in the world. He was part of the very Temple superstructure.

Imagine then the consternation in Grecian Temple circles when this Egyptian-trained superior arrived and began by apparently throwing the rule-book out of the window, teaching young men who had not even been given a proper initiation.

They appreciated that perhaps the real and basic secrets of Temple geometry were not revealed, but it was enough that this Pythagoras should have the blasphemous audacity to encourage non-initiates to occupy their minds with similar types of problems.

Intrigue on the part of the Temple and back-stabbing by certain citizens who saw their powerful positions threatened by the Pythagorean school had its desired effect. Pythagoras was forced to shut down his influential school, and flee the country at the age of 80 years. His teachings were forbidden. But the seed he had sown was too healthy, too strong to die.

Soon Pythagorean schools and societies were sprouting up in countries and states bordering Greece. They were set up and supported by faithful Pythagorean scholars, and within an amazingly short time had crept back over the border and become underground movements all over Greece. And a new aspect of secrecy developed. It was not only forbidden to be an associate of Pythagorean teaching; it was punishable by death.

About the year 300 B.C. (roughly 250 years after the death of Pythagoras) the situation regarding mathematics and its teaching was as follows.

The old Temple Mystery societies still existed, with tradition and ritual as strong as ever. They had staged a bit of a comeback after the closure of the Pythagorean brotherhood, and appeared once again to be monarchs of all they surveyed.

They were headed by priests, who virtually lived and died by secrecy. Within the priesthood were an even closer brotherhood of mathematical experts, with a full knowledge of and familiarity with ancient geometry's origin and application.

The Pythagorean school was totally wiped out of Greece by the time the old master died, but his followers had started similar schools and mystery societies outside Greece, and – as noted above – had worked their way back over the border again under different names.

One of the best-known of these latter schools was Plato's Academy. Without actually putting it in words, Plato was an avid follower of and believer in the work of Pythagoras. His Academy was inspired by the latter.

Those were the two groups, the Temple and the various Pythagoreans. They stemmed from the same source, i.e. Egypt, where they had collected their knowledge of ancient geometry. The former enjoyed the principle of secrecy as an age-old tradition, the latter had secrecy forced upon it in the fight for survival.

The ruler of Greece at this time was Ptolemy Philadelphus, a leader who well appreciated that education was vital to the nation's advancement at home and abroad.

But as the Temple schools were very much a "closed shop" and the others that

existed were partly in hiding and partly (such as Plato's Academy) too small to cope with the problem, Ptolemy set up a place of learning across the Mediterranean at Alexandria. It was a centre where wise men and students could mingle and discuss scientific and other matters for the benefit of public listeners.

This place was called the Museum.

Alexandria at this time was a city-state and as such came under the wing of the Greek empire. The city was rich and influential and naturally the Museum had to be the finest centre of learning in the world.

The building itself was impressive, with dozens of shady courtyards and cloisters where visitors, students and professors could gather in large or small groups to muse upon problems of the day.

It is said that the Museum was the most remarkable and most important institution of its kind in antiquity, and contributed towards the preservation of Greek literature and learning. It has also been called "a great university".

The library alone is reported to have contained 400,000 papyri obtained by Ptolemy's researchers from every corner of the learned world.

The Museum demanded no pledge of silence and no initiation or process of admission.

It represented the third school of geometric thought in Greece, but in spite of its extensive library and excellent tutors it failed to register the same progress or success as its two senior counterparts.

The old Temple would not dream of lifting a finger to help the Museum towards participation in their traditional knowledge. They were suspicious but, compared with their jealousy towards the Pythagorean school, saw no competitor in the Museum, knowing that the new institution had no access to the real ancient spring of learning, and knowing that the papyri would be unable to pass on any part of the Temple's ancient geometry. Geometry was at that time very much a verbal subject, with few written books or papyri to support the teacher. And the little that was in fact entrusted to print was written in such an ingenious manner that it told the non-initiated nothing.

And it is not at all improbable that Temple brethren, on account of their high standing and powerful influence in society, had actually been able to examine the papyri before releasing them to the Museum.

The Pythagoreans were equally guarded in their approach to the Museum and jealously retained their superiority where possible. As Pythagoras's disciples scattered to the four winds after his death, it is unlikely that more than a few were in possession of the full Pythagorean course of learning as regards geometric subjects. What was passed on may therefore have lost some of its Pythagorean character. This latter thought is however only conjecture on the part of the author. But it is accepted fact that, where possible, Pythagoreans made every effort to visit Egypt in order to supplement their basic level of learning.

That is how I see the geometric situation in Greece at the time (about 300 B.C.) of Euclid.

Euclid belonged undoubtedly to the third group of geometers, and his work-table was equally certainly the Museum at Alexandria. Little is known about his personality, few personal details have survived history.

It is quite remarkable that we should have full biographies on most leading Greeks, not only those of Euclid's era but from earlier centuries, but only a slim file on Euclid himself.

The explanation may very well be that the people who led the field of mathematical thought and speculation and his-

torians who recorded the day's events attached a minimum importance to Euclid's stature. It is generally accepted nowadays, for instance, that Euclid's personal and genuine original contributions to geometry were few.

The Dane, Professor H. G. Zeuten, wrote in a postscript to a Danish translation of *The Elements:*

"One should not be mistaken into assuming that Euclid created precise mathematics. It would be doing the subject a great injustice to underestimate it as the work of one man. Euclid has on the other hand the honour of having produced the building on which later work was conducted."

J. L. Heiberg, in his book *Den græske og den romerske litteraturs historie,* Copenhagen, 1920, wrote:

"Euclid included in *The Elements* the fundamental propositions of Pythagorean and (Plato-influenced) Athenian mathematicians.

"He produced little new material, with the exception of Book X which deals with the study of irrational sizes.

"The significance of *The Elements* lies in the supremely-knit system which is built up, proposition upon proposition, on a handful of axioms.

"The first six books deal with plane geometry, including (in Book V) the work of Eudoxus on proportion. Books VII to IX contain details of such Pythagorean arithmetic as is necessary to study irrationality in Book X. The last three books provide the groundwork on stereometry, which was fashioned for the first time into a regular system, and conclude with the construction of regular polyedra.

"In other preserved examples of Euclid's work we see that he used for educational purposes the mathematical basis of optics, astronomy and music. Among those copies of his work which have been lost is a book on the elements of the geometry of conic section.

"Whereas Euclid therefore was principally a collector and arranger of the original work of others, Archimedes was on the other hand . . . ."

And in his *Story of Civilisation* series the American, Will Durant, wrote:

"All we know of his (Euclid's) life is . . . . that he was a man of great modesty and kindliness; and that when, about 300, he wrote his famous *Elements,* it never occurred to him to credit the various propositions to their discoverers, because he made no pretense at doing more than to bring together in logical order the geometrical knowledge of the Greeks. Books I and II summarize the geometrical work of Pythagoras; Book III, Hippocrates of Chios; Book V, Eudoxus; Books IV, VI, XI and XII, the later Pythagorean and Athenian geometricians. Book VII-X deal with higher mathematics."

These are three representative comments that illustrate the opinion that Euclid collected as much geometric and mathematical material as he could lay hands upon, and arranged it in the form known as *The Elements.*

The French researcher, Edouard Schuré, wrote of Euclid:

"He was the son of a merchant from Miletus and for a time was employed in his father's business. The son demonstrated a fine business sense and produced many revolutionary ideas.

"One of these was to publish documents on mathematical subjects, since he considered that these would be bound to fetch high prices in a Greece where the urge to know, to learn, to be educated was powerful.

"To procure first-hand knowledge of the material in question, Euclid gained admittance to a mystery temple. But he

never progressed beyond the level of a very junior brother because the Temple soon discovered his real intention, and excluded him from their midst.

"Undaunted, he continued with his task and collected as much mathematical material as possible. He derived considerable benefit from the documents which had been assembled at the Museum in Alexandria."

Whether this account is true in detail is impossible to establish since, as already noted, information on Euclid's background is rather sparse. But it fits admittedly very well into the picture one forms. In addition it explains perhaps to a lesser degree the almost complete failure on the part of Greek writers to mention Euclid's name. It is seldom that a publisher makes his name as an individual. Normally this falls to the author. The writer gets the public acclaim and recognition, the publisher puts up the money for printing. The Greeks would appear to have regarded Euclid as the publisher.

His object was to collect, weed out and tabulate every scrap of available material. But the question remains: Was the *available* material suffice to reveal completely the heart of Greek geometry?

Euclid belonged to the third and latest school of thought. He did not have the knowledge and tradition of the Temple for support, nor did he have the benefit of Pythagorean experience. He had to content himself with what little had seeped through the wall of secrecy that surrounded both groups.

Speculative mathematics was a new subject and, as such, unfettered by the occult. Arithmetic, stereometry and many other subjects were emerging in their own rights, and were matters in which anyone could join and add their contribution to public debate.

But geometry, the study of the *sacred cut,* the study of the circle's circumference and area, and associated problems, this was still the sphere in which the Temple ruled supreme.

We see that Euclid in his 13 books of *The Elements* has written not a word of speculation on the area of the circle. Not that he fails to make use of the circle. He does in many instances. But not once does he offer a theory or thought on the circle's most vital statistics: circumference and area.

Of course, he had tried to pinpoint concrete factors on the circle, but unsuccessfully. He does however describe a number of its other facets:

Book IV,
- Prop. 6: In a given circle to inscribe a square.
- Prop. 7: About a given circle to circumscribe a square.
- Prop. 8: In a given square to inscribe a circle.
- Prop. 9: About a given square to circumscribe a circle.

We see him here wander around the problem in four propositions, without reaching a conclusive solution. And these are not the only instances in which he employs the circle.

He does so in Books I and II, Props. 1, 2, 3, 12 and 22.

In Book III, Props. 1 to 36 inclusive. In Book IV, Props. 1 to 16 inclusive and 33. In Book XIII, Props. 7, 8 and 18.

His first five books comprise 158 propositions, and in these he uses the circle 58 times in his geometric constructions without once delving deeper than the surface into the circle's properties.

This was Zeuthen's view on the phenomenon:

"*The Elements* .... lack certain factors now considered part of elementary geometry, i.e. calculation of and propositions

in respect of the surface and volume of the sphere. Despite the fact that the Greeks, just as the Egyptians before them, were acquaint with approximate values for the circle's area and circumference, it is part of the plan of Euclid that this should be omitted—precisely because they *were* approximations and therefore not exact and accurate enough for inclusion in his work."

Zeuthen's personal opinion is that Euclid refrained on idealistic grounds from clarifying the mystery of the circle's dimensions because existing calculations were not well enough defined.

Is this not shouldering Euclid with just a little too much ideal? How accurate can one be about the circle?

If Euclid was aware, as Zeuthen suggests, that an inaccuracy was involved, then his attitude must have been influenced by the power or inability of proof to clarify the situation.

If Euclid was in a position, from a geometric construction, to discover a discrepancy in value, it must be inferred that he possessed two points or values between which to place the inaccuracy. If he could illustrate that the error lay between a plus and a minus value, then he should in turn have been able to formulate the error and produce a picture of either the circumference or the area.

And if able to illustrate the extent of the error, he should have been able to compensate for it. In other words, even with an approximate value he should have been able to put forward an account of the known facts and situation——if indeed he knew the approximate value.

It is much more likely, I believe, that Euclid's "oversight" of the circle's dimensions throughout his entire collection of 13 books was due to his realisation that the problem had already been solved by Temple geometricians. He could not very well produce an answer that was less perfect than that already in existence. He would have been a laughing stock in mathematical circles (no pun intended).

Euclid was probably acquaint with the fact that Temple builders employed the circle's area and circumference in their plans, but he did not know how. He was possibly aware, too, that a concept existed with the grand name of "mother of all harmony" or something to that effect, but did not know the details.

I think therefore that Professor Zeuthen is wrong in implying that Euclid's 13 books contain the full mathematical and geometric knowledge possessed by Greece at the time of Euclid.

Euclid wrote of all the material that was accessible at the time, but this does not exclude the possibility that certain information and knowledge escaped him because it was the very substance of Temple mystery and ritual.

Euclid had searched high and low, through jar upon jar and pile upon pile of papyri to get his hands on the inner secrets of the Temple—but met with a solid wall of bland silence.

He had soaked up everything possible from the Museum's libraries, but at the same time had been aware that the Temple and the Pythagoreans held some vital links of information just beyond his reach.

In Chapter Ten we saw that Euclid was familiar with the existence of the five-pointed star, a symbolic figure used as a sign of recognition by all initiated Pythagoreans.

He and his fellow scholars at the Museum suspected that this particular star concealed a geometric factor, and examined it in great detail.

He first approaches the problem in:

Book V,

Prop. 11: In a given circle to inscribe an equilateral and equiangular pentagon.

Prop. 12: About a given circle to circumscribe an equilateral and equiangular pentagon.

Prop. 13: In a given pentagon, which is equilateral and equiangular, to inscribe a circle.

Prop. 14: About a given pentagon, which is equilateral and equiangular, to circumscribe a circle.

Book XII,

Prop. 1: Similar polygons inscribed in circles are to one another as the squares on the diameters.

Book XIII,

Prop. 7: If three angles of an equilateral pentagon, taken either in order or not in order, be equal, the pentagon will be equiangular.

Prop. 8: If in an equilateral and equiangular pentagon straight lines subtend two angles taken in order, they cut one another in extreme and mean ratio, and their greater segments are equal to the side of the pentagon.

Prop. 10: If an equilateral pentagon be inscribed in a circle, the square on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.

Prop. 11: If in a circle which has its diameter rational an equilateral pentagon be inscribed, the side of the pentagon is the irrational straight line called minor.

Prop. 18: (Final lemma and comment in the complete series of 13 books)
That the angle of the equilateral and equiangular pentagon is a right angle and a fifth we must prove thus . . . .

We see here Euclid's deep (almost obsessive) interest in the pentacle. He occupies 10 propositions with its investigation, the investigation of a geometric construction which in effect has no special characteristic to distinguish it from other polygons and stars.

The obvious assumption (and cause of interest) by Euclid was that the Pythagorean five-pointed star was of regular design, and that a regular pentagon would result when the tips of the five arms were united.

Euclid's attempt to unearth the pentacle's secret was probably the essence of the many similar attempts by other mathematicians, but, as we saw in Chapter Ten, one of the star's secrets is that it is not regular in form but irregular and incorporates the *sacred cut,* the acute-angled triangle and the square, i.e. the main (apart from the circle) ingredients of ancient geometry.

The story is similar with the *golden section.*

It was gleaned by an outside agent, or perhaps related by a mathematical traitor, that one of the great secrets of ancient geometry in Temple and Pythagorean society comprised the construction of a square and a rectangle with the same area.

The intersection of the square's vertical sides by the rectangle's horizontal side was one of the keys to the geometric system.

The information passed on by the spy was correct, but the construction then produced by Euclid and his colleagues in Alexandria was inaccurate.

As we saw, the rectangle they constructed has the same area as the square but is longer and narrower than the rectangle

treasured by true ancient geometers. The outside investigators discover the peculiar fact that A is in the same ratio to B as B to A + B.

This ratio is at once correct and a geometric curiosity—but there are an infinite number of such curios. None of them is significant to ancient geometry. There is only one *sacred cut*. It alone (and its ratio) is found in the design of every structure erected by Temple builders. At the same time it was the key that unlocked the mystery of calculating the circle's area and circumference. In fact, the main-door key to ancient geometry.

Euclid spent time and trouble on cutting a given line in extreme and mean ratio, and immense experimentation must have preceded his conclusions. He must have tried all types of rectangles. But the division of the straight line was emphasised as the "answer" to his various theories and experiments.

Normally Euclidean geometry is regarded as the system prevalent in Pythagorean societies, but this must by my reasoning be an erroneous conclusion.

Plato discusses the *sacred cut,* but Euclid does not. But Plato does so in a manner that must be understood in order to appreciate its latent meaning.

Euclid and his friends tried to discover the *sacred cut,* and this was precisely why Plato's dialogue was veiled in confusing double-terms.

Euclid was able to lay his hands on several verbal truths, but could not match them with numbers or quantities. In his effort to convert words into unknown numbers he created a new concept, to which he gave no name but to which he returned again and again. This continued bombardment by Euclid of the problem, and his interest in extreme and mean ratio persuaded observers in later centuries to believe he had discovered a special geometric factor. And if they also believed, through sheer incomprehension, that Plato is discussing the same factor, then its significance is (wrongly) emphasised still further.

I believe the *golden section* is a geometric ratio arrived at by Euclid and his followers in their unsuccessful search for the *sacred cut.* The *golden section* bears no relation to the geometric system that emerged and operated in earliest history, nor has it any connection whatever with the practical application of geometry, i.e. building and planning.

That is why it is not surprising that its existence in ancient buildings has been difficult to prove. It is not there. If it appears to be worked into the odd building here and there, it is thanks to coincidence, which proves nothing.

When broadcast, the concept of the *golden section* mushroomed so rapidly in the hungry public mind that it established itself as a reality without in fact having the necessary solidity. Attempts to illustrate it in building design are forced into numbers and calculations on cutting a line in extreme and mean ratio. That is the only real factor about the *golden section.* To demonstrate its existence in building is impossible, so researchers confine themselves to mathematical calculation, satisfy themselves that it exists—and pass on without practical proof.

But how can something like the *golden section* be accepted as a reality if in fact it has no such claim? Euclid's own theories on this particular aspect of proportion and ratio give no reason to identify them with any outstanding concept, nor to connect them with the art and sphere of building. Euclid never mentions building design. He sets out his postulate on the division of a straight line in extreme and mean ratio, proves the accuracy of his statement—and says no more, apart from returning several times to drink at the same well. The statement differs slightly

each time, but retains its basic sameness —and simplicity.

No, it was not Euclid who boosted the qualities of the *section* or the *golden section*. This was done for him much later by historians and writers.

Theories on the application of the *golden section* are many, and range from being the main principle in ancient building design (even though this was never proved) to being the proportional unit on which the entire plant life is based. The latter theory is described by Franz Xavier Pfeifer in his work *Die Proportione des goldenen Schnittes an dem Blättern und Stängelen der Pflanzen,* Zeitschrift für Matematik und Naturwissenschaft, 1884.

Or the theory that the *golden section* represents the unit laid down by nature on which the human body is based, set out in Adelbert Goeringer's *Der goldene Schnitt (göttliche Proportion) und seine Beziehung zum Menschlichen Körper und anderen Dingen mit Zugrundelegung des goldene Schnitt,* Munich, 1911.

The literature of the Middle Ages contains several references to a beautiful proportion or ratio from which harmony itself was supposed to stem.

Names that have been associated with this concept include Perfection, Solomon's Seal, Solomon's Ring, Saulus Pythagoræ, the Harmony of Harmonies, etc. They are all terms for *The Mother of Harmony,* in other words geometric harmony.

In *Ad Quadratum* Lund tried long, hard and unsuccessfully to track down the origin of the *golden section:*

"The actual name *Golden Section* was first mentioned in literature towards the end of the Middle Ages by Campanus's Italian translator, Luca Paccioli, in his *De Divina Proportione,* Venice, 1509. But even Euclid himself foresaw prominence for this ingenious division of the straight line. It formed the central element in his entire geometric thought process. Euclid's Book XIII, Prop. 6 is reproduced by Campanus as follows:

"'Directa linea rationalis extrema et media ratione secta fuerit utrumque segmentorum irrationale est appellaturque apotome.'"

This latter quotation is translated by Heath from Heiberg's version:

"If a rational straight line be cut in extreme and mean ratio, each of the segments is the irrational straight line called apotome."

And in *Der goldene Schnitt* Pfeiffer's comment was:

"Bei Euklides für die Proportion selbst in abstracte noch keine besondere benennung sich findet, sondern bloss für die nach solcher Proportion geteilte Linie, also für das concrete Substrat der Proportion."

At some early stage these and all such literary references were associated with Euclid's speculations simply because nothing else in the field of mathematical theory seemed to fit the picture.

All written material that touched on the proper concept, the *sacred cut,* was so slanted that it could be understood and read only by an initiate. An outsider could not—as we have done—discover the thread with which Plato wove his tale.

Euclid's writing on the other hand on the so-called section or extreme/mean ratio cutting of a line is literally an open book. It is clear in its explanation, and simple to accept. It shows how to construct a special geometric arrangement which turns out to be something of a curio.

At some (perhaps very early) stage a non-initiate has connected the two: a beautiful ratio' and Euclid's proposition. At the same time scholars were unable to disprove Euclid's theory.

Macody Lund was unable to discover the term, *golden section,* in print before 1509. It would be an impossible task to trace the stage at which literary references to a harmonious factor in building were first linked with Euclid's speculation, but Lund at any rate identifies without question these references with Euclid's line-division.

It would appear to be beyond doubt that many of those writers and historians who hinted at a harmony of design in ancient buildings were familiar with or knew something of the actual situation, but the fact that they knew of it infers to a certain extent that they learned about the *sacred cut* within the Temple, and were therefore forbidden by their pledges of secrecy to reveal it openly.

Others had been aware that something or other on the lines of proportion and ratio had existed but did not have the full facts. Naturally they were unable to write of more than guesses and theories. But researchers of a later period found it difficult to distinguish between a man who wrote of a subject on which he was not allowed to speak freely and expertly, and a man who wrote on a subject on which he was unable and unqualified (because of ignorance) to speak freely and expertly. In literature it would be simple to confuse the two.

If the belief that Euclid discussed a special and beautiful ratio was established at an early date, it is more than likely that one group of writers referred to and spoke openly of the Euclidean concept as Perfection, while another group continued to glorify a concept which had not at that time been made public.

When Paccioli wrote in 1509 of the *golden section* he must have referred either to the term identified with Euclid, or to the still secret concept of harmony which we have called the *sacred cut* and which is identifiable with Plato.

The massive organisation of the Temple and its many departments (of which the building group was one) naturally attracted philosophers, writers, thinkers, mathematicians, scientists, etc., who bore either direct or indirect associations with the Temple itself.

These are the people among whom we must look for concealed references, veiled hints and involved "explanations" on the secret concept of harmony and perfection. Just as Plato wrote.

Plato applied it not only as a descriptive part of temple structure, but as an expression for the perfection of the universe, and few appear to have dared—as he—to write in such detail of esoteric subjects.

We spring forward in time now by about 1200—1500 years, into the Middle Ages. Man is trying to uncover the secrets of his earlier life by poring through surviving literature, books, documents.

Often the researchers come across references to a factor that seems to have formed the perfect harmony round which ancient art and buildings were designed, a key to classical art. But the key is described only in words and clues, never by direct concrete information.

Further digging on the subject brings the literary archaeologist to the work of Euclid. Admittedly, Euclid himself does not state that his propositions and postulates contain terms that were lauded for centuries in ancient literature, but the searchers think nevertheless that they can identify Euclid's theories with the proportional beauty referred to, perhaps because the thoroughly discussed propositions on the subject appear to have no other purpose than to allow the user to draw a pentagon inside a circle.

Added to this is the fact that examination of esoteric structures throughout the ancient world has brought to light the pentacle, or five-pointed star, carved into

the stonework. Five-sided figure, five-pointed star? Surely there is a connection. Obviously, Euclid was describing geometrically what literature only mentions mysteriously.

Later Plato's dialogue, *Timaeus,* is also identified with Euclid, or rather Euclid and Plato are assumed both to be discussing the properties of the newly-discovered and impressively-named *golden section.*

Difficulties in fitting Plato's text to the concept are overlooked, explained away by the suggestion that it is part of a special Pythagorean mysticism.

Euclid's concrete geometric constructions are generally approved to be "the real thing", and all future references in literature to harmony, perfection, proportion, etc., are immediately and unquestionably supposed to be the concept discussed by Euclid.

In a way, the reaction is a natural one and as such, understandable. Euclid provides a term that can be shown and proved mathematically, something that can be drawn geometrically, something simple enough to be appreciated.

Only one link is missing: illustration and proof that the *golden section* had anything whatever to do with harmony, that it was ever employed to harmonise building design.

The term has been worked in and accepted without the slightest proof apart from Euclid's geometry, and no proof has been offered since that the *golden section* in its original proportions (i.e. to within a reasonable degree of accuracy) has provided the theme for design of one single building far less *a whole series.*

A different story with the *sacred cut,* which has been shown and proved time and again to be the principal factor in the design of building after building.

My conclusion is that in the same way as his successors, Euclid sought the secret of harmony which even in his era was an ancient concept. It defied his search and that of later scholars. Only with this book has it been revealed in its entirety.

CHAPTER SIXTEEN

Ancient Geometry and Figurative Art

As far back as we can trace Man's history we discover him attempting time and again and increasingly to depict objects or events within his immediate vicinity. We think at once of the renowned cave paintings of southern France and Spain, or the old Nordic rock engravings, the best-known of which are perhaps those at Bohuslen in Sweden.

One can only guess as to the motive behind these works? Did they spring from a natural urge by Early Man to create pictures, likenesses, etc.? Or were they part of an ancient religion, bound in ritual? It is seldom that such early work is linked with recognised cultural eras in Man's history. It belongs to a much older epoch.

But it is not with these extremely early, primitive samples of human art that we shall concern ourselves. It is not these works of art in which we shall find evidence of ancient geometric symbols. Instead we shall look to the period during which construction work in accordance with ancient geometric principles can be illustrated: from early Egyptian times up to the Middle Ages.

There can be few "civilised" countries today without some form of museum or tangible association with the past. A store for ancient artistic and everyday items.

These museums house priceless treasures of art and history, not only from the country in which they are built, but also from neighbouring and distant countries. The numerous collections represent a large part of the riches and society of bygone civilisations. Each geographical area and era has its own distinct department.

Usually cultural treasures were recovered from their original site, often after long searches and periods of excavation. Over the past century or two major European nations have sent archaeological expeditions to the ancient centres of civilisation. The earliest birds collected the fattest cultural worms. The mass of material at the Louvre in Paris and the British Museum in London illustrates this point.

The material varies considerably from the tiniest, almost negligible, splinters of pottery to massive statues in marble, basalt or granite.

Not everything, of course, in a museum can be termed "art", if by art we mean something created for its own beauty. Most objects in a museum have earlier served another function: value as a practical tool, implement, weapon, means of transport, and so on.

This practical value was the primary reason for the object's creation. That the item also possessed an attractiveness (of proportion, style, fabric perhaps) which singled it out from among its fellows improved its value as a thing of beauty. But not as a thing of practicability.

In certain areas of the world native art (usually in the form of handicraft) was relatively highly developed at an early stage. Peasant families spent many long winter evenings making and carving useful objects for the home.

These objects frequently have an honesty and simplicity of form and function that assure them of a place as genuine *objets d'art*. They are exchanged for large sums of money because the trend is towards collection and appreciation of such things. More people than ever have time and money to spare in expressing their interest in art. But one need only look back over a very short period to see precisely the same simple, old objects being thrown from the home as trash and space-stealers. Artistic value of such objects is thus a question of whether one is able to appreciate the item for its own sake.

If we go a little further back in time—to Europe in the Middle Ages—we find the idea of art and artistic value was then too a relative concept. Not that beauty of form was not appreciated. It certainly was—among people who could afford the time and cash for it. But in those days art and handicraft were synonymous. Art was created by the skilful craftsman.

The clever carver was occupied with every type of carving work: on chairs, cupboards, doors, pillars, everywhere the opportunity presented itself.

Not all the work of these craftsmen could bear the label "art", but a handful of individuals rose head, shoulders and carving chisel above the average, and carried out great work. A few of the average masters, too, occasionally had inspired periods when their efforts were more successful than at other times.

But prime in his class usually was the craftsman who had practised and trained so well that the border between craftsman and artist simply fell away.

The earlier we turn our faces in history, the stronger is, I believe, the combination of craftsman-cum-artist. This is also based on the fact that the artist, in order to procure the practical abilities necessary for any form of art, must have a training.

There were no academies of art or private art colleges in ancient times. And yet we find innumerable examples of magnificent art, in such quantity and over such long periods that there at any rate can have been no shortage of artists or craftsmen.

The ability to create statues, reliefs, mosaics, paintings of outstanding quality naturally implied a prolonged prior training during which time of course the scholar had to live and eat. And there was also the student who failed his course as a stone- or wood-carver. He too had to be assured a comfortable living though his capabilities left much to be desired.

This is where the craftsman's brotherhood and organisation found its origin. There was lots of work in the trade for a carver even though he could not boast from birth the natural ability to create independently.

In the field of carving, for example, hours—perhaps days—of rough work had to be done on a log or chunk of timber after the motif had been decided. This was probably carried out by apprentices or journeymen who never progressed beyond this stage. When the preliminary shaping had been completed, along came the master carver to take care of the intricate and finishing details.

The same would have been the case with sculpture. There is a lot of chipping, chiselling and drilling on a piece of granite or marble before the master gets down to detail.

And who in those past times ordered and paid for the great majority of these works of art and craftsmanship? None other than the Temple and Church.

Most surviving material of this kind has

shown evidence in some way of a religious link or significance. Churches of the Middle Ages are frequently real-life museums of the craftsmanship of those days.

We are again under the wing, the control, the influence of the Church. The inner sanctum of craftsmanship.

To begin with, the Temple or Church was probably the only patron for this type of creative art, and it is perhaps correct to assume that craftsmen's organisations were affiliated to the Temple's powerful building brotherhood, likely a minor part of the training scheme.

But the training obtained by a carver or sculptor was something that could be practised independent of other construction work. It was more an individual work of creation than, for example, the effort of other brethren who worked in teams to raise a structure from foundation to roof.

As other groups of society developed an interest in art and found the wherewithal to satisfy their requirements (I am thinking here of royalty and aristocracy) craftsmen were perhaps "lent out" to execute work for people outside the Temple's immediate circle. Gradually more and more craftsmen found it more and more lucrative to foster this form of work and client than to labour under the Temple.

This tendency of freeing oneself of the harness of religious doctrine and Temple influence is a development that began late in the Middle Ages. Prior to this change of status, the tendency becomes weaker the earlier we progress backwards in time. Similarly it gains momentum the nearer we approach our own era.

The entire tradition of building "clans" was inherited from Temple predecessors, and was a concept that swept from country to country through successive stages of civilisation.

Part of the Temple's sacred and secret knowledge was the existence and application of ancient geometry, which was guarded jealously. And if, as seems most likely, the Temple and Church were behind the army of craftsmen who decorated the respective houses of god, then we must expect to discover with care some representation in creative and figurative art of ancient geometry. For just as we turn to our multiplication tables when we require to calculate a sum, so the architects of olden days turned for assistance to the diagrams of ancient geometry when they wished to determine the proportions and dimensions of an object they were designing.

In our search then we may come across two main groups of craftsmen specialising in creative and figurative art. One was based within the Temple and the latter's building clan. In the work of this group we should without fail be able to trace signs of ancient geometry, perhaps even more than anticipated. Whether the individual spent his entire life in the service of the Temple, or strayed from its ranks at some stage in his professional career, it is most probable that his work will bear the stamp of geometry.

The second group was made up of individuals who, without a formal training and driven purely by love of creation and natural ability, devoted themselves to the creation of art and artistic subjects. In the work of this second group we can trace no more of ancient geometry than the individual was able to glean from a study of the work of the first group. In this respect they stood at precisely the same stage as we stand now: prepared to study the work of the masters and to discover as much as possible of the motive and background for their art.

The late Middle Ages was the period of dissolution. The Church's extensive army of builders broke up relatively rapidly, and many former "soldiers" set themselves

up as independent craftsmen. But since the majority retained traces of tradition and formed protective secret societies, *guilds* came into being as a natural successor to the religious orders of craftsmen.

Ancient geometric rules of proportion and dimension (together with the trade's other secrets) were handed down from master to apprentice, and the latter were initiated in the more advanced degrees of work as the older men retired or died and senior posts became vacant.

The principle of secrecy was maintained. It now served a practical as well as a symbolic purpose: to exclude untrained impostors.

Master craftsmen (sculptors, carvers, builders, stonemasons, etc.) in a particular area appointed from among themselves a representative council headed by a senior craftsman, and this council ruled who was worthy and who unsuitable to be a member of the local guild.

The occult geometric rules passed to their successors were neither the former religious symbolism of ancient geometry nor its wide application in other spheres. It was purely and simply the use of geometric symbols and diagrams as a guide to certain proportions in their field of work. They lost over the years the geometric knowledge contained by the symbols in question.

This general picture of professional and outsider, the latter striving for admittance to the ranks of the trained worker, is a familiar and recognisable scene. The nearer we come to the present time the more bitter and determined becomes the fight, for the privileged group diminishes constantly in proportion to those on the outside peering in.

Natural development led to the rejection of established and acknowledged rules of proportion, which were replaced by new units and standards accessible to everyone interested in a particular craft. Gradually the barrier between the two forces crumbled away until today it is more a social than a professional factor.

If we directed our searchlight back through time however we would find the reverse to be true: the earlier the period, the more firmly established the Temple craftsmen and builders, and the less well organised and influential the "outsiders".

It is hard, if not impossible, to believe that the main patron of art in olden days, i.e. the Temple, would buy or procure work that was not created or associated with a religious and (with particular reference to our study) a geometric background. Man is (and certainly was in those days) too intolerant to admit the merits of a stranger. I believe therefore that artists of that older period who were outside the influence of the Temple or Church were forced to make and create work either for their own enjoyment or for that of their closest circle of acquaintances, while everything required for a more ceremonial or public purpose was created by the recognised group of Temple craftsmen.

Naturally this theory can only be applied generally. It need not have been the case everywhere, all the time. Exceptions may have been made. Both in the case of "official" and "unofficial" craftsmen.

It is highly possible that Temple builders carried on a secondary business of creating, building and making items not directly associated with either Temple or Church. And there was probably the odd individual who did work "on the side", as we say today when a trained journeyman employed by a master craftsman works on his own account during his time off and receives payment for it. And it is not unlikely that occasionally artists and craftsmen emerged outside the Temple with such masterly skill and inspiration that they could not be ignored. They had to be accepted for certain work—in spite of

strong protests from the "trades union". These exceptions, however, serve only to emphasise the clear-cut rule.

If we consider surviving material on this basis, we can expect to find art created by both groups, the one showing obvious signs of ancient geometry, the other entirely (or almost entirely) free from geometric influence simply because they did not know the rules.

There should be periods during which a similar geometric pattern is reintroduced time and again, depending to an extent on the particular period and geographic situation. And varying with the individual peculiarities of the artist. For just as there was ample opportunity in the construction of temples for the architect to develop a personal theme and variation, so there was the same, possibly greater, flexibility in figurative art.

The jobs of proportioning a building and a human figure differ considerably, of course. In planning a building one must take into account more than just a sense and appearance of beauty. The building must, for example, have strong enough walls to support its roof. It would be no use planning an attractive twin-towered church if the gable-end is structurally too weak to bear the weight of the towers. Or if spires are so slim that they may blow down in the wind.

A building moreover must meet another demand: it is normally a place where people meet, walk and live. Consideration must therefore be paid to the inside dimensions. These dictate up to a point the structure's size.

These factors do not apply to an artist as he sits down to proportion and plan a human figure, whether drawing or sculpture. But he has other limitations.

The work of art must resemble something, perhaps a certain person, perhaps an event in history. The figure or relief intended for a particular wall is at once bound in size to the dimensions of the space to be filled. Maybe the sculpture is to be erected in front of a building. Then its proportions are to a degree dictated by the dimensions of the building.

Examples of works of art tailored to fit a special space or area are numerous. Think, for instance, of the Parthenon. The triangular gable or pediment above the entrance had at one time an imposing decoration of carved stonework, which fitted the gable exactly. Today the whole triangular decoration is to be found in the British Museum.

The question has probably already risen in the mind of the reader: How can ancient geometry be adapted to the world of art? How can the straight lines and right angles of geometry be likened to a piece of sculpture that consists only of rounded, soft lines? How can geometry be used in art without introducing conformity and dullness?

In the same way as an ancient temple was planned in height, width, length, etc., by the proportions of geometry, so was figurative art made up of certain, easily recognisable, proportions that formed a skeleton within the figure and indicated, for example, its height.

Once the rough skeleton of the figure had been determined by one or two points and lines, it was given a body and shape by the artist. We cannot in the human body, for instance, pick out the bones of the concealed skeleton. In the same way, unless one knows the make-up and rules of ancient geometry it is impossible to uncover them in figurative art.

Art in Egypt

Let us start our study in ancient Egypt. Apart from maintaining a general chronological order this brings us to a period and place in history when art was much more uniform, more conformable than the art of later civilisations. A geometric sameness, one might say. The nearer one moves to the origin of such a geometric background, the more closely bound by its rules are the subjects.

Subsequent civilisations developed within a wider framework (but still *within* the framework), and gained valuable artistic freedom. The framework grew, too. And gave the artists increased geometric scope.

My approach turned out to be well founded. Not only have I succeeded in confirming my theories on numerous samples of pictorial art, but I have also found that in certain examples of surviving material the guide-lines of ancient geometry *can still be seen quite plainly* on half-finished work! They have never been painted over.

The detail and evidence contained in this material are of such lucidity that the observer, with a familiarity with and knowledge of ancient geometry, can spot not only the unit of proportion used to construct the particular figure but also the variations introduced in the same theme by the artist. We are able to follow the development of art from its primitive, geometric, mathematically handicapped stage to the point where it bursts into a floodlit arena of creative freedom. This applies to both Egyptian and Greek art. Proportions gradually attain a grace and beauty hitherto lacking.

Our survey is directed first at the Sakkara area of Egypt in which the Sakkara Pyramid stands. To the north and south we have numerous Egyptian *mastabas:* ancient burial chambers or tombs. A mastaba was originally carved out of a rocky cliff or into the terrain. Over the cave or hole was a roof of clay tiles.

A short description of this kind may sound primitive, but in fact the mastaba was often in reality an underground house, with several rooms and corridors. Many were finished internally in intricate mosaic. Often they were so deep that they required columns to support the "roof".

It was true of almost all that they were richly decorated and equipped—often to a finer degree than their occupants' earthly dwellings. The Egyptian belief was that this permanent "home" should be the most luxurious possible since the owner would spend his eternal life here after death. The home in which he lived during his short, temporary stay on earth was of considerably minor importance.

A number of mastabas had their inside walls smoothly plastered, and on the plaster were drawn and engraved various pictorial scenes and writing.

The tombs, which date approximately from the 1st and 2nd Dynasties, were usually constructed during the lifetime of the future occupant. Many examples have been discovered of mastabas being incomplete at the time of the owner's death—and left unfinished by his next-of-kin because of the considerable expense involved.

We shall probe about in the darkness of some of these unfinished tombs, because it is here that we find the artist's guide-lines still free of paint. This is an excellent place to start our study of Egyptian art.

Who decorated the walls of these tombs? If we accept that the mastaba was build before the death of its owner, and that

the family of the deceased occasionally omitted to complete the tomb because of the great expense attached to the work, then we may take it for granted that a mastaba was a costly means of ensuring after-death comfort. It was not presumably the poor who indulged in such luxury.

The mastaba was a link in the *death cult,* and the leaders of the cult both in life and in death were more than likely to be found within the Temple.

It has been established by historians and archaeologists that the Egyptian Temple was an extremely properous organisation. It collected taxes on its own and the Pharaoh's behalf, and in this way (among others) took an active part in the material life of society.

What would be more natural than that the Temple should organise tomb-construction to provide an extra source of income. A private temple of death (which in effect was the purpose of the mastaba) would be unthinkable without the blessing and dedication of the society's religious leaders. And considering the undoubtedly sound mercantile background of the Temple in Egypt, such consecration was not handed out for nothing. It meant a large sum of money for the holy coffers.

I doubt whether the Temple in such circumstances would be satisfied with a mere consecration fee. The whole arrangement presented a wonderful business opportunity. Prototype for Forest Lawn?

The Temple had at its disposal a huge staff of workers devoted mainly to the planning, construction, decoration and maintenance of temples. But this army was not fully occupied all the time. What to do in periods of idleness? That was the "problem".

One possible answer was to direct the workers' energy towards the erection (or digging) of mastabas. Here was a field in which the Temple was without the slightest competition. A man wanted an after-death mastaba. The Temple arranged to build and consecrate one for him. It is most unlikely that the religious hierarchy would agree to the consecretation of a mastaba built *outside* the Temple framework.

If the theory is sound, then we can assume that the Temple's builders erected all the mastabas, and that her "painters and decorators" carried out the ornamentation. This in turn means that it is likely that we shall find, worked into the drawn figures, the Temple's secret geometry as a factor of proportion.

One of the major works on Egypt and Egyptian art is by the German scientist, C. R. Lepsius. His research culminated in the publication in Leipzig in 1897 of a vast collection of 27 volumes. Most of these are made up of plates and illustrations of precisely measured drawings from ancient Egyptian culture. The book is *Denkmaler aus Aegypten und Aethiopien,* and was patronised by King Friderich Wilhelm IV of Prussia.

In one of the early volumes one finds the figure reproduced in *Fig. 261.*

It was copied from an unfinished mastaba and is described as "Proportione im Grabe" (Sakkora No. 17).

The figure in Lepsius's book is 24 cm high, and he writes that it is exactly four-fifths the original size, which means that the figure on the mastaba wall measures approx. 30 cm.

Lepsius gives no indication whether the figure was traced off the original or drawn, but in view of its relatively small proportions it is most likely that it was traced if it was situated in a convenient position.

The German scientist has this to say of the figure:

"Im Grabe des Pyramide bei Sakkora (no. 17) sind Proben aus alten Stadien der Wandskulptur und Wandmalerie zu

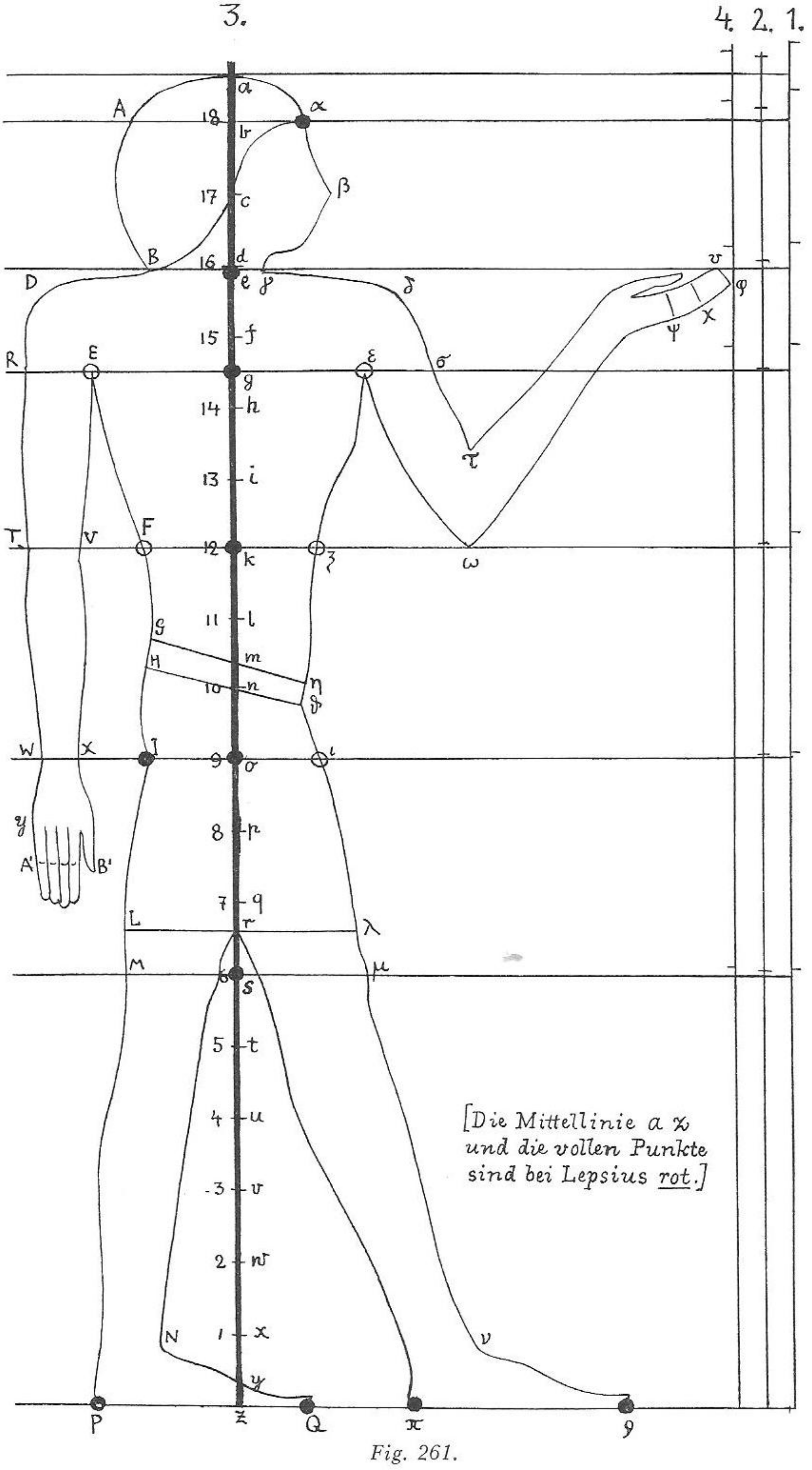

3.
4. 2. 1.
[Die Mittellinie a z
und die vollen Punkte
sind bei Lepsius rot.]

Fig. 261.

finden. Viele stellen sind erst angefangen, mit dem Meisel umzogen zu werden, anderen Stellen sind gar nicht gemeitselt sondern nur ausgemalt; an der Wand gerade dem Eingange gegenüber (LD II 68) ist eine Reihe Figuren z. T. nach mit den ursprunglichen Proportionslinien versehen, von denen immer eine den ganzen Korper von oben bis unten durchschneidet, 6 andere ihn horizontal durchschneiden und meht oder weniger sichtbar durch die ganze Reihe der Figuren durchgezogen sind. Sie voren zuerst rot angelegt, dann aber schon schwartz übergangen, wodurch aber die ursprungliche matematische Anlage der Proportionen ungenauer wurde.

"Nach langer Untersuchung kann ich doch zu keinen anderen Ergebnis, als das so = ok = ½ kb war, und = ½ sz sein solte; keine Einheit war zu finden, noch die Punkte P und g zu bestimmen.

"Um so wichtiger war eine Reihe von 4 Figuren, die zweite von oben, an der westlichen laibung des Einganges (LD II 65). Hier war nur die Linie a-z, die die ganze Figur von oben bis unten durchschneidet angegeben; die übrigen Proportionen waren durch rote Punkte angegeben, meistens ein wenig neben der Linie, um nicht mit ihr zusammenzufallen, einige Punkte gaben auch zugleich Seitenproportionen an. Dadurch dass jeder Fuss durch 2 rote Punkte genau destimmt war, ergab sich mir unmittelbar, dass der Fuss selbst die Einheit des Ganzen war, und dass sich die Punkte g und e durch die Unterabteilung des Füsses im Teile ergaben indem kg = 5/6, ge = 3/6, eb = 4/6, zusammen kb, also 2 Fuss sind."

This ancient Egyptian drawing, scratched on the wall of a dusty tomb, naturally gave rise to innumerable interpretations as soon as the material was published. The first meditation on the figure's proportions was by Lepsius himself.

But both he and later commentators who tried to solve the riddle of the figure's guide-lines were hindered too much by mathematics and proportion as they themselves understood the subject. They worked exclusively with numbers. They ignored geometry. Lepsius tried fitting in the length of the figure's foot as a unit of proportion. A Danish researcher, Erik Iversen, later put forward a theory that the hidden unit was contained in the width of the hand. His theory was based on the fact that the Egyptians operated a small standard of measure called 4 fingers (discussed in some detail in Chapter Nine). The width of a hand is in reality the width of one's four fingers. The thumb is "stuck on" at the side.

But none of the theories so far presented has catered fully for every one of the geometric lines that cut across the figure.

Explaining the presence of the lines on the basis, for example, of foot-length or hand-width one ends up with several awkward numbers and odd fractions. And I think it would be difficult with these to draw or proportion a given figure in a given space.

Let us assume a figure has to be drawn in a particular position on a certain wall. The first dimension that has to be marked out is the height. According to hitherto existing theories, we face a fairly complicated calculation in order to build up and fill in the respective guide-lines. The calculation demands knowledge of and practice in such finely graduated standards of measure that we can rule out any ability in this direction on the part of the Egyptian. He simply did not have that kind of mathematical knowledge.

Egyptian artists produced works of art ranging from the tiniest miniature to massive statues the like of which have not been created since. And this variation in the size of subject would undoubtedly present insurmountable difficulties if the artist relied on measurement and number

calculation for his proportions. The system would be so troublesome and long-winded to work with that it would defeat its own purpose: speedy and convenient rules for remembering and reproducing (human) proportions. No, existing theories in this respect *must* be wrong!

Another factor that is pushed somewhat into the background and passed over in silence is that the man (?) in Fig. 261 has other markings than just horizontal guide-lines. He is "pinned" to the wall by a series of dots or points that lie on the horizontal lines like so many knots in a string.

Of the 16 points, 11 lie on either side of the figure's vertical axis. It would appear that they were placed at the intersection of a horizontal and a vertical line. But no effort has been made to explain their presence and origin.

The reason for the refusal of this figure to yield to all previous investigations of its proportioning is simple: the "detectives" used the wrong apparatus, numbers instead of pure geometric areas and lines.

Not every geometric situation and term is expressible in finite numbers and fractions. We need look no further for such "awkward" geometric shapes than the square. If we call the side A and enter the diagonal (for example, for construction of the square's double-size version), we must admit that with our advanced knowledge of geometry and mathematics we cannot today state the length of that diagonal in a finite number or fraction, whether simple or decimal. The length of the line is $A \times \sqrt{2}$, and since $\sqrt{2}$ is an irrational number, i.e. a number whose root cannot be exactly determined, we are forced to admit that only by using a special mathematical sign ($\sqrt{\ }$) can we find a means of expressing the line's length. And the sign actually expresses more a geometric term than a mathematical.

An attempt to state the length of the two lines accurately in relation to each other fails .... whether we use our own system of calculation or any other.

It is not in the world of numbers that we must look for the secrets of Egyptian proportion. It is in geometry. Ancient geometry was the natural and obvious tool for the job—just as it was in building.

Let us start our geometric analysis of the silhouette in Fig. 261. We see that the figure is crossed horizontally by seven lines, including the base on which he stands. I am ignoring for the time being the line that appears to mark his height. The reason will be given shortly.

Line 1 marks the Forehead
Line 2 marks the Throat
Line 3 marks the Armpit
Line 4 marks the Elbow
Line 5 marks the Pelvis or Hip
Line 6 marks the Knee
Line 7 marks the Foot.

The intersection of the vertical axis with five of these lines is in the Egyptian original indicated by a tiny red circle. In addition to these main points there are

1 point at the Forehead
2 points on line 3 marking the Armpits
2 points on line 4 marking the Waist width
2 points on line 5 marking width or depth of Hips
4 points on the base-line (7) showing position and length of Feet.

These 11 points do not touch the vertical axis. Their positions are apparently determined by other (invisible) vertical lines. If therefore we are to prove our theory and justify our belief in ancient geometry we must be able to construct a diagram which horizontally coincides with the figure's horizontal guide-lines

and vertically cuts through the 11 points mentioned.

Setting out on a geometric analysis, it is essential to know how accurate the research material is. If it is not extremely near perfection, then no geometric diagram will fit. As surmised earlier, the figure was probably traced from the original, and was reproduced in Lepsius's book four-fifths the original size. So any degree of inaccuracy in converting measurements would appear to be excluded.

Our preliminary survey nevertheless reveals an inaccuracy: the vertical axis is not exactly at right-angles to the seven cross-lines.

The error is approximately ½ % but this is sufficient to affect an analysis in which importance is placed in the intersections of two sets of lines.

The error seems to have been made by the ancient Egyptian artist whose job it was to decorate that particular mastaba. Perhaps the Egyptians were not so fussy in laying out artistic creations as in the planning of buildings? Perhaps the artist was careless in his work?

In any event I considered it most correct to compensate for the ½ % error and to bring the vertical axis square with the seven horizontals. We see the figure and the first geometric diagram in *Fig. 262*.

Once again we must find the basic square. This is discovered by taking as the side-length the distance (along the vertical axis) from forehead to the base-line. The middle point of this line, we find, is marked by one of the red dots on the Egyptian original. After constructing a circle on the basis of this line, we describe the basic square around it: 1-2-3-4.

Why, the reader may ask, do we construct a square that cuts the top off the man's head? And in the same connection: Why did we take the forehead line as no. 1, and ignore the line that marked the top of the head?

The explanation is logical. From the sole of the foot to the forehead an artist can recreate the ideal human proportions. Particularly with the aid of geometry. But he cannot take into account what lies *above the forehead*. He must ignore tall head-dresses, high hair-styles, etc. He therefore takes his guide from forehead to foot. Later he can add any kind of headgear he wishes to the flat top of the head. Thus the importance of the dot painted at the forehead.

We have the square's horizontal axis as 5-6, and the vertical axis is number 7-8. This is one of our old friends, symbol "C", and provides us with three of the seven horizontal lines, nos. 1, 5 and 7.

We develop the symbol a little further by entering the diagonals and acute-angled triangles. The latter are 3-7-4 and 1-8-2. This permits us to add the circle's rectangle 9-10-11-12. Now we complete symbol "U", which contains the square on the circle's rectangle: 13-14-12-10.

We observe immediately that the top of this square, line 13-14, coincides perfectly with line 3 in the silhouette, marking the armpit. Four down, three to go.

The positions of the acute-angled triangles enable us to enter the 3-part dividing lines in our diagram, a geometric procedure we have seen often in our earlier investigations. The lines are 15-16 and 17-18 vertically and 19-20 and 21-22 horizontally.

We see at once that the upper horizontal line of 3-part division, 19-20, coincides exactly with line 4, marking the elbow. And the lower horizontal, 21-22, matches line 6, which marks the knee.

Now we have traced accurately the origin of six of the seven guide-lines. Only no. 2 remains.

A further point we may record is that the vertical 3-part dividing lines seem to box in the silhouette without actually touching it except where line 17-18 cuts

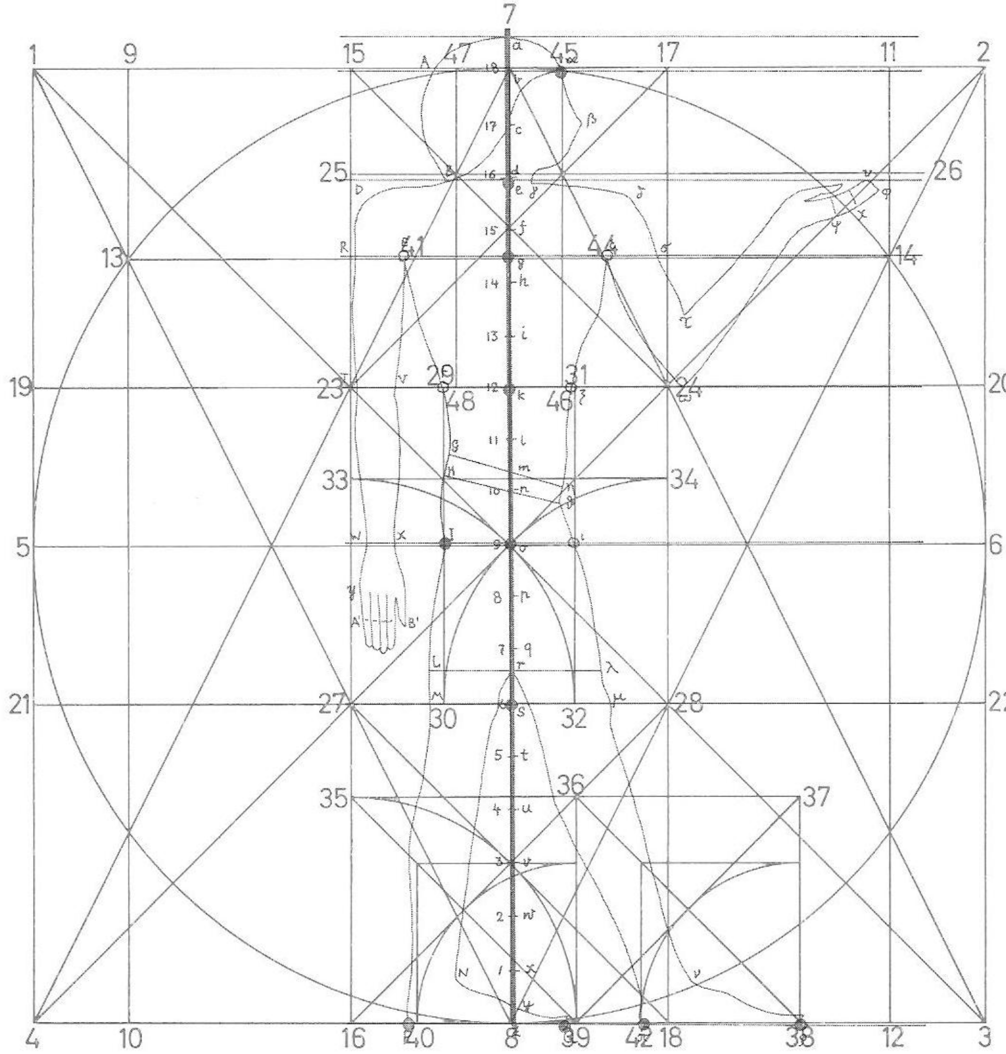


Fig. 262.

through the bent arm and the outstretched foot.

The upper part of the body and the head are framed by one of the nine squares into which the main square is now divided: 15-17-24-23. This square, too, is split 3 × 3 by means of the acute-angled triangle and the diagonal cross. And now we have our missing seventh line! It is marked by line 25-26, which is an extension of the upper horizontal 3-part dividing line. It has been produced to meet the circle's rectangle in order to show how it also marks the height of the raised hand.

All seven lines (which of course mark various heights in the human frame) have been traced. We lack only the 11 dots, which mark the vertical lines of the body. But already we can see how the ancient

Egyptian artist, sculptor and carver was able to lay out the ideal proportions of the human body. He simply took the familiar (though secret) symbols of geometry, found suitable guide-lines in them, and reproduced these each time he wanted to draw a figure.

The method possesses certain advantages over more complicated (e.g. arithmetical) systems. The geometric symbols can be constructed very quickly to allow the figure to be sketched in, and the proportions are constantly correct regardless of the size of the geometric diagram or required human figure.

It naturally meant a fairly strong conformity of work, but the aim of the Egyptian artist was not so much to produce an inspired creation with personal qualities as to recreate a picture or scene resembling as close as possible the ideal proportions previously approved.

The basic square in Fig. 262 was split vertically into three strips each of three smaller squares. The central strip, indicated by 15-17-18-16, stretches from forehead to foot-sole and in effect frames the whole figure except for parts of two limbs.

The upper of these three small squares is, as we recall, divided 3×3. Line 25-26 indicated the throat. The lower horizontal dividing line is not included in the diagram since it does not appear to have had any special purpose.

The vertical dividing lines are 45-46 and 47-48. The former cuts through the first of our 11 dots, i.e. in the forehead, and appears to cut the face off the figure.

The restriction of the face to a single vertical line is motivated by the same principle as ruled that the head should be cut off flat at the forehead: interchangeability.

Just as the artist was able to fit or draw any type of headgear he wished to the flat top of the head, so he was able to reproduce any form of face or facial expression on the flat front of the head.

We are well aware that Egyptian art frequently portrays human figures with the heads or masks of animals and birds.

There is, for example, the god Anubis with his wolf's face, or Thot with his face of the long-billed ibis, not to mention Horus, the clear-sighted god of daylight. His characteristic cranium was the face of a large-beaked falcon.

The silhouette of each of these figures was instantly recognisable and from the neck upwards had no resemblance to a human being. But the body was invariably the same—that of a human.

If the artist created his figures according to a system of modules, i.e. units for the feet, the arms, the torso, etc., the system had to be flexible enough to suit every type of figure. It is thus a natural course that he should indicate the face by a specific line—and on the right of that line reproduce whatever face (human or animal) was required. In this way the figure retained all its proportions. The head would always be the correct size in relation to the rest of the body.

So much for line 45-46. The other vertical, 47-48, would appear to have served a similar purpose. It marks the junction of head and shoulders but takes no account of the figure's hair arrangement. Here the artist could add the desired hairstyle.

It is interesting in our examination of the upper part of the body to note the position of the arm that points to the right. The diagonal lines 7-24 and 24-2 would appear to indicate the angle of the raised arm. One can almost feel the attraction of the artist to that angle as he filled in the shape of the arm along the geometric lines.

The next two points in the diagram are those marking the armpits. These, we see, are determined by the intersection of the acute-angled triangle 4-7-3 with the top

of the square on the circle's rectangle (line 13-14, which is also the armpit line). We have thus three of the 11 guide-points.

Moving on to the middle square, 23-24-28-27, we enter the sacred cut. The vertical lines are 29-30 and 31-32. Observe how they run exactly through the two points marking the width of the waist under the chest, and the two points marking the hips or pelvis. This brings us to a total of seven points of the 11. Four more to go, and we shall find them in the lowest of the three central squares.

But before moving to that square, we can record that the upper horizontal sacred cut in 23-24-28-27 runs through the figure's belt-line.

The sacred cut has again been entered in square 27-28-18-16, partly to indicate the position of the feet, and partly to mark their length.

The procedure is as follows: First the half-size version of the small square is constructed (16-35-36-39). Corner 39 meets the base-line exactly at the point that marks the toe of the rear foot. Then we lay this square out again along the base-line to form a rectangle 35-37-38-16. This rectangle has the same area as square 27-28-18-16. The corner of the rectangle 38 meets the base-line at the point that marks the toe of the front foot.

Now we require to find the marking for the two heels. This is achieved by entering in the two small squares just constructed the sacred cut from the bottom right corner, i.e. the toe. We see that points 40 and 42 coincide with the heels and with the final two guide-points in the figure.

Sticking rigidly to the rules of ancient geometry, we have thus traced all seven guide-lines and all 16 guide-points in the figure.

One might have expected to find the geometric diagrams displayed on the walls of the half-finished mastaba. But this would have been demanding too much. Non-initiated workers or outside parties might have had a chance to examine the lines and grasp their secret. Perhaps the craftsmen or artists who actually drew the figures did not themselves know the source of their proportions.

The procedure was probably as follows: The master craftsman sketched out a rough of the required figure in the sand, on papyrus or on a clay tablet. He then marked on the wall a vertical axis at the place where he wanted the finished figure to be drawn. On this axis, with reference to his preliminary sketch, he marked the necessary 16 guide-points and seven guide-lines. Then he or his trainees set to work filling in the figure's shape and colours.

The conversion ratio from sketch to wall was probably restricted to whole numbers. For example, if the desired height of the finished figure was four times the length of the vertical axis in the preliminary sketch, all dimensions were multiplied by four and transferred to the wall.

It was unnecessary to have any approved form of standard measure. Such would have been useless in any event unless it could be graduated to a very fine degree. The sketch's dimensions may have been transferred by means of a length of straw, twig or even knotted string. The actual method cannot be established with certainty, but the aforementioned means is logical and workable.

Fig. 263 shows a series of figures from a mastaba of the 2nd Dynasty, i.e. roughly the same period as the figure examined in the previous analysis. The new picture is also reproduced from Lepsius's collection of Egyptian pictorial art.

We can plainly see that the figures have been "constructed" in accordance with the various guide-lines discussed above. Some of the lines are still visible,

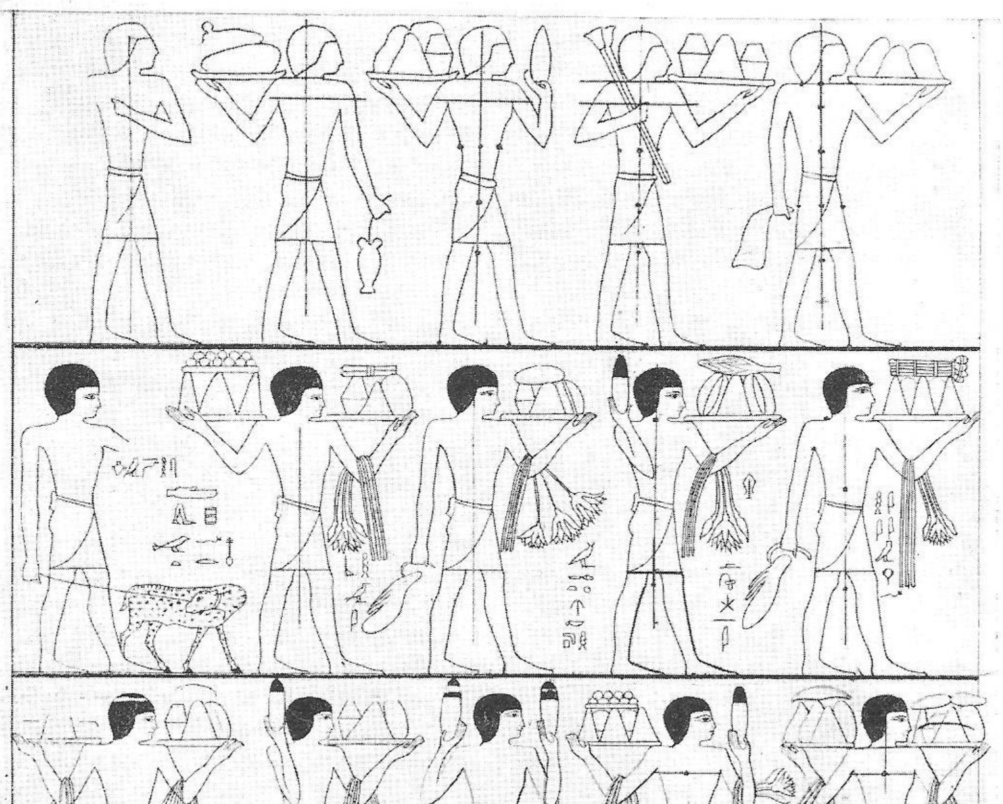


Fig. 263.

others have been painted over. A trial analysis on several of the figures showed complete and unswerving agreement with the theory demonstrated in Fig. 262, as long as the line of the forehead was taken as the point of origin. It is interesting to trace—about five thousand years later—the constructive background to the artist's creations as he decorated the dimly lit walls of the mastaba.

The technique by which the artist drew a figure around a vertical axis and an arrangement of guiding lines and points was in force over a considerable era. Later, however, a more advanced method of reproduction was brought into use.

According to Lepsius, the oldest surviving examples of Egyptian figurative art show traces of a vertical axis, guide-lines and guide-points.

But from the 17th or 18th Dynasties pictorial art was modified slightly. Lepsius reports finding figures from that period onwards drawn on a "squared" background, i.e. similar to present-day graph paper. The most common size of figure filled 19 squares in height. Later dynasties however allotted a different number of squares to the figure's height. Some were 22 squares high, others 25. But 19 remained the most common of all. In thinking of these dimensions of 19, 22 and 25 squares, we refer only to those figures which are standing erect, and ignore the various head adornments which could increase the figure's total "height" considerably. Our calculations in the following examination will be based on the figure's effective height from forehead to the soles of the feet.

Where is the missing link between the system of proportions allotted by the symbols of ancient geometry and the new system of the squared background? Investigators in the past have been unable to detect any system of dimensions or proportions in the "graph paper" method adopted by the Egyptian artist. And why do the figures occupy in height a different number of squares?

The explanation must surely be that the squared background was used not as a means of determining the figure's personal proportions but of reproducing it from a rough or preliminary sketch. The procedure was probably something like this:

The master craftsman or artist, having a knowledge of both drawing and ancient geometric symbols, started by producing a sketch of a suitable size for reproduction on the wall or column for which it was intended. Perhaps in a 1:4 scale.

The figure was proportioned according to ancient geometric diagrams, the lines of which determined the figure's height, width, measurements, etc. The completed sketch probably resembled the one in Fig. 262.

This diagram shows the figure placed vertically in the three central squares that make up one-third of the basic square.

The uppermost of these three small squares is also divided 3×3, as we saw. We then proceed to repeat the process of division, the square being broken into 6×6 small square. The same thing is done in the two other central squares. The rectangle in which the figure is placed is thus now divided in length (i.e. height) into $3 \times 6 = 18$ squares.

The squared effect extends from the sole of the feet to the line of the forehead—while the top of the skull occupies part of a new (19th) square. Thus we see a connection between the 19 squares of the preliminary sketch and the 19 squares later to be occupied by the figure on the wall.

The diagram complete with appropriate lines of division is shown in *Fig. 264*.

If we now imagine that the same three central squares, instead of being sub-divided into 6×6 smaller squares, are split

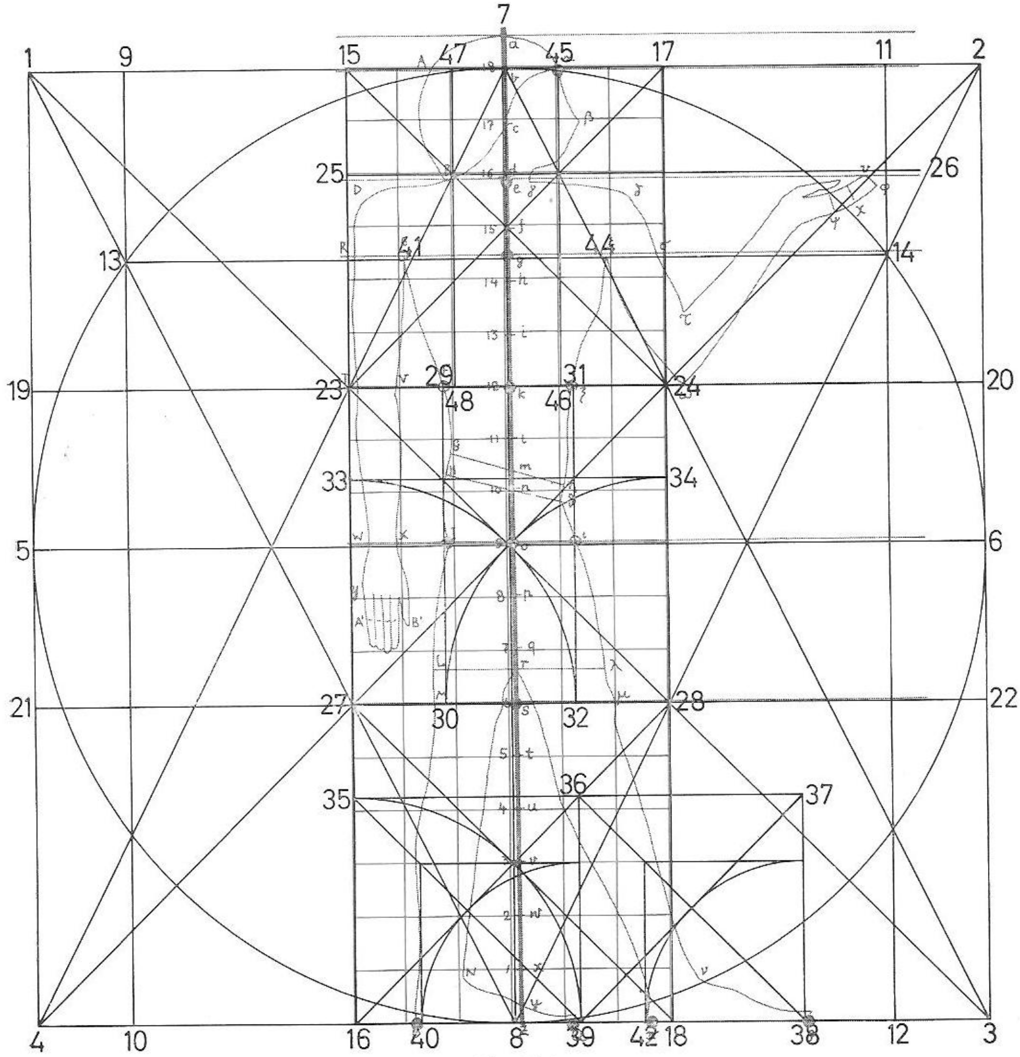


Fig. 264.

into 7 × 7 squares, the figure then occupies 21 squares from feet to forehead. And with the square intended for the top of the skull we have 21 + 1 = 22 squares.

We find this particular division in *Fig. 265*.

Similarly, when the three central squares are divided 8 × 8, the figure has a height of 24 + 1 = 25 squares. We see this in *Fig. 266*.

Thus we see how these recognised and natural divisions of the square may well have accounted for the particular arrangement of squared background found in many examples of surviving art.

While the master was drawing the figure and sub-dividing the geometric diagram as we have done in the past three illustrations, his assistants and apprentices were preparing the wall on which the

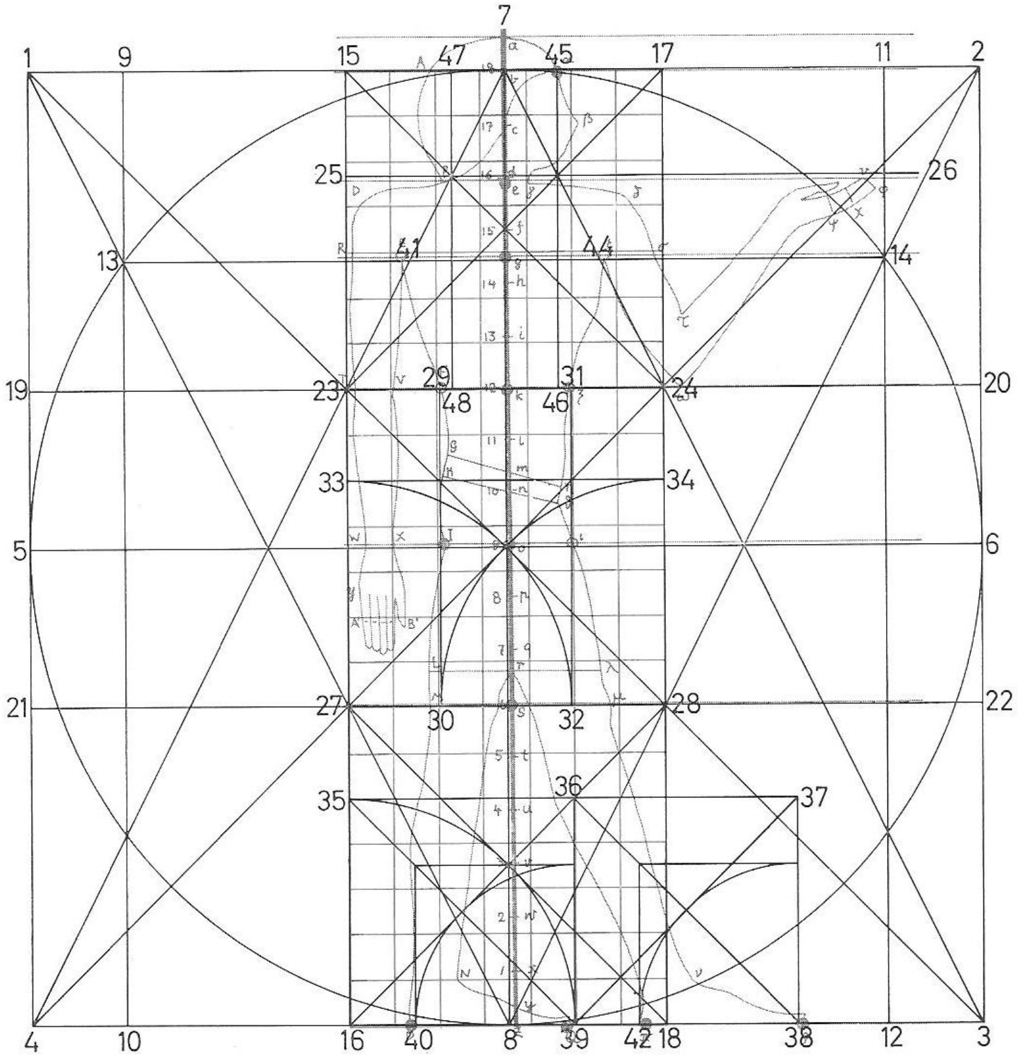


Fig. 265.

final figure should be painted. This included dividing the appropriate area into a pre-determined number of squares. The number and size of these squares was decided by the master. He it was who measured the wall and allotted the finished figure a height from feet to forehead of 18, 21 or 24 squares.

In the earlier inspection of the wall he perhaps concluded that the lower half should be occupied by a figure (or procession of figures) while the upper half should be devoted to hieroglyphics. Accordingly (depending on requirements and personal wishes) he ordered the lower (pictorial) half to be divided in height into 20 squares. Nineteen of these would be occupied by the figure, the remaining square forming a kind of breathing space before the hieroglyphs.

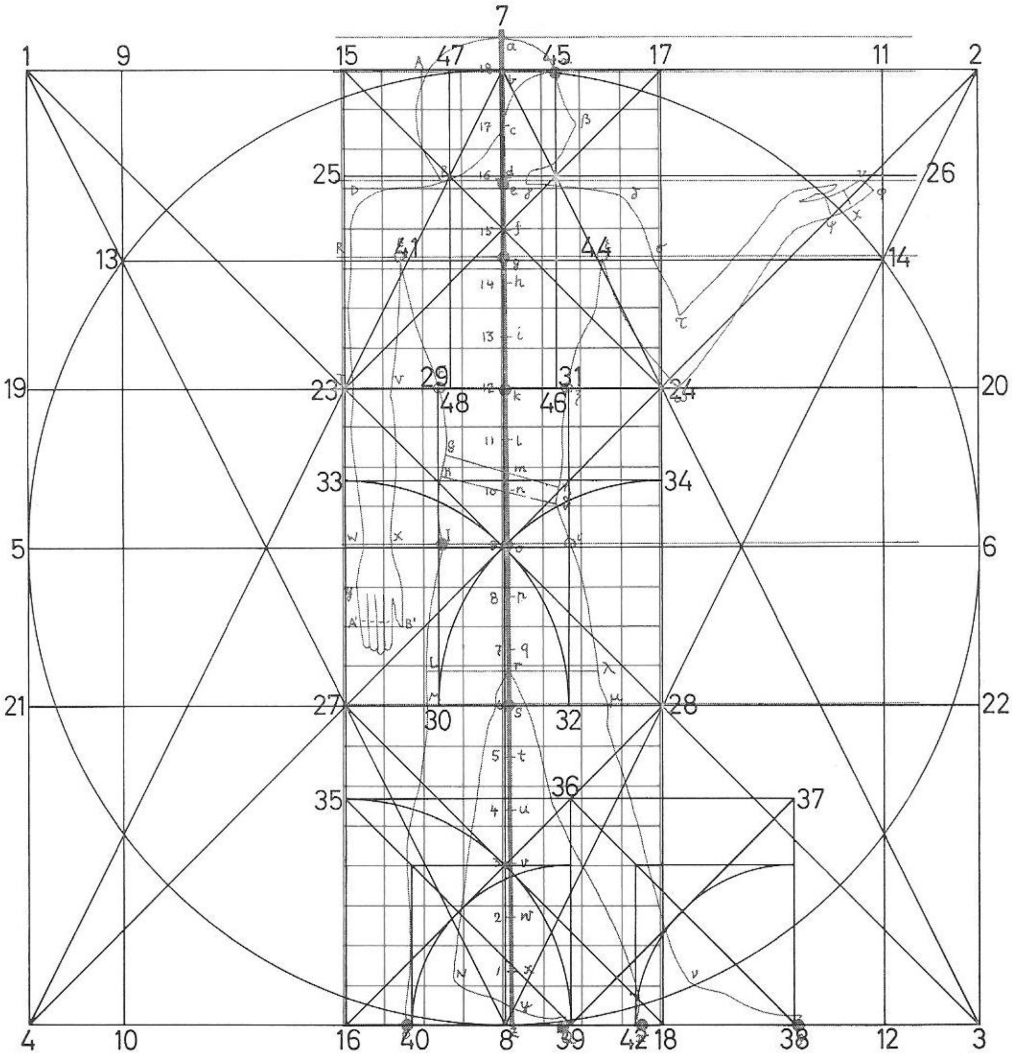


Fig. 266.

Once the wall was ready to receive the figure, it only remained for the master to provide the apprentices and lower-graded journeymen with sufficient illustrative material to let them get on with the job.

The master's finished preliminary drawing carried both the squared background and the ancient geometric symbols. The latter had not of course to be revealed to uninitiated tradesmen. He therefore covered the sketch with a piece of thin papyrus —and traced off the figure and squared background, omitting the sacred symbols.

The traced copy of the figure was handed to the team of assistants and their task comprised simply counting off the required number of corresponding squares and drawing in the figure. They had no idea whatever of the figure's proportional origin. But nevertheless they were "creat-

ing" a work of art that would puzzle and interest investigators when the mastaba—finished or not—was rediscovered thousands of years later.

Instead of tracing a copy of the original, the master may instead have handed over the actual ancient geometric diagram —after clipping away the two outer strips of the drawing and leaving only the central three squares. This would destroy any attempt to trace the origin of the figure's proportions. This method of reproduction —used in many spheres to this day—was greatly to be preferred to the old system of horizontal lines and guide-points. The actual process of squaring the background had thus no significance in laying down a figure's proportions. That was still done by geometric symbols. Squaring merely improved the job of reproduction and made it possible to produce a finished figure nearer to the master's original in appearance than by the old method. As the process of squaring the background probably also confused any search into the figure's proportional basis, it possessed an added advantage.

Ancient geometry has revealed to us not only the basis on which Egyptian pictorial and figurative art was proportioned, but also the reason for the subsequent introduction of a squared background. Logic also indicated why individual figures had a differing number of squares in height.

We have moreover established that the system of determining and fixing a figure's proportions had nothing as far as the ancient Egyptian artist was concerned to do with numbers, whole or part. Proportions were selected from the symbols of ancient geometry.

Squaring a pictorial background, it has been estimated on the basis of surviving material, was started around the 17th to 18th Dynasties, i.e. approx. 1600 B.C. and about a thousand years before the bloom of Greek art.

The first figure we examined (in Fig. 262) was from the 2nd Dynasty, i.e. approx. 2900 B.C., which means that 1300 years at least went by without any significant change taking place in the technique of Egyptian art. Even then the change we have examined affected the process of reproduction, not of proportioning the figure. For example, in *Fig. 267* we have a figure from the 20th Dynasty, i.e. about 1200 B.C. and approx. 1700 years later than the first figure shown.

It is another sample of the work published by Lepsius, and the Egyptian original was drawn on a squared background but did not of course show a geometric diagram.

The figure occupies, from feet to the crown of the head, 22.5 squares, which is a deviation from the pattern of 19, 22 or 25 squares we might have expected.

The reason for the slight modification is that the artist has now begun to take into account the fact that the figure must have a neck.

Checking back at Fig. 262, we observe that the head rests squatly on the shoulders and has no neck. We recall that the line of the throat was indicated by the upper line of 3-part division in the top centre square.

Fig. 267 has been given more or less the same geometric treatment and has the same proportions.

The basic square is 1-2-3-4, the circle's rectangle is 5-6-7-8, and the latter's square is 9-10-8-6. The basic square's lines of 3-part division are (vertically) 11-12 and 13-14, and (horizontally) 15-16 and 17-18.

We observe first that the basic square cuts through above the eye, not—as in the earlier figure—through the line of the hair. This alteration was caused by the new feature—the neck. Thanks to the neck, the figure's head was raised slightly from the body, but the basic square

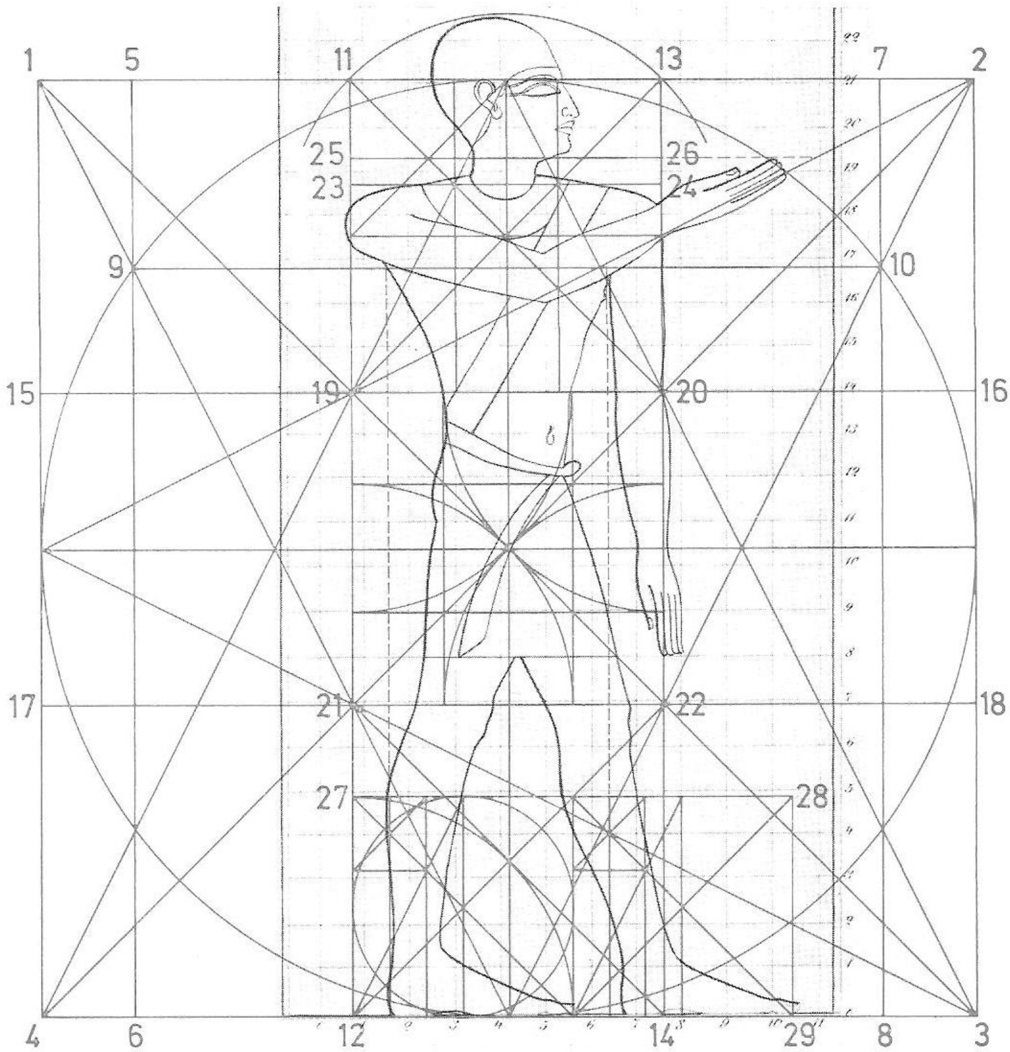


Fig. 267.

was kept at the same size to avoid a complete change in the figure's remaining proportions.

When the head is raised and the square retains its original size, the figure then exceeds the normal 22-square type by about half a square. The extra length is in fact the length of the neck, which we can examine in square 11-13-20-19 in Fig. 267.

Line 23-24 is the upper line of 3-part division and in Fig. 262 was termed the Throat line. In effect the line serves the same purpose in the new analysis: it marks the actual throat at its junction with the chest. But the new figure also has a new line. It is the upper horizontal line of 4-part division, 25-26, and marks the top of the throat or neck. And as the figure's face and head more or less retain their original proportion, the top of the basic

square, line 1-2, is moved down from the hair-line to the eye or root of the nose.

There has also been a minor modification in determining the length of the feet, although the broad geometric basis is the same.

Square 21-22-14-12 is converted to a rectangle 27-28-29-12, where line 27-28 is the sacred cut in the said square.

The right-hand "end" of the rectangle, line 28-29, indicates the extreme tip of the front foot. This corresponds with the procedure in Fig. 262.

The rectangle is in effect two component squares and, whereas in the earlier analysis we executed the sacred cut in order to find the heel of the foot, each square is now divided 3×3. The left-hand strip in each square is further split in two in order to mark the heel. The length of the foot is thus $5/6$ of the small square instead of $\frac{\sqrt{2}}{2}$. It would appear that the Egyptian artist gradually came to regard the old proportion of foot as too small, and altered it accordingly.

Lepsius, too, mentions the difference of foot-size in his commentary. He had experimented with the figure's proportions, using as the constructive factor the length of the foot. In the older example of figurative art (Fig. 262) he found that the length of the foot could be divided six times into the figure's height from foot-sole to forehead. A similar attempt with the later figure (Fig. 267) showed that the foot had grown slightly and no longer divided six times into the whole body.

His discovery was perfectly valid, and we are able now to check it by comparing the geometric sizes. In the same way we can see that the ratio of 1:6 no longer fits the figure's proportions.

These minor changes apart, the analysis is precisely the same as that in Fig. 262 and it is quite amazing to reflect that a procedure for allotting proportions to the human body was evolved and maintained for at least 1700 years virtually unaltered .... strong evidence that the system was governed by a strict and well-founded set of rules that withstood any possible attempt at amendment.

Such discipline in art is inconceivable today. Far from retaining a particular style for 1700 years (even 17 years!), we find fashions and methods altering with each generation. The explanation for the firm discipline must have been, as in so many other spheres, the domineering position of the Temple. The hierarchy tolerated no outside influence, and within the Temple itself it was as good as impossible for younger or revolutionary minds to introduce anything new in a tradition that was as old as the Temple institution itself. The senior priests stood fast.

Not only did the priests resist any attempt at revisionism, but they were assisted by innate religious belief.

It was part of the belief that geometry was derived from the gods, and that working with geometry was the privilege of the sons of the gods, i.e. the priesthood. In fact, with the circle as its basis geometry was no less than a picture or image of the principle god of those times, the Sun.

As we saw in the two preceding analyses, each comprised a large circle described by an outer, basic square. The proportions of the human body were built up within that framework, which in turn meant that the body was proportioned within the picture or image of god, within the circumference of the circle.

It is not at all incredible that this may have been the date and original basis of the Old Testament statement that "God created man in his own image". If by "god" the religious leaders of those days meant the Sun, and if (as was the case) the Sun's image was a circle, then the statement is perfectly logical and sensible.

A statement of this sort may well have been brought out of Egypt as part of the religious system of that time, and spread throughout Europe and the rest of the Old World. It passed into other religious systems, too, but its original meaning was lost in time .... if indeed more than a handful of priests had ever known from where it stemmed.

It has always proved more effective in laying down bans or laws to cloak them with a religious significance than to issue them through a civil authority. And if it was the belief of the Temple Fathers that god had ruled that the human body should be thus proportioned, then it would be utterly impossible for any individual, however enthusiastic, to break the habit of belief. This is perhaps the reason that artistic proportions of the human frame remained unchanged for 1700 years.

If the Egyptian artist wished to draw two or more figures side by side, he had several procedures to choose from. For example, he could draw each figure within its own individual basic square on a separate piece of papyrus, later placing the figure wherever required. Or he could draw one or two figures within the same basic square; the various heights obtained in the central three squares could be extended to right and left to cover the extra figures. Or he could select a recognised area within his basic square, remove it and place it adjacent to the square, and therein draw a smaller (perhaps seated) figure.

We have seen the one-square/one-figure technique. Now in *Fig. 268* we shall examine an example of the technique by which two figures are proportioned from the same square. The illustration shows a man at the centre of the square and a woman on the left.

The figures date from the 17th Dynasty, and we observe first of all that the top of the basic square lies at the line of the hair, as in the first analysis (Fig. 262).

Like that analysis, too, the two figures lack a real, well-defined neck. The throat is marked by line 25-26 which, as previously, is the upper line of 3-part division in the small, uppermost central square.

They are placed on a squared background and cover 19 squares from the roof of the head to soles of the feet. This means that the reproductive process used was the 6×6 division of each of the three central squares as discussed earlier. This provides $3 \times 6 = 18$ plus the extra "skull" square.

The dating and our knowledge of the drawing's geometric background tell us a little of the development of the technique of reproduction.

According to Lepsius the squaring process did not appear until the 17th Dynasty. It would seem therefore that the Egyptian artist began experimenting with the new system of reproduction in the form of a 6×6 division, progressing later to the much finer 7×7 and 8×8 divisions.

It was perfectly natural to initiate the system with a 6×6 procedure of division: the uppermost of the three central squares was already divided 3×3, and it was logical that the process should be continued in sequence (to 6×6) rather than to jump in at the deep end with a 7×7 or 8×8 adaptation.

We can moreover establish that it is after the 17th Dynasty that figures are supplied with necks (or rather that necks are allowed for in a geometric plan), and in this connection that the uppermost marking line, the top of the basic square, is lowered from the hair-line to the root of the nose.

The actual analysis of the two figures is identical to the two preceding. The basic square is 1-2-3-4. Entry of the circle and the two acute-angled triangles allows

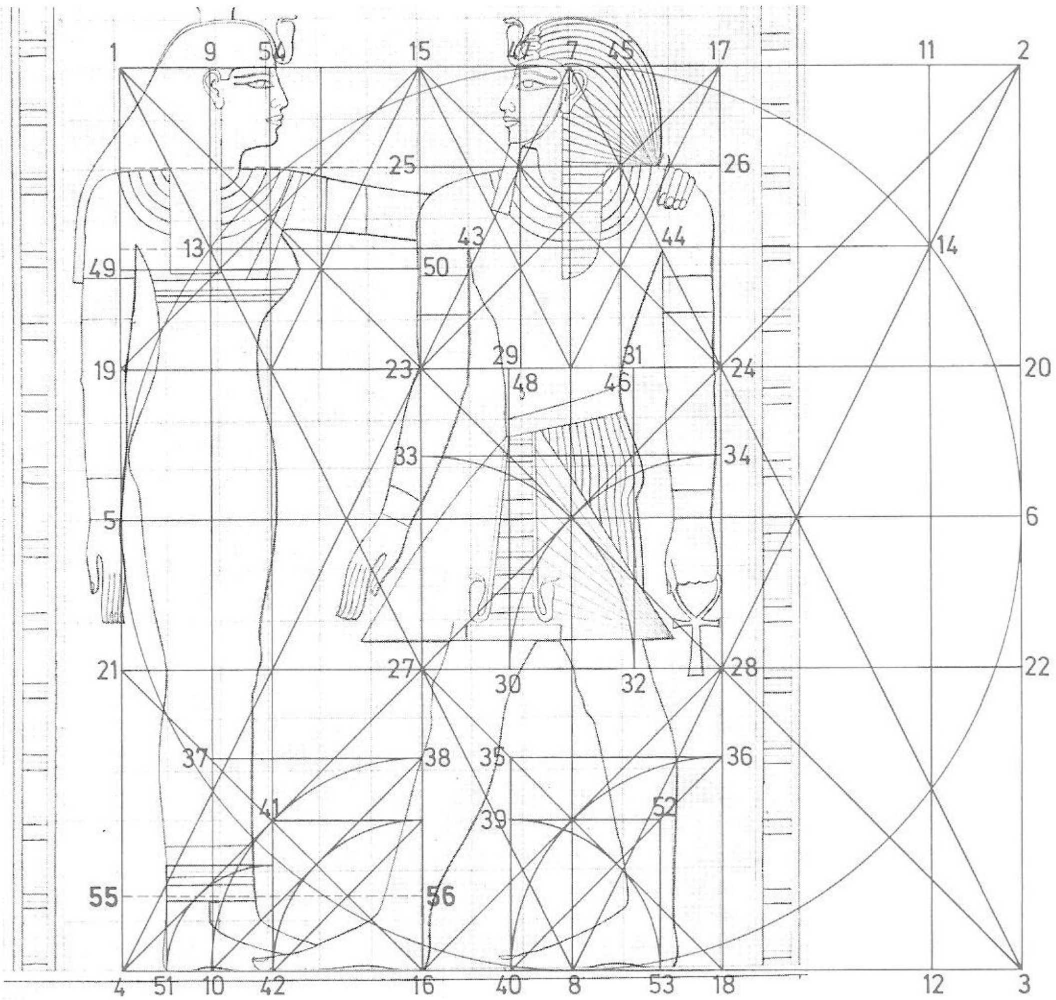


Fig. 268.

us to place the circle's rectangle 9-10-11-12 and its square 13-14-12-10. The basic square is sub-divided as in previous analyses.

The man is placed centrally in the diagram and occupies the width of the three centre squares. The top square 15-17-24-23 has been divided 3 × 3, and the figure's throat-line is the upper line of division 25-26.

The vertical 47-48 is the line that "cuts off" the face, and 45-46 similarly slices through the back of the figure's head, cutting off the hair-style.

The top of the square on the circle's rectangle, 13-14, indicates at its intersection with acute-angled triangle 4-7-3 the man's armpits (points 43 and 44).

The central small square 23-24-28-27 is, as before, divided by the sacred cut, the two vertical lines of which (29-30 and 31-32) indicate the man's width. The edge of the cuff on both the man's sleeves and on the woman's right sleeve is marked

by line 5-6, which is the pelvis line and the horizontal axis in the main square.

So far the analysis is the same as the previous two, but now the Egyptian introduces a slight variation in the position of the feet. They are placed (in the case of the man) more widely apart than before.

But this deviation is not in fact a deviation. It is still part of the same system, illustrating the suppleness of the geometric principle.

Within the lower central square 27-28-18-16 we enter the half-size version, placing it on the right: 35-36-18-40.

Line 35-40 indicates the toe of the man's rear foot. When (with point 40 as the centre of our circle) we enter the sacred cut in square 27-18 we see that line 52-53 indicates the heel of the same rear foot.

To position and dimension the other foot we have the same three-square process, only this time they share a common side: 27-16. The length of the foot is indicated by line 16-42.

The new feature in this analysis is that the basic square contains two figures instead of one. The woman occupies part of the three squares on the left.

The top left square 1-15-23-19 has been split in two vertically, and its vertical axis produced downwards through the diagram as line 51-42. This line marks the side of the figure nearest the centre of the symbol, and at the same time it is the line that cuts the woman's face away from the head.

Line 1-4 marks the other side of the figure, apart from the arm which is placed vertically just outside the square.

Thus the figure of the female is drawn in the rectangle 1-51-42-4 and is given all the respective horizontal lines of height from the man's figure at the centre of the symbol.

Square 1-15-23-19 is divided 3×3 in the normal way, one of the dividing lines (9-10) acting as the vertical axis for the woman's body and at the same time indicating the position of her head-dress below the ear.

The lower horizontal line of division, 49-50, marks the middle of the woman's breast and simultaneously indicates the lower end of the head-dress which is also seen hanging behind the arm that extends beyond the main square.

The position of the woman's feet is indicated by square 16-41, which was also responsible for marking the position of the man's feet. The half-size version of this square is laid out along the base-line, ending with point 51 at the woman's heel. Her toes are hidden behind the man's foot.

We have now derived sufficient information from our geometric diagram and analysis to permit us to draw in the two figures with the exact proportions allotted by the original Egyptian artist—without the least need for measurement.

We would be able to enter all the main vertical and horizontal lines of proportion for the two figures. It would then be a question of exercising an artistic ability to complete the illustration. The dimensions would be correct despite any possible mistake the artist might make, but it is undoubtedly the artist who would be responsible for making the finished picture beautiful or mediocre.

Ancient geometry presented the creative artist, designer or craftsman with ample opportunity for variety. It is only natural that the greater number of variations appear in figurative art. In the first place far more pictures were painted than buildings designed, and this led to a search on the part of the artist for an increasing realm of variety.

We saw how the first primitive prototype was created within the proportional framework of ancient geometry, and we discovered how the same technique was

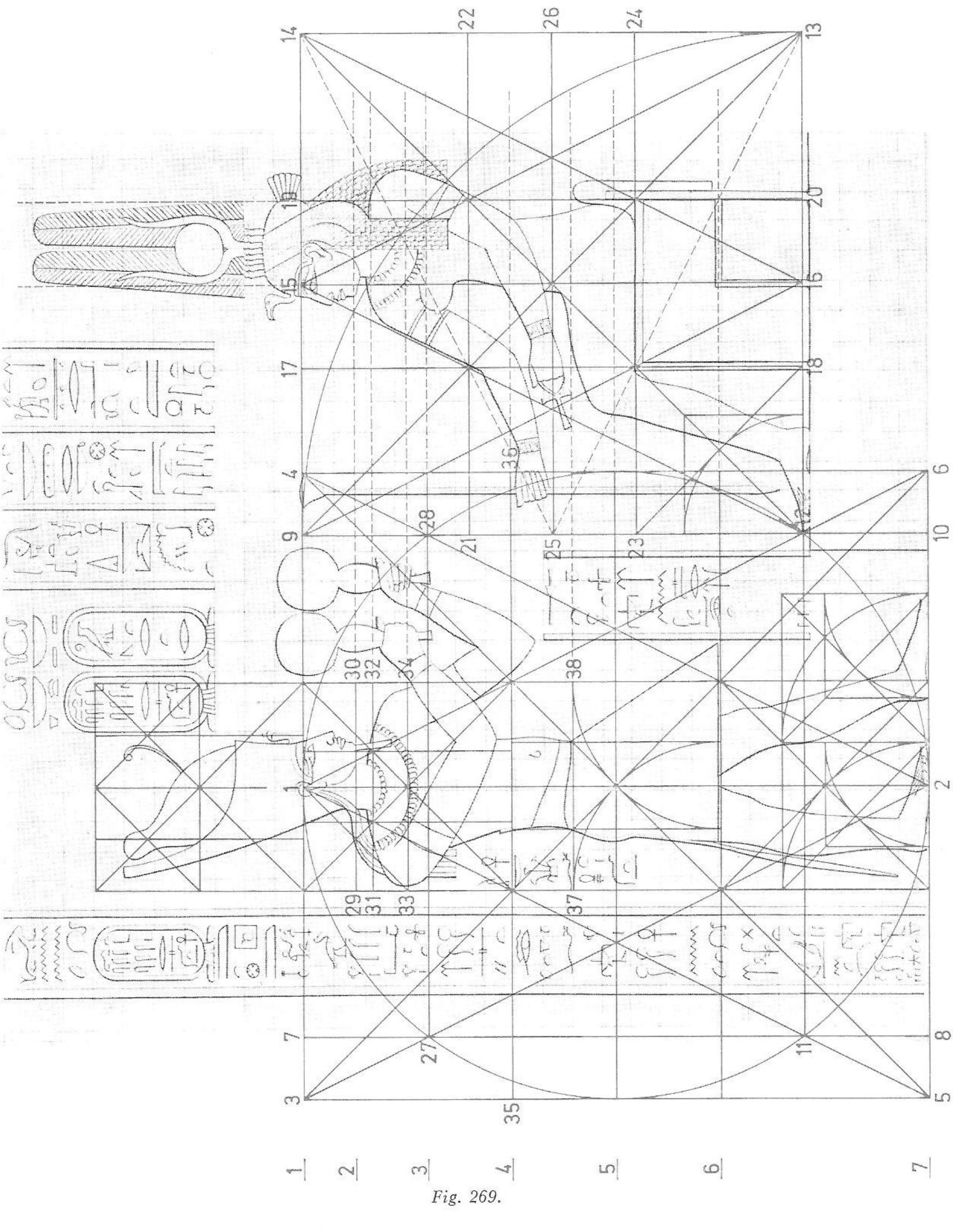


Fig. 269.

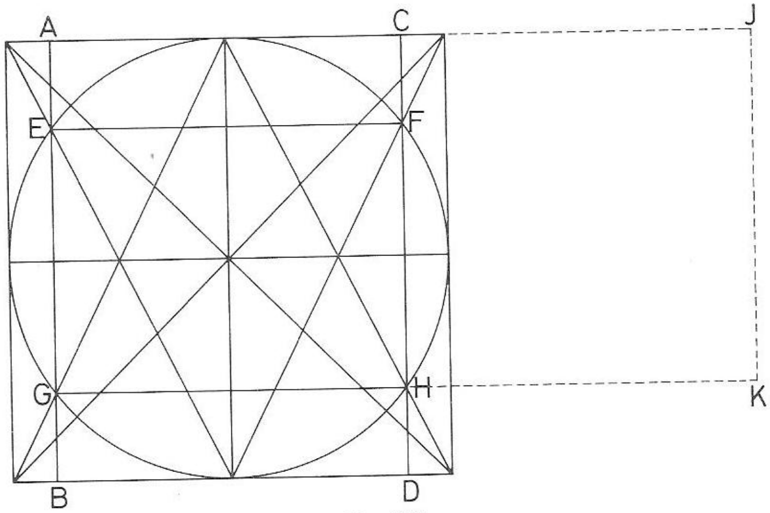


Fig. 270.

employed 1700 years later to proportion two figures in the same basic square.

In *Fig. 269* we find yet another variation in which a standing and a seated figure are both proportioned according to the dictates of the same square but in which one of the figures is outside the basic square. The picture is another from Lepsius's collection.

He quotes an original date of around the 20th Dynasty, which certainly fits in with the type of squared background used. As can be seen the standing figure of the man measures 21 squares from the sole of the feet to the root of the nose, i.e. indicating that the smaller squares within the basic square were divided 7 × 7 instead of 6 × 6 as in the preceding illustration.

This was how the Egyptian planned the picture in Fig. 269: He constructed the basic square and, after entering the necessary guide-lines, placed the square on the circle's rectangle at the top and at the base of the main square. In other words, constructed a combination of symbols "R" and "U".

We see his next step in *Fig. 270*. The circle's rectangle is ABCD and the latter's square is shown at the roof of the square as ACHG and the base as EFDB. The artist took square ACHG and flipped it over towards the right, providing a new square with a common side (CH). The new square is CJKH.

Turning back to Fig. 269, we recognise the geometric lay-out. The square in which the seated woman is placed is 9-14-13-12 and is thus equal to the square on the circle's rectangle in the basic square.

The basic square is split 3 × 3 in the normal manner to provide the various lines of proportion for the male figure. The square on the circle's rectangle, 9-14-13-12, is also divided 3 × 3. It thus marks the various proportions of the woman's body from the hips downwards. From the hips to the level of the forehead she is proportioned by the same lines as determine the man's corresponding proportions.

It is a peculiarity very often encountered in Egyptian art that a seated figure was lifted upwards in the motif to bring the head level with the heads of standing figures. Sometimes the seated figure

was placed on a small rise or hillock on the ground, but sometimes—as here—was simply left to float above ground-level for no apparent reason.

I believe the reason was not one of wishing to achieve beauty or symmetry, but of sheer practical necessity. Once the Egyptians had approved certain geometric proportions as suiting those of the human body, then it made no difference whether the figure was standing or seated, the proportions remained unaltered.

The artist constructed a square and provided a standing figure with the appropriate proportions, extending from forehead to feet, as we have seen earlier. But when he required to place a seated figure near the standing figure and in the same proportion, the problem arose: how to transfer the respective proportions to the seated figure?

The simplest method—and apparently that selected by Egyptian artists—was to raise the seated figure until the head came level with the head of the standing figure, extend the appropriate lines outside the basic square, and draw the new figure accordingly.

In Fig. 269 the figures are placed as described above. The vertical axis of the male figure is 1-2 and the basic square is 3-4-5-6. The circle's rectangle is 7-8-9-10, and its upper square is 7-9-12-11, and lower square 27-28-10-8. The upper square is laid out again as 9-14-13-12.

It is unnecessary to go over in detail the description of the man's proportions; these are identical to the previous analyses, and we see the position of the feet identical to that of the figure in Fig. 262.

The male and the female both have a little more neck length than the figure from the 1st Dynasty, and the small square that contains the male's head and shoulders has been divided 3×3 and 4×4 to illustrate the origin of the neck.

It is difficult because of the man's head-dress to calculate exactly how tall he is in terms of the squared background, but the height of the woman is more readily observed. We see once again that when the figure is given a proper neck the top of the basic square rests at the root of the nose, and the figure occupies not 22 but 22.5 squares in height.

The top of the man's neck is marked by line 29-30, which is produced as a broken line to pass through the same level of the woman's neck. The lower line of the neck is 31-32 and indicates the same part in each of the two figures.

Line 33-34 is the horizontal axis of the upper of the three squares, and marks the bottom of the man's necklace. An extension of this line and of line 31-32 marks the position and posture of the man's raised hands.

The next horizontal line in our diagram is 27-28, the top of the square on the circle's rectangle. It marks the height of the armpit in both figures, although on account of the position of their arms it is difficult to see this clearly.

The upper line of 3-part division in the basic square, line 35-36, also indicates part of the female's posture. It helps to place the hand which holds the upright staff. The position of that same arm, too, is interesting. The upper arm appears to have been place parallel with line 15-12, the forearm with 14-25, and the hand with 35-36.

The sacred cut in the centre square of the basic square indicates the level of the man's belt and, bearing in mind the woman's seated position, it in fact marks the same level on her body, too. Moreover, it marks the height of the back of her chair.

The 3-part process of division was also executed in the square occupied by the female (9-14-13-12), and it is plainly seen that the vertical lines of division (17-18 and 19-20) accurately mark the depth of

the chair on which she is seated. Line 19-20 indicates the woman's back and the rear upright of her chair.

Line 15-16 is the vertical axis of the woman's square, and we see that as well as forming the inside limit of her body it cuts her face off from the head as seen in earlier illustrations.

The lower horizontal line of 3-part division, 23-24, is actually the upper surface of the chair. Obviously the chair was based on the 3×3 division of square 9-14-13-12.

The position of the woman's feet was governed by square 23-18. Her toe meets line 9-12 and her heel is indicated by the half-size version of the square. This particular mode of proportion in fact provides the woman with a larger foot than the man's. But there was no place in the Egyptian artist's mind for gallantry—the rules of geometry had to be obeyed! If he had adopted the same procedure when drawing the woman's feet as he did in drawing the man's, the ratio would have been out of order. Her feet would have been disproportionately small. Instead of 3-part dividing, then entering the half-size version, and again the half-size version, the artist made only one half-size version in the case of the woman. The fact that she received slightly larger feet than her partner's was immaterial in relation to the importance of following the geometric rule-book.

Further study of the diagram would certainly bring to light other points and lines of guidance used by the Egyptian artist. And the addition of one or two more geometric lines would release a few more proportional secrets.

For example, in the square occupied by the woman it would appear that the artist produced lines 16-15 and 20-19 in order to help proportion the woman's head-dress. Similarly an additional square constructed above the main basic square would seem to indicate some of the lines of the man's head-dress.

But we cannot devote an excessive amount of space to any one painting. Our aim is to discover and illustrate any links that existed between ancient geometry and the art of Ancient Egypt generally.

Naturally it would be impossible to include in a work of this type a geometric analysis of every drawing that has survived from the days of the ancient Egyptians. Those examined in the past few pages must serve—as did the handful of buildings investigated earlier—as representatives for others of their kind from the same period.

I cannot say with certainty that every known Egyptian drawing was constructed and can be proved to be constructed according to the outlines of ancient geometry. I can merely assure my reader that I have not yet found a single drawing or painting from that period which has refused to fit as neatly into the geometric symbols and outlines as those in the preceding pages.

But Egyptian art was more than rows of paintings on the walls of mastabas and pyramids.

Art treasures recovered by archaeological expeditions have been lodged in museums all over the world, and they include many samples of work which demonstrate the expertise and ability of the Egyptian in creating sculpture and statues in stone. Every imaginable size has been found—from groups of huge figures carved into the face of an entire cliff to tiny, exquisitely formed miniatures.

If a strictly applied system existed to lay down the proportions of the human body in drawings and paintings, we must naturally expect to find the same rules of proportion in planning a piece of sculpture. How else would the figures in drawings and sculpture so closely resemble each other?

In the sphere of Egyptian sculpture we meet the same conformity, the same established postures, as encountered in painting and drawing. Perhaps the most noticeable feature in both spheres is that the figures lack altogether any flow or movement in composition.

I firmly believe that this stiffness (almost squareness, one could say) was due to the fact that at no time did the Egyptian artist create or derive inspiration from real-life models. He built up his figures and compositions within the symbols of geometry, the intersections, lines and points of which only to a limited extent allowed a personal influence to creep into art.

In preparing to analyse a piece of Egyptian sculpture we must bear in mind the advantages and problems of modern photographic aids. Quite apart from the distortion of perspective, much depends upon the angle from which the subject is photographed. There are normally two possibilities: from the front and from the side. But it must be directly from the front, and precisely 90° from the side-view. The slightest deviation alters both perspective and one's true view of the subject.

It has proved extremely irksome locating material accurate enough for a geometric survey for a normal photograph is taken with a different purpose in mind, e.g. to emphasise a particular characteristic of the subject, or to show off its beauty in a special light.

I was unable to procure, among readily available material, a view of any piece of Egyptian sculpture taken from the side at precisely 90°. We must therefore conduct our analysis on face-on material.

The first sample of sculpture on which we shall turn our geometric magnifying glass is a group of three figures, a male and two females, dating from the 4th Dynasty. We see it in *Fig. 271.*

The group represents King Mykerinos standing between the goddess, Hathor, and a local goddess from a district in Upper Egypt. Originally the sculpture was part of the adornment at Mykerinos's burial temple at Gizeh, but is now housed in a Cairo museum.

The group is not of the large variety, being about 93 cm in height. It is executed in dark, almost black, basalt and the three figures stand with their backs to the stone from which the whole composition was carved. The trio stand on a form of platform which is also carved out of the main stone. The platform slopes forward slightly, the result being that a direct frontal view—instead of being level with the soles of the figures' feet—reveals the whole arrangement of feet from above.

The figure of the king, which is the main one in the group, stands slightly ahead of the two goddesses, with one foot before the other.

In conducting a geometric analysis of the group in this photograph, taken at very close quarters, we must take account of the position and angle of the feet on the sloping base.

We require the horizontal level of the feet, not the lowest tip of the toes shown in the photograph. This latter would provide a greatly distorted proportion, by requiring too large a basic square.

One can only assume that turning the feet forward on a sloping base was a refinement of sculpture beyond the scope of our analysis. In the same class as the head-dresses examined earlier in the wall-paintings.

We therefore take as the base-line of the analytical diagram line 5-6 which passes through the group at the equivalent level of the horizontal line of the man's rear foot.

The basis of the diagram is line 1-2, which as usual extends from the sole of the feet to the root of the nose. We then

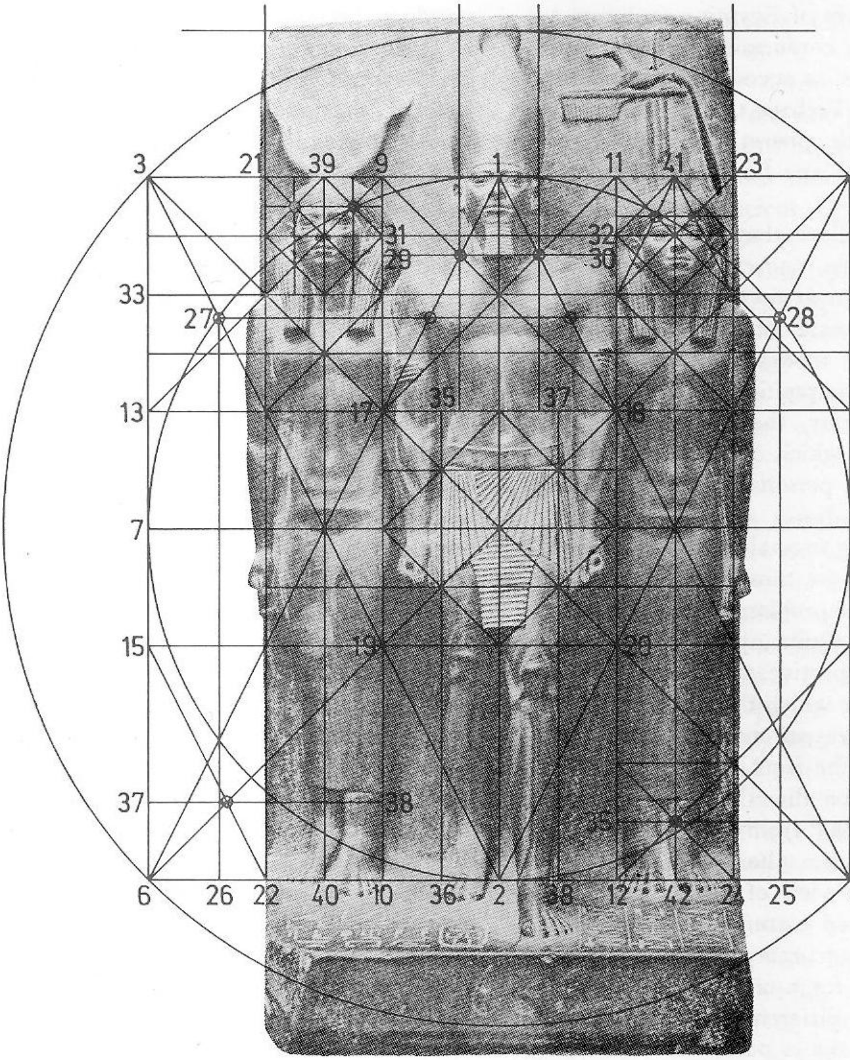


Fig. 271.

enter the constructive inner circle and the basic square (3-4-5-6), as well as the horizontal axis, 7-8. Thereafter we add the acute-angled triangles and the diagonal cross, enabling us to enter the vertical lines of 3-part division, 9-10 and 11-12, and their horizontal counterparts, 13-14 and 15-16.

This division creates a square in the centre of the diagram: 17-18-19-20.

The circle's rectangle is entered, as is its square on the base-line, 25-26-27-28.

The uppermost of the three central squares, 9-11-18-17, was the square which in the wall-paintings provided the proportions of the head and upper body by means of 3-part division. The same division is entered here.

Thus far the diagram resembles exactly the earlier analyses of Egyptian art, and

we can trace in the proportions of King Mykerinos the same proportions and markings as discovered previously.

Line 3-4 marks the height of the figure at the forehead. The upper line of 3-part division in square 9-11-18-17 marks the shoulders. Line 27-28 the armpits, line 13-14 the elbows, line 7-8 the pelvis, line 15-16 the knees, line 5-6 the sole of the foot.

We have thus traced the same levels of height as discovered in the wall paintings. The inference must be that the sculptor followed the same rules of proportion as his brother artist.

As far as the width of the male figure is concerned there is a similarity of treatment here, too. The total width of the figure is marked by the two vertical lines of 3-part division, 9-10 and 11-12.

The top of the square on the circle's rectangle is intersected by the acute-angled triangle and as observed in the earlier analyses marks the position of the armpits.

We have become accustomed to seeing the width of the body determined by the sacred cut in the centre square, but a modified approach was adopted in the case of King Mykerinos. Instead the artist chose to make the determinative factor the vertical lines of the 4-part division—in fact making the figure rather wider than normal. Could this be an indication that good King Mykerinos was more portly of stature than the young gods and goddesses in his immediate vicinity? The two lines are 35-36 and 37-38. They have been produced to meet the base-line, where we observe that they also indicate the dimensions of the legs. Line 35-37 is split by the vertical axis, which produces two rectangles in the lower part of the diagram, and this is the placing of the legs.

The horizontal lines of 4-part division have also been entered in the central square, and we notice how the uppermost of these marks the belt-line of the male figure. The line also marks the elbow level of the two female figures, which are of course rather shorter than the male.

The lowest of the 4-part dividing lines indicates the bottom of the king's kilt or skirt, and at the same time marks the middle of the women's clenched hands.

It is worth noting that the line of the man's kilt follows the sweep of the diagonal of the square.

We can record, with regard to the figure of King Mykerinos, that he is proportioned in exactly the same manner as the earlier wall paintings, apart from one or two minor variations.

It is also true that he is the main figure in the group (proportionately speaking), and this tallies with customs and beliefs of that period. Although the trio includes two goddesses, the pharaoh is on level footing with them being himself a god.

The female figures are proportioned according to the dimensions of the male, with slight modification to compensate for the difference in their respective heights. The two women also differ in respect of each other. The one on the right is not quite as tall as her sister goddess.

If we regard first the female on the right, we find that her dimensions are based on the upper right square (of 3-part division) 11-4-14-18. This square is divided 4 × 4 but only the vertical lines have been entered for reasons of clarity. The vertical axis is 23-24, and we see how this line follows the outer dimensions of the woman's body, while cutting the right arm from the body—precisely as we saw in the wall painting in Fig. 268.

The left-hand vertical line of 4-part division has also been produced through the diagram, and we observe how it forms the female's vertical axis.

The goddess's head is framed in a small square, the top of which is line 11-23. Entry of the horizontal lines of 3-part division in this square shows how the up-

permost coincides with the figure's forehead.

The height of the woman's shoulders is indicated by the horizontal axis of square 11-4-14-18. This line, 33-34, is extended across the diagram and has a place in both female figures. On the right it shows the line of the chin, on the left the upper edge of the shoulders or the lower part of the throat.

The same means of allotting proportions to the right-hand female was employed in building up the form of the goddess on the left. Her arm, too, is cut away from the main group by the corresponding line, 21-22. The figure's vertical axis is seen as line 39-40, and the height of the left-hand figure is the only basic difference from her sister. The one on the right we saw had a forehead height determined by the 3×3 lines of division. The lady on the left has had the same level determined by the 4×4 lines of division.

If we could imagine that the respective divisions were executed in the height of the two female figures, we would probably recognise the "squaring" process practised in the wall paintings. The left-hand vertical axis 39-40 would be split into $3 \times 8 = 24$ sections or squares, while the right-hand axis would be divided into 3×6 squares $= 18$.

At the base or lower part of the two female figures the same procedure of division was adopted: the bottoms of the long dresses were governed by respectively the 3- and 4-part lines of division. But the process employed at the head of the figure was swopped at the base, e.g. the height of the right-hand female was decided by the 3×3 division, while the length of her skirt was decided by the 4×4 division. In square 20-16-5-12 the edge of the skirt is marked by 4-part dividing line 35-36.

The goddess on the left is related in height to the male figure in the ratio $^{23}/_{24}$ to $^{24}/_{24}$ while the female on the right is in the ratio $^{17}/_{18}$ to $^{18}/_{18}$. A firm illustration of the existence of the previously observed squaring system in the sphere of sculpture.

Before leaving this godly trio we can enter one final element: the basic square's outside circle. We see how its upper horizontal tangent marks the total height of the group.

Further examination of the symbol and the addition of extra guide-lines would no doubt reveal more detail of the sculptured trio, but the taste acquired from the already conducted analysis should be sufficient to demonstrate a similarity in proportions between Egyptian sculpture and Egyptian wall painting.

From this 4th Dynasty group (approx. 2500 B.C.) we jump forward 1200 years to the period of Egyptian history referred to as the New Kingdom. Finally, after generations of warring and struggle the Egyptians have thrown aside the influence of the Hyksos, or Shepherd Kings, and other Asiatic rulers.

The period is that of the 18th Dynasty, and the subject of our next analysis is a statue of King Tut Ankamon who occupied the Egyptian throne around 1350 B.C.

In *Fig. 272* we see the statue. The original is a full-size version and like the last group is housed in a Cairo museum.

A geometric diagram has been superimposed on the figure in the normal manner, and the result is similar to that achieved in analyses of the wall paintings and of the preceding piece of sculpture.

The basis of the diagram is the well-known vertical axis, 1-2.

The figure stands on a horizontal base, with one foot slightly in front of the other. The rear leg is vertical and we must thus make the rear foot the level of the base-line as the basic square would otherwise be too large.

Stretching the vertical axis from the

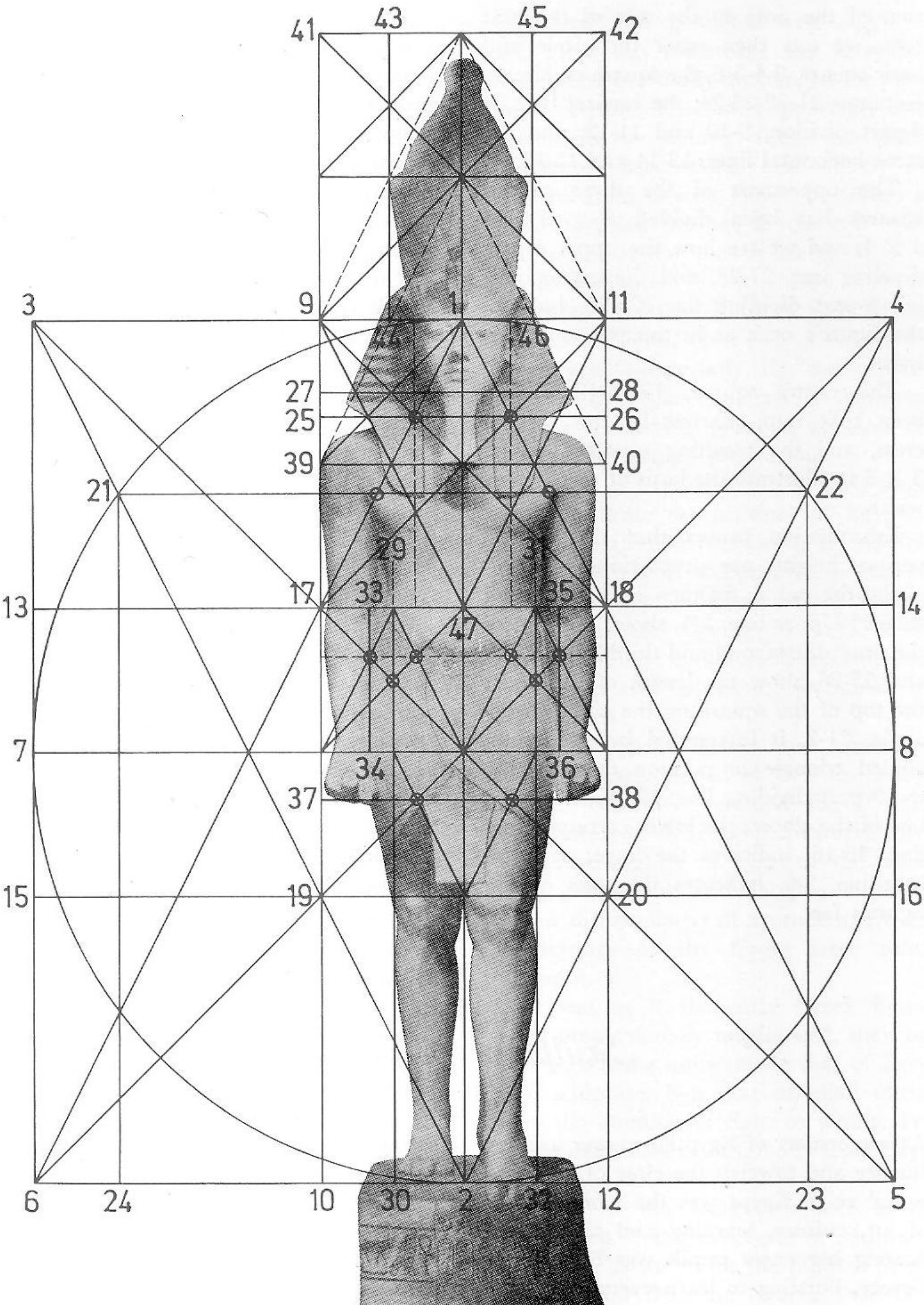

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Fig. 272.

root of the nose to the sole of the rear foot, we can then enter the circle and basic square, 3-4-5-6, the square on circle's rectangle 21-22-23-24, the vertical lines of 3-part division, 9-10 and 11-12, and the same horizontal lines, 13-14 and 15-16.

The uppermost of the three central squares has been divided 3×3 and 4×4, and we see how the upper 4-part dividing line, 27-28, and the corresponding 3-part dividing line, 25-26, indicate the figure's neck as in the previous analysis.

The centre square, 17-18-19-20, has been split into quarters by the vertical cross, and the resulting squares divided 3×3 to illustrate the basis of the figure's width.

Examination proves that the figure's various heights are proportioned in precisely the same manner as experienced earlier: Upper line, 3-4, shows the root of the nose; the second and third lines, 27-28 and 25-26, show the length of the neck; the top of the square on the circle's rectangle, 21-2, is intersected by the acute-angled triangle to position the armpits; the 3-part dividing line, 13-14, shows the line of the elbow; the lower corresponding line, 15-16, indicates the knee; and the base-line, 5-6, indicates the soles of the figure's feet.

The geometric analysis and markings of this figure are so precise that it may be regarded as a copybook example of the proportional lay-out of ancient Egyptian sculpture.

The only effective deviation from the procedure already examined is in the square above the basic square in order to proportion Tut Ankamon's head-dress.

The square in question is 41-42-11-9, and it is the same size as the nine component squares into which the basic square is divided. Notice how the two vertical lines of 4-part division, combined with the acute-angled triangle, indicate the top of the figure's headgear.

Entry of yet another acute-angled triangle from the horizontal axis of square 9-11-18-17 to the same axis of square 41-42-11-9 marks exactly the slope of the figure's peculiar head-scarf.

As maintained previously, further details of the figure's proportions would yield to a closer examination and to the addition of a few more lines. But we have obtained enough information about the life-size statue of Tut Ankamon to reproduce his proportions in any ratio. It would then be up to the individual artist and his creative dexterity to make the statue attractive or ugly. He would at any rate have achieved the proper proportions.

Sculpture in Greece

At the height of Egyptian power and influence and towards the close of the pharaohs' reign Egypt was the world centre of art, culture, learning and civilisation. Among her eager pupils was bright-eyed Greece, bursting to learn everything, asking, searching and experimenting.

The Egyptian Temple with its mystical wisdom assembled over thousands of years, with inspiration from Babylon and the East, was the pivot of Egyptian superiority. The Temple was respected and regarded with deep awe. Greeks came here to learn.

We examined earlier the strength of the Temple in both Egypt and Greece, and

observed how the Greek intelligentsia were fascinated by esoteric Temple life, and frequently travelled to similar religious refuges in Egypt to supplement their relatively meagre store of Grecian know-how. The Greeks may have had a word for it, but it was the Egyptians who showed them how to do it!

One of the many subjects in which visiting Greeks soaked themselves while living with their Egyptian Temple brethren was ancient geometry. Once back in the vibrant atmosphere of an emerging Athens they injected geometry with their own characteristic design and form of temple-planning.

But how was figurative art treated? Did the Greeks build upon the experiences and ability of the Egyptians, firmly established over a period of thousands of years? Or did this form of art develop in Greece quite independently of the Egyptian Temple?

We today consider Grecian art as the finest effort Man has made in this field, the perfect climax to a well-founded, well-practised training. The Greeks, we say, were in an infinitely more advanced class than their Egyptian predecessors.

The art of the two nations is seldom referred to in the same context, their links are sparse, they belong to quite different epochs. Each in its period was supreme. But there the comparison is normally ended.

I think this view is wrong. Precisely at the time when art was making a primitive beginning in Greece, that country was enjoying extensive cultural association with Egypt. It was roughly at this stage that Pythagoras, for example, spent more than 30 years in Egypt and Babylon. And his was certainly no isolated case. Hundreds of his contemporaries, philosophers, mathematicians, writers, etc., made similar trips to the Egypt well of learning.

These Greek travellers undoubtedly encountered and pondered over Egyptian art. Here was something the Egyptians had fostered virtually unchanged for thousands of years. There was apparently a set pattern in the pictorial friezes and tomb or temple decorations. Instinctively art scholars from Greece joined the ranks of their Egyptian masters to discover what they could of the ancient secret. It is invariably simpler to learn from the experiences of others than to start at the bottom and build up a web of experience for yourself—especially if "the others", i.e. the Egyptians, had spent thousands of years perfecting pictorial art and the drawing of human proportions.

A study of Grecian art in its early stages would indicate the presence of Egyptian influence. In many respects, it still had the conformity of composition so characteristic of Egypt. The scholar was obviously content with copying the work of the master until he could build up sufficient confidence and experience to branch off on his own.

Figures (both statues and drawings) from the early period of Grecian artistry around 600 B.C. retained the upright, stiff posture of their Egyptian forerunners. One foot slightly in front of the other, arms gently bent and hanging loosely.

But this similarity of attitude apart, the proportions of the figure were subtly different.

It was as if the early Greek figures were proportionally unbalanced, their appearance being quite unlike that of Egyptian sculpture. Not that we shall discuss here the question of right or wrong. Perhaps Greeks *looked* differently from Egyptians!

Where is the basic difference in the dimensions of the two types of sculpture? Assuming as we do for the purpose of our investigation that the figures were built up according to the same system of units, then they should look alike not only in

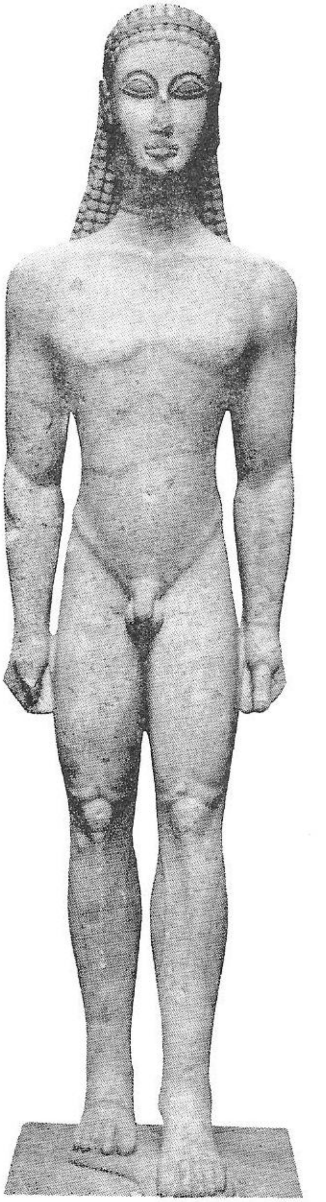

Fig. 273.

posture and composition but also in proportions.

As far as I can detect, the difference lies in the fact that the Greeks learned of the geometric symbols used to lay out human proportions but in transferring the knowledge to their homeland they applied the lines of the diagrams wrongly either in ignorance or in experiment.

It is possible that they were given a description—but not a sufficiently full one—of the symbols in Egypt. When in practice they tried to fit the human frame into the symbols' lines they did not achieve the same result as the Egyptians.

Alternatively the secrets of human proportion may have been carried from Egypt to Greece by a Temple brother not fully acquaint with art. Perhaps his time in Egypt was spent learning mathematics or astrology. He was unable therefore to recall exactly the artistic instructions given him (or overheard) in the Temple.

In any event we shall find that early Greek art—pictorial and sculptural—was based upon the same ancient geometric symbols as used in Egypt, only with a modified application.

In *Fig. 273* we have a piece of Greek sculpture dating from the earliest recognised period of that country's artistic career. It is from approx. 615 B.C. and is housed in the Metropolitan Museum in New York.

Immediately we observe a general similarity with Egyptian sculpture. The figure stands erect, one foot rather ahead of the other.

A close study of the figure reveals that one of the differences in proportion between this and an Egyptian statue is that the Greek figure from the shoulder axis to the roof of the head is longer than the Egyptian. Moreover, the Greek figure appears to be somewhat slimmer in proportion to height.

These factors will be examined in our geometric study which starts with *Fig. 274*.

The red lines show that the basic square is the same as used by the Egyptians. The top passes through the figure's hairline. A line (1-2) from that point to the sole of

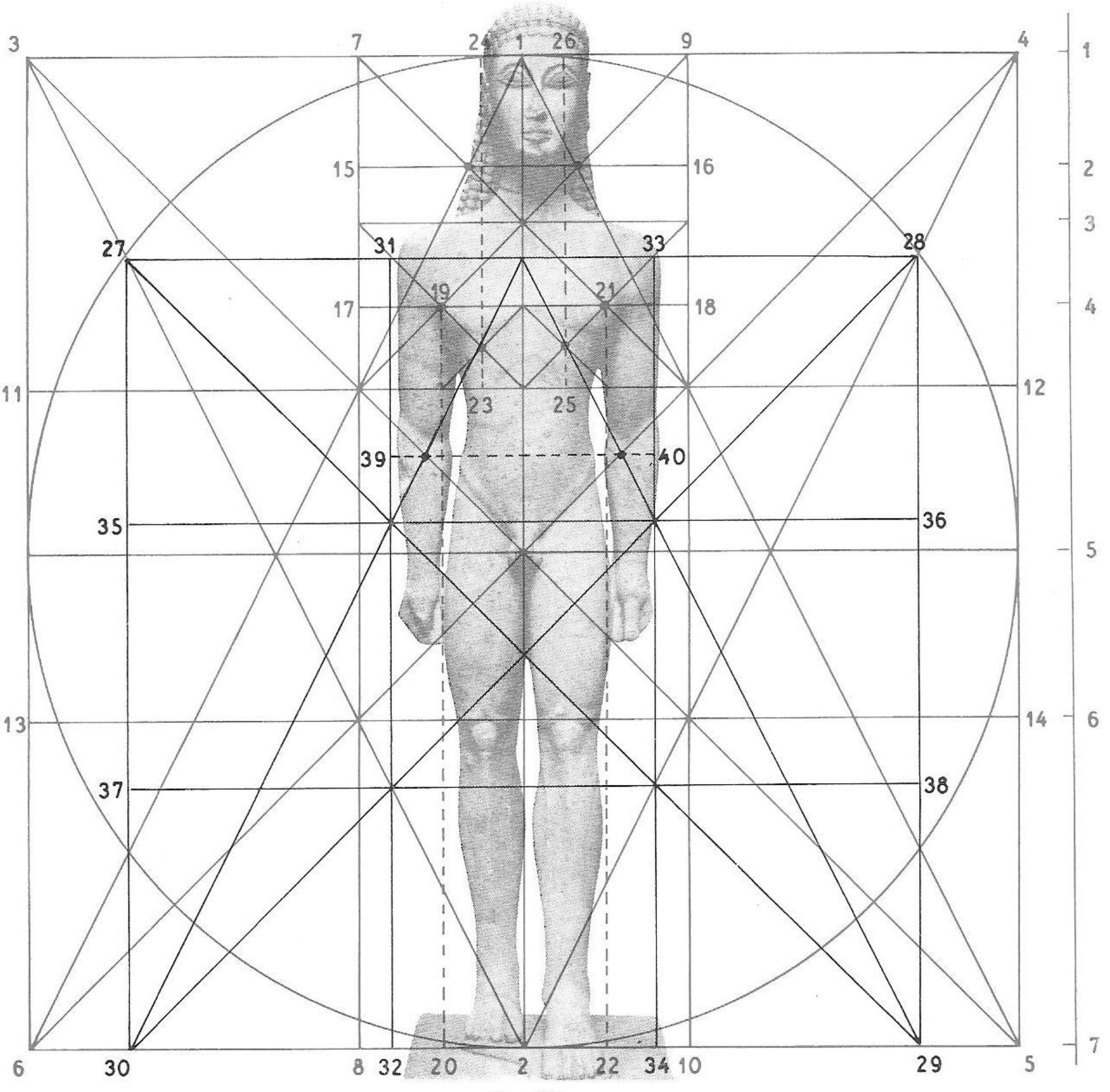

Fig. 274.

the feet provides the diameter for the constructive circle, and the basic square, 3-4-5-6, is also entered.

As with the Egyptian figures, we divide the basic square 3 × 3. The lines are (vertically) 7-8 and 9-10 and (horizontally) 11-12 and 13-14. Immediately we can compare the difference mentioned above in the width of the two figures. Whereas the Egyptian filled out the central vertical rectangle, the Greek is much narrower than this rectangle.

The uppermost central square is also, as before, divided 3×3 but only those lines we require here have been entered.

The upper horizontal line of 3-part division, 15-16, marks the top of the throat. In the Egyptian figure the same line marked the bottom of the throat.

Thus another difference: The Egyptians took $1/9$ of the figure's height for the face and neck, the Greeks used the same $1/9$ for the face only, which altered the figure's proportions considerably.

The next horizontal line employed as a point of proportion by the Greeks is the horizontal axis of the small upper square, line 41-42. It was applied to mark the line of the shoulders. The Egyptians made no use of that line at all. It fell well below the level of the shoulders.

By using this line as the shoulder axis, the Greek sculptor gave his figure a long neck. These dimensions were in turn deducted from the length of the body, which naturally ended up shorter than the Egyptian.

The Egyptians fixed the level of the armpits as the top of the square on the circle's rectangle, but for this proportion the Greeks went lower in the diagram, selecting the lowest of the lines of 4-part division in the small square: line 17-18.

The 4-part division of the same square was also the basis of determining the figure's width: lines 19-20 and 21-22. Subdividing the square vertically one stage further, i.e. 8-part, we see the width of the face between lines 23-24 and 25-26.

On counting our progress so far, we find that we have traced seven horizontal lines of proportion (the same number discovered in Egyptian sculpture): 1. Forehead, 2. Throat, 3. Shoulders, 4. Armpits, 5. Pelvis, 6. Knees, and 7. Feet.

But we lack a use for one important geometric figure: the square on the circle's rectangle.

We observe it entered in the diagram in black: 27-28-29-30. As is our normal procedure, we split it 3×3. The lines are (vertically) 31-32 and 33-34, and (horizontally) 35-36 and 37-38.

We see that the central rectangle in this square was taken as the total width of the figure, whereas in Egyptian art the same width was given by the central rectangle in the *basic* square.

No specific line has yet been discovered indicating the line of the elbow and it would appear that for this dimension the Greeks took the line that joins the two intersections of the acute-angled triangle in square 27-28-29-30 with the diagonals of square 3-4-5-6. The line is shown as 39-40.

From the pelvis to the feet the dimensions and proportions are the same. The difference between this Greek figure and the Egyptian proportions lay in the upper body and the figure's width.

The two sets of proportion agree on major issues, e.g. the same symbol was used in both cases as was the same procedure of division, and there were seven main height-lines in each case. The difference in the two periods of art and their respective proportions lay in the application of the geometric lines. And we have seen why the variations appear to have been introduced.

It is interesting, I think, to note that whereas the Egyptians employed $1/3$ of the basic square to indicate the figure's width, the Greeks chose this from $1/3$ of the square on the circle's rectangle. If we turn back to our earlier analyses of Grecian temples, we discover that this was precisely the same part of the symbol used by the Greeks to transfer the facade dimensions to the ground-plan and (by 3×3 division) to lay out the arrangement and spacing of columns.

How can it be that the Greeks made a mistake in applying the Egyptians' proportions to the human body?

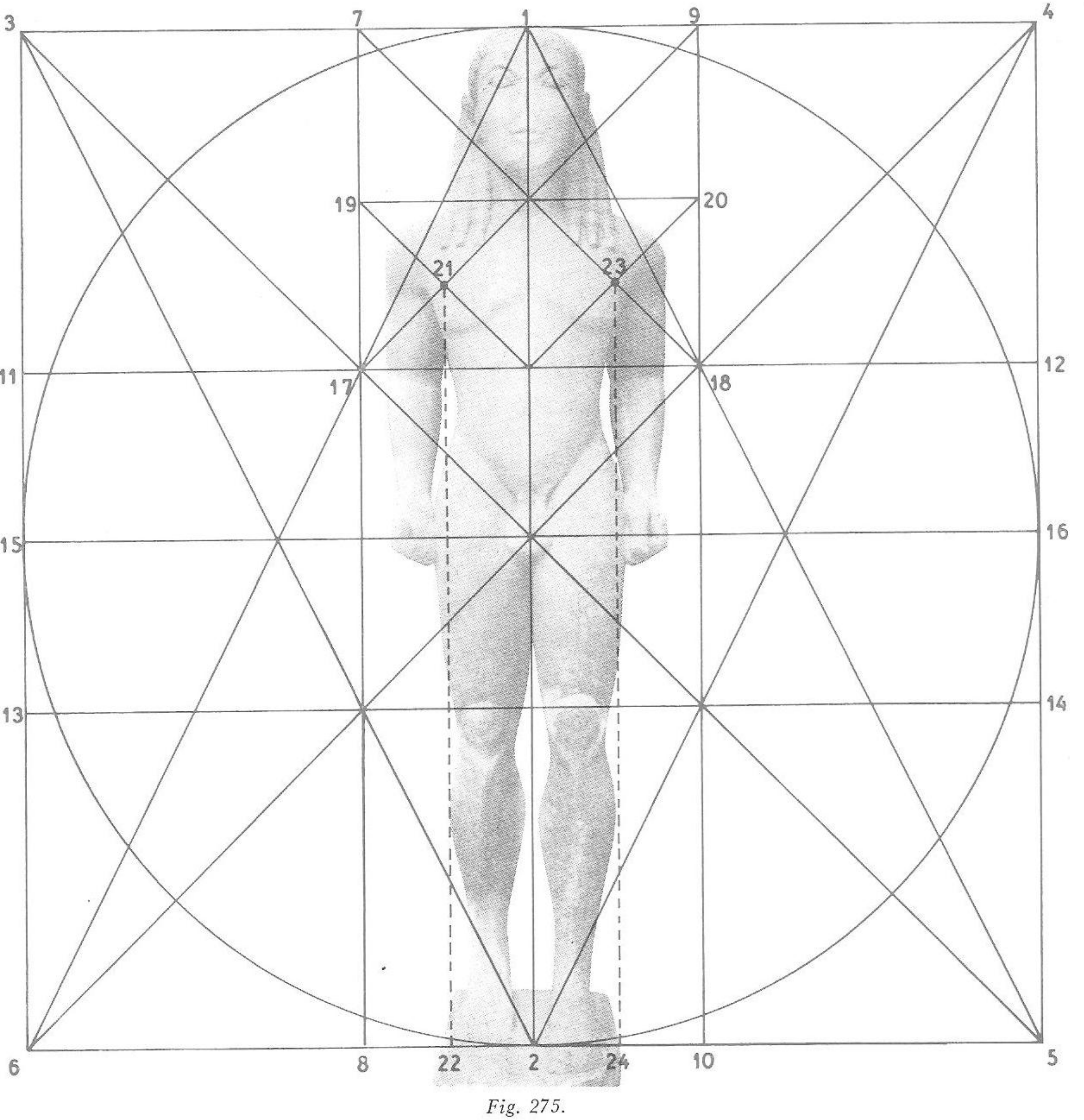

Fig. 275.

In practice it is extremely easy to make such an error. If the man or men who transferred from Egypt to Greece the ancient system of human proportions recalled that seven lines were necessary in allotting the height but forgot the precise position of one of them, putting in another instead, then the remaining lines would be similarly misplaced upward or downward.

Any artist not satisfied with the result and suspecting that something was amiss with the proportions would probably try to find the correct geometric dimensions. The next few analyses will indeed show that the Greeks recognised the imbalance in proportions, and that they searched within the geometric symbol for the correct answer.

And here—with this search—we have the real beginning of Greek's independent career in the sphere of art. It is here that she started building upward from the foundation laid by the Egyptians. Perhaps forced to progress on her own because of a lack of information on the procedure used by the Egyptian artist.

Acknowledging eventually that they were using the Egyptian symbol wrongly, Greek artists discovered that their statues (principally of young men) had elongated faces, much longer than was natural. In their later work and experiment they compensated for this fault by placing the whole figure inside the symbol, starting their proportions at the roof of the figure's head rather than at the root of the nose.

We see the result in *Fig. 275* which is a statue of a young man from about 590 B.C.

We start with the vertical axis which extends from the soles of the feet to the roof of the head. With this line (1-2) as the diameter we construct the circle and enter the basic square, 3-4-5-6. The square's horizontal axis is still the figure's pelvis line.

The square is divided 3 × 3, and the horizontal lines of division, 11-12 and 13-14, as in the past cut through the elbow and knee respectively.

Entering the vertical and diagonal crosses in the upper central square, 7-9-18-17, we are able to add the horizontal axis 19-20. The lower rectangle (half) of this square of course forms two squares, and the diagonals are entered in these, permitting us to indicate the 4 × 4 division of square 7-9-18-17.

This division at points 21 and 23 marks the position of the armpits and when lines 21-22 and 23-24 are produced downward through the diagram, we see that they indicate the width of the figure at the hips.

This tallies exactly with the preceding diagram. The only difference so far is that the head has been shortened slightly to bring it within the basic square.

Optically the difference is considerable; the first statue is long and slim, the second appears short and dumpy. But apart from one or two minor anatomical amendments, we discover surprisingly that the proportions of the two are identical.

We move to a new diagram, *Fig. 276*, in which the old lines are shown in black, the new in red.

The square on the circle's rectangle is 25-26-27-28, and as before is divided 3 × 3. The vertical lines are 29-30 and 31-32 and the horizontal lines 33-34 and 35-36.

We find, as in the previous analysis, that the 3-part division of this square provides the total width of the figure. The top of the square, line 25-27, lies across the figure's shoulders: rather higher in the Greek statue than in the Egyptian where the line marked the armpits.

The diagram lacks constructive guidelines for the proportions of the head, and for this purpose the Greeks placed on top of the square on the circle's recatangle a small square equal in area to the nine into which 25-26-27-28 is divided.

This was another link between Egyptian and Greek procedure. Whereas the Egyptians in order to proportion the head and head-dress added a square equal to $\frac{1}{9}$ of the basic square (see Fig. 269), the Greeks chose to use $\frac{1}{9}$ of the square on the circle's rectangle. The evidence would make one suspect that the Greeks, on be-

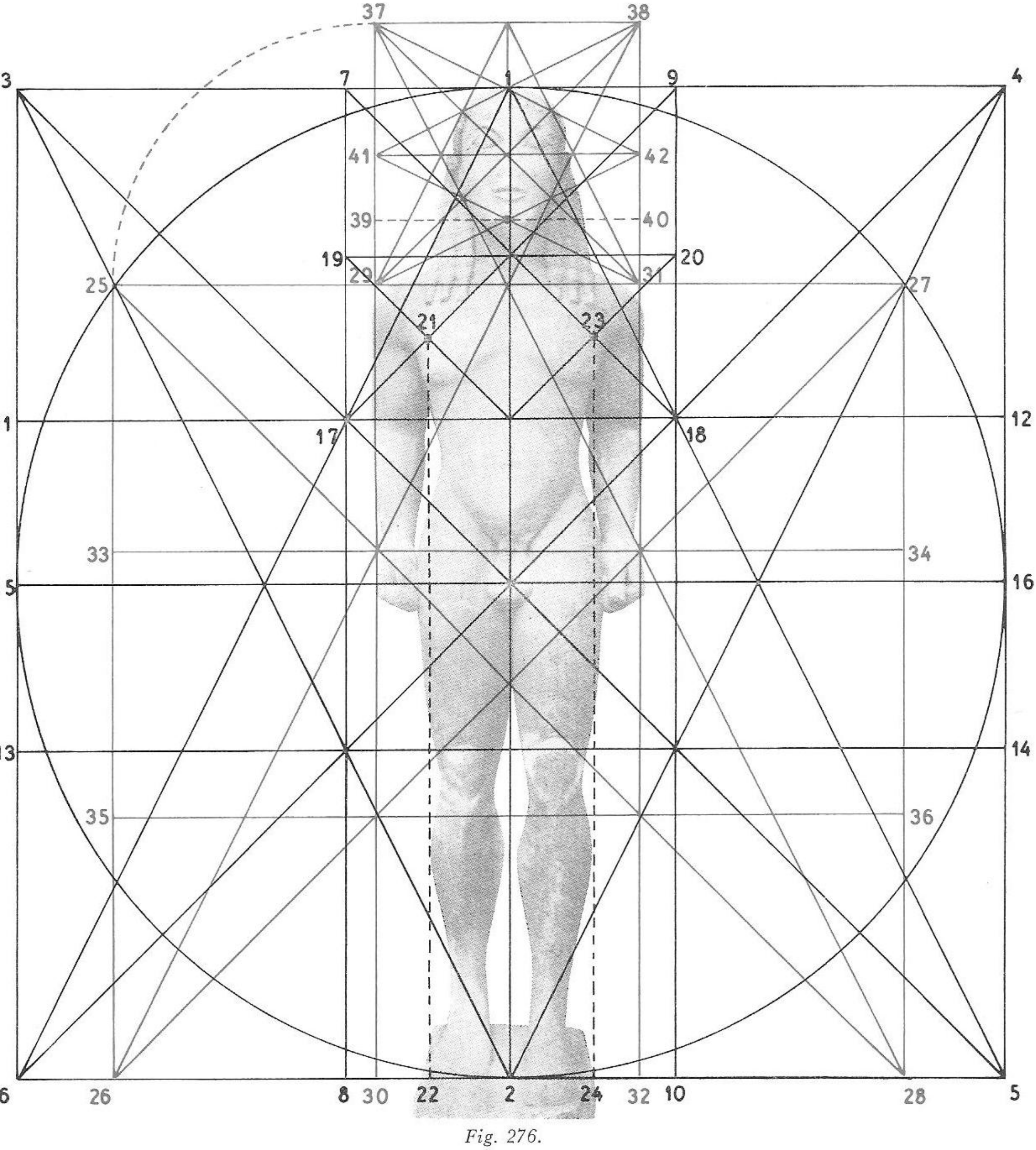

Fig. 276.

ing told of the square from which the body's width was determined, confused the two squares.

The Egyptians took the central ⅓ of the basic square as the total width of the human form, the Greeks applied the cor-

responding part of the square on the circle's rectangle, achieving a slimmer figure in relation to the height.

The new, extra square in question is seen as 37-38-31-29.

The usual basic guide-lines (vertical and diagonal crosses and the acute-angled triangles) are entered and form the eight-pointed star which, as we saw in Fig. 57, was the climax of a long period of geometric experiment and speculation, e.g. the secret of the division of a given square into a number of smaller component squares.

The horizontal axis (41-42) cuts through the centre of the square, of course, and it is interesting to note how the centre of the square lies (accidentally?) on the youth's nose.

The lower line of 4-part division (39-40) shows the position of the chin, and the inner area of the eight-pointed star frames the face, which makes the face equally as wide as long and gives it an almost circular form.

Two other lines worth mentioning are 42-29 and 41-31. Note how they govern the slope of the shoulders.

The symbol is by no means fully developed, there are lots of other possibilities. But I think the survey we have carried out on this statue from 590 B.C. illustrates that the Greeks applied the same procedure of proportion as did their Egyptian forerunners. We have traced the sculpture's main dimensions.

Where is the basic optical difference between early Greek figures and corresponding sculpture from Egypt. It apparently lies in the fact that although determining the height by the same principle (the basic square) the Greeks fixed the human body's width as $\frac{1}{3}$ of the square on the circle's rectangle instead of the same part of the basic square. Thus the body appeared slimmer.

Added to this change, the fact that they selected a shoulder-line that was too low, the upper part of the body could not avoid being shortened disproportionately as the pelvis was at the right height. Consequently, as we saw, the shoulders, neck and face, were at the outset too long.

They made up for dimensional errors in composition by shortening the figure

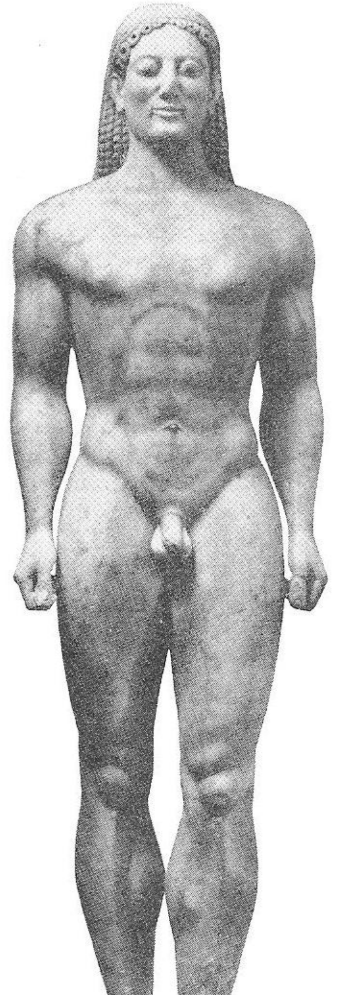

Fig. 277.

in relation to the basic square, i.e. bringing the top of the square level with the roof of the head instead of the forehead.

The figure is thus shortened by about $\frac{1}{19}$, and the visual effect is quite noticeable. No change has been made in the width of the figure; the head is merely lowered to procure a better balance in the proportions—and to come closer to their image, the Egyptian model.

We move forward in time by nearly a century in order to see what modifications the Greeks have made in their human proportions in art. Our geometric diagram will be tested on a figure from ca. 515 B.C., i.e. a statue modelled about a hundred years after the period normally accepted as the birth of Greek art.

In *Fig. 277* we see a well-proportioned sculpture of a young man. It is exhibited in the National Museum in Athens.

The figure gives the impression of strength, and the sculptor appears to have achieved an excellent balance between head and shoulders in relation to the rest of the body.

As regards the pose, there is no mistaking the typically Egyptian stance. Head up, shoulders back, left foot slightly forward!

In *Fig. 278* the geometric diagram applied to the previous figure (Fig. 276) has been constructed. With the exception of one single point the proportions are identical to those of the preceding sculpture.

The figure is placed completely within the basic square. The square on the circle's rectangle marks (line 5-6) the horizontal shoulder axis, and the 3-part division of the same square marks the width of the figure.

The basic square's horizontal axis (37-38) is the line of the pelvis, and the two horizontal lines of 3-part division 39-40 and 41-42 again mark the elbow and the knee.

On top of the square on the circle's rectangle is a square which in area is $\frac{1}{9}$ of the latter. The same as constructed in the preceding analysis. The head and face are proportioned by the framework of this additional square.

Whereas previously the centre of the square was placed at the man's nose, here it is raised to the root of the nose. The width of the face is indicated by the two vertical lines (31-23 and 32-25) of 3-part division, while the height of the head is determined by two of the 4-part dividing lines. The chin rests on line 35-36.

The head is hardly as wide as that of the figure in the preceding analysis, resembling more an oval shape than a circle. As in the earlier analysis the slope of the shoulders follows the two diagonals (33-13 and 11-34).

The only real difference between this figure and the preceding is in selection of body width. Previously the width of the body (minus the arms) was determined by a sub-division of the upper central square produced by the 3×3 division of the basic square. In this case however the width was determined by the same division of the corresponding square in the square on the circle's rectangle, 11-13-20-19. This latter square was first split into 3×3 smaller squares and further down into 6×6 (via the diagonals of the 9 small squares).

We observe that the two vertical lines of 6-part division, 27-28 and 29-30, run from the armpits down through the diagram, and in so doing mark the width of the body without the arms.

Considering the diagram and our analysis as a whole, we find that although nearly a hundred years had passed extremely few and minor amendments were made to the system of proportion.

The head and shoulders occupy the same part of the geometric diagram as before, the only alteration being a slight

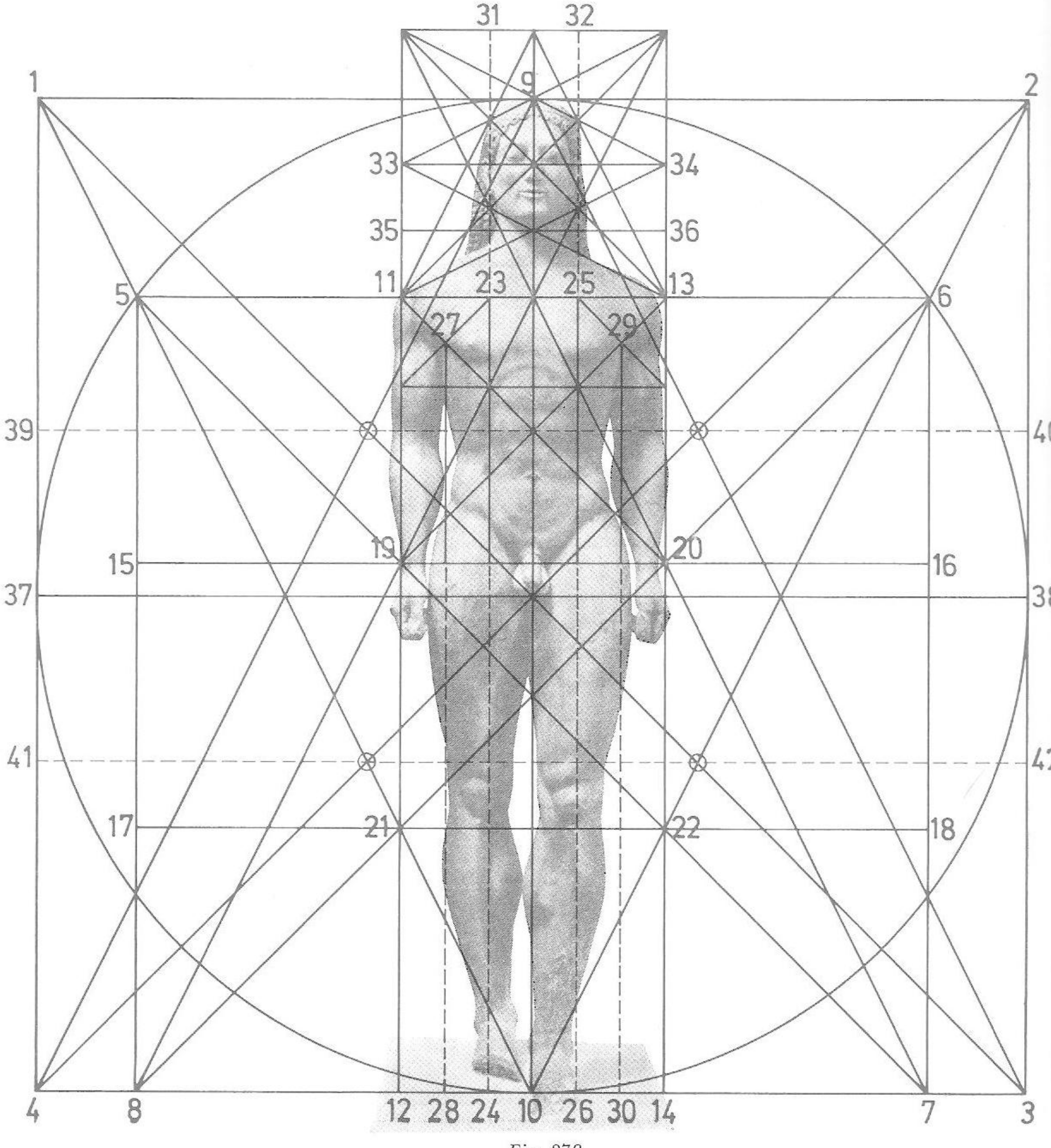

Fig. 278.

narrowing of the head thanks to the use of the 4-part division instead of the 3-part.

The general dimensions of the body were retained. The width of the body was determined by factors within the square

on the circle's rectangle and not directly within the basic square.

Our series of analyses tends to illustrate the selection of more and more factors from the square on the circle's rectangle. Apparently artists were content to let the basic square become a mere aid to the construction of the diagram.

And this fits in with our knowledge of Greek design and proportions. If we examine the diagram closely and compare with earlier analyses, we find that it is precisely the same diagram as applied to the proportions of all the ancient temples inspected in Chapter Nine.

In building, the square on the circle's rectangle contained considerable instruction for planning the respective temple facades. When that figure was such an important feature of temple-building (from both a proportional and symbolic standpoint) it is not really surprising to discover the Greeks trying gradually to give it an equally vital place in the proportions of the human body and in art generally.

The Greeks evolved a form of proportion based on the symbols of ancient geometry which had the property of harmony and beauty. It also matched what could be described as the ideal proportions of the human body.

The Egyptians had also, within the same symbols, discovered their ideal human form. But whereas they stuck rigidly and stubbornly to their original human proportions for a period of *several thousand years,* the Greeks were more adventurous and developed the symbols to the stage where they reckoned was geometric and proportional perfection, i.e. the perfect harmony.

Once they had found the "perfect" human proportions, the urge came to give the figures movement. Previously the Greeks had copied the standard Egyptian pose: an erect figure, facing straight forward, with slight movement of the arms. But as their expertise increased, so did their appetite for more realistic sculpture. And it fired off the Greek flair for advancement. It brought them infinitely further than the Egyptians had reached.

This urge to see figures move registered about 125—150 years after the arrival in Greece of figurative painting, sculpture, etc., as near as surviving works permit us to ascertain.

Movement, and the departure from straight, stiff, angled figures, presented many a headache to the artist intent on retaining his geometric principles. But in spite of the difficulties he succeeded more or less in fulfilling the demands of ancient geometric symbols and yet achieving the desired movement and vareity.

I think his modus operandi was something like this:

He was accustomed to proportioning a human body standing erect within a square and a circle, the figure occupying the square's vertical axis. We see this in *Fig. 279 A* in which the height of the figure is from a to b. The centre of the diagram and the line of the figure's horizontal axis is at c.

In *Fig. 279 B* we see an alternative placing. It illustrates that there was nothing to prevent the artist transferring the figure's proportions to another part of the circle. The total height, d-e, is the same (= circle's diameter), and the various nose, throat, neck, elbow, pelvis, knee lines, etc., can be transferred by means of compasses from the centre of the diagram.

If the figure can lie from d to e, reasoned our Greek artist, then he can also take the pose d-c-b, leaning forward from point c.

We see this in *Fig. 279 C,* and a natural extension of this stance might be to raise one leg backwards to balance the body. The figure would thus be shaped d-c-b and d-c-e, *Fig. 279 D.*

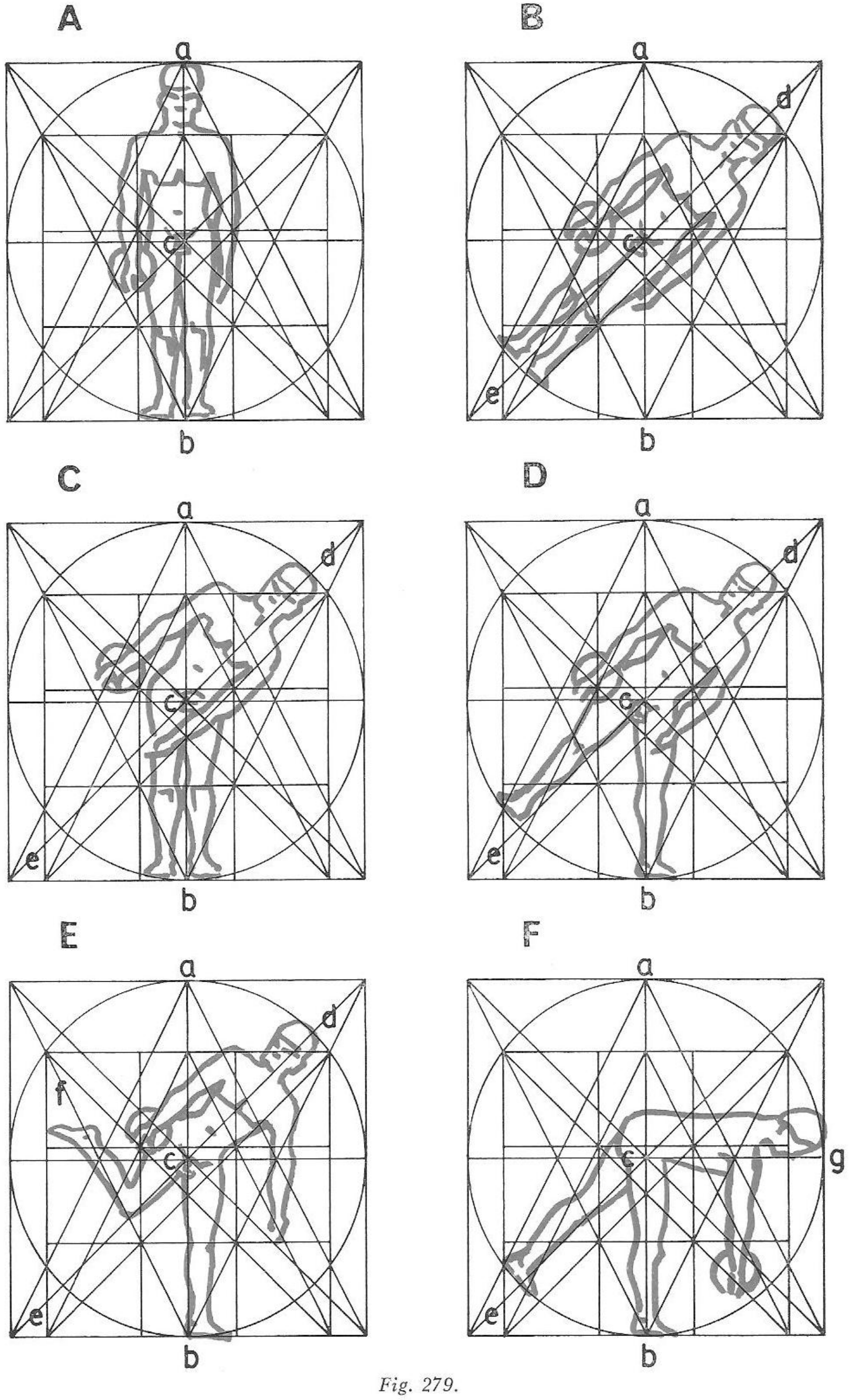

Fig. 279.

The artist perhaps went on to bend the raised leg to point f. The bend would occur at the point in the diagram indicated by the knee-line. This is illustrated in *Fig. 279 E*.

And as a final example we have in *Fig. 279 F* the figure bent forward at a 90° angle. The upper body is horizontal (c-g) and the legs are perpendicular (c-b).

The apparently severe laws of geometry could therefore become flexible enough to permit artists to introduce movement in their compositions.

I regard a typical example of this development of motion to be one of Greece's best-known works of art: the sculptured figure of the discus-thrower. The most complete surviving copy of this work by Myron is in the Terme Museum in Rome.

Experts have frequently discussed the attributes of this discus-throwing Greek youth, and one school of opinion maintains that in spite of the figure's undoubted harmony of motion it is in fact anatomically impossible to take up this precise stance. The hips, it is claimed, are twisted in a manner that no youth (Greek or otherwise) could adopt.

If this is the case, it supports my theory that *diskobolos* was constructed rather than modelled from life. An artist working from a real-life model would be able to identify the possible and distinguish it from the impossible: particularly an artist with the mastery of Myron. For regardless of the basis of the sculpture, the discus-thrower is a magnificent piece of work.

We begin our analysis with *Fig. 280* in which we find the figure placed within a circle, but instead of standing erect on the vertical axis with the middle of the circle as the line of the pelvis, the figure has been bent forward 45° with the upper body pointing from the centre of the diagram to corner 5 of the basic square. The head remains flush with the diameter of the circle.

The symbol also has the lines of 3-part division: 9-10 and 11-12 (vertically) and 13-14 and 15-16 (horizontally).

It is worth noting first that diagonal 5-6 cuts through the sculpture's chest. And one of the vertical lines of 3-part division, 11-12, now forms the figure's new axis. Moreover, it is interesting to observe how the artist placed the lower arm along line 5-2 of the acute-angled triangle. And the chest muscles were developed around the intersection of lines 5-6, 1-8 and 11-12.

The position of the legs was planned as follows:

The lower horizontal line of 3-part division forms in the normal standing posture the line just above the knee-cap which we call the knee-line. The length of this line is thus from the pelvis (horizontal axis) to the line of 3-part division.

The centre of the diagram and therefore the pelvis line has been labelled O, and the line normally regarded as the thigh is OA.

Using the length of that line as the radius of a circle, the artist swung it up to the required angle, indicated in the diagram as OB: which was where he ended the thigh and began the knee-cap.

The vertical height of the foot, shinbone and knee is normally the distance between the 3-part dividing line and the base of the square. We therefore take this distance and place it downward from point B. This (BC) marks the sole of the figure's right foot. When we swing BC round (with B as pivot) to point D we mark the sole of the other foot, pointing to the rear.

The analysis is continued in *Fig. 281* in which we have entered the square on the circle's rectangle and the small square above it, used to proportion the figure's head and shoulders.

We observe first that the diagonal (29-31) of the upper central square follows the figure from armpit to hip, marking

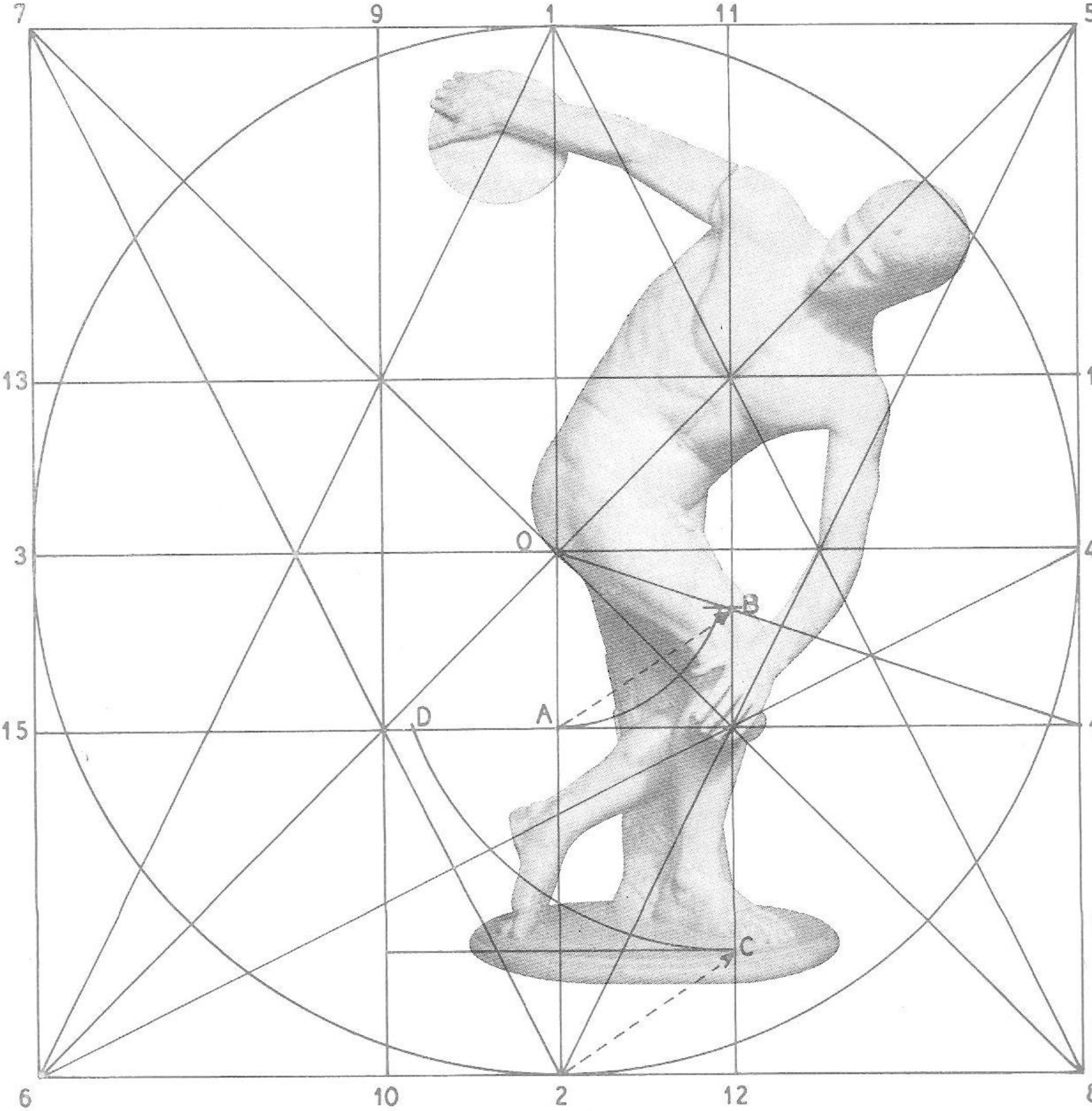

Fig. 280.

one side of the body, while the diagonal (22-18) in the square on the circle's rectangle marks the other side. In modern mathematical terminology the width of the body was made to equal $\frac{1}{\sqrt{2}}$, which is close to the net width of earlier figures.

As regards the head, its dimensions are conveyed round to appropriate points in the symbol. The roof of the head, for example, is part of the arc of the constructive circle (arc EF).

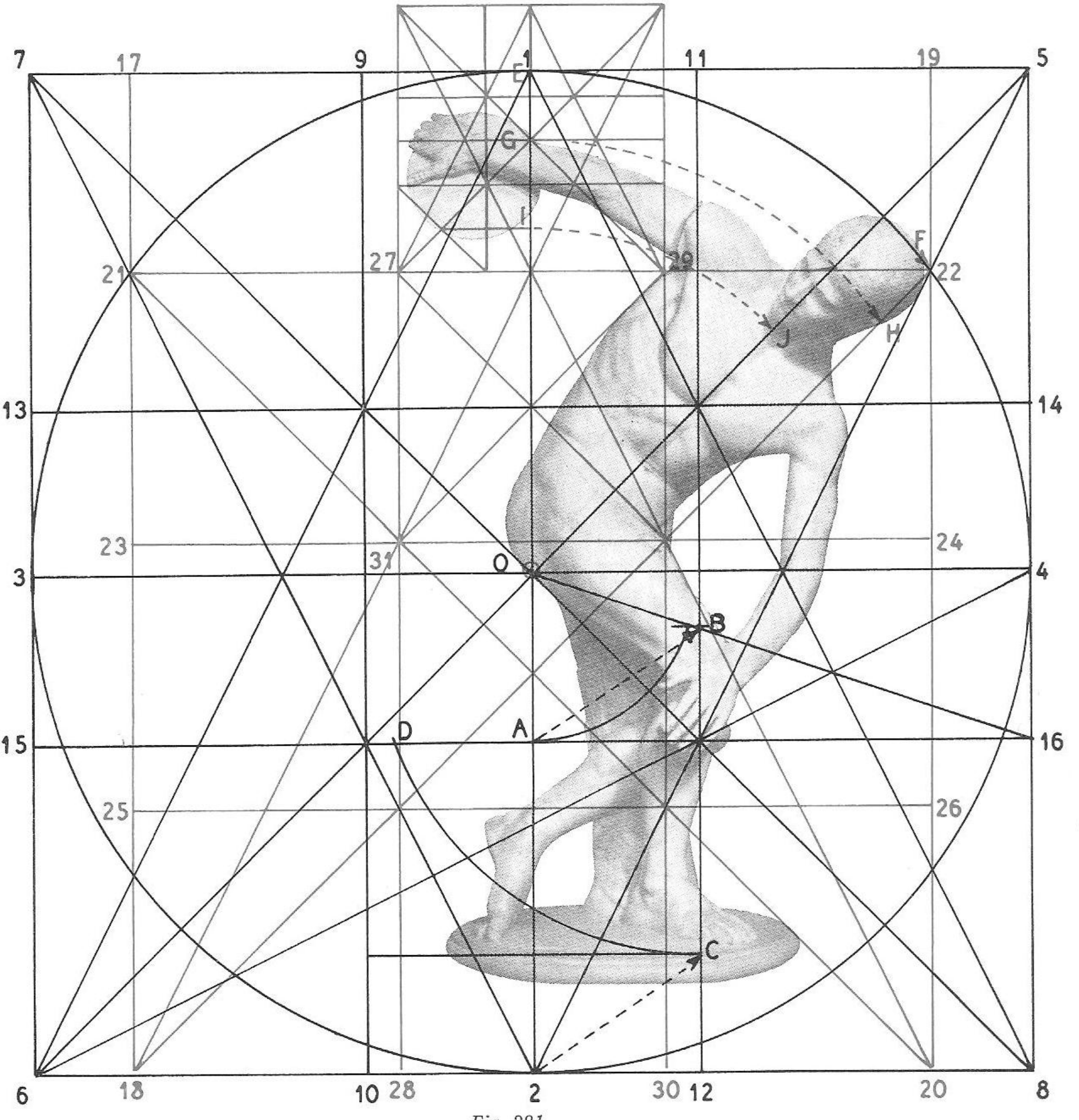

Fig. 281.

In this sculpture the horizontal axis of the small square marked the figure's hairline (shown by arc GH). This infers that the face is lower than the last figure examined. The reason is however that the discus-thrower bent his head and tucked in his chin slightly. Hence his marking lines vary from those of a standing figure.

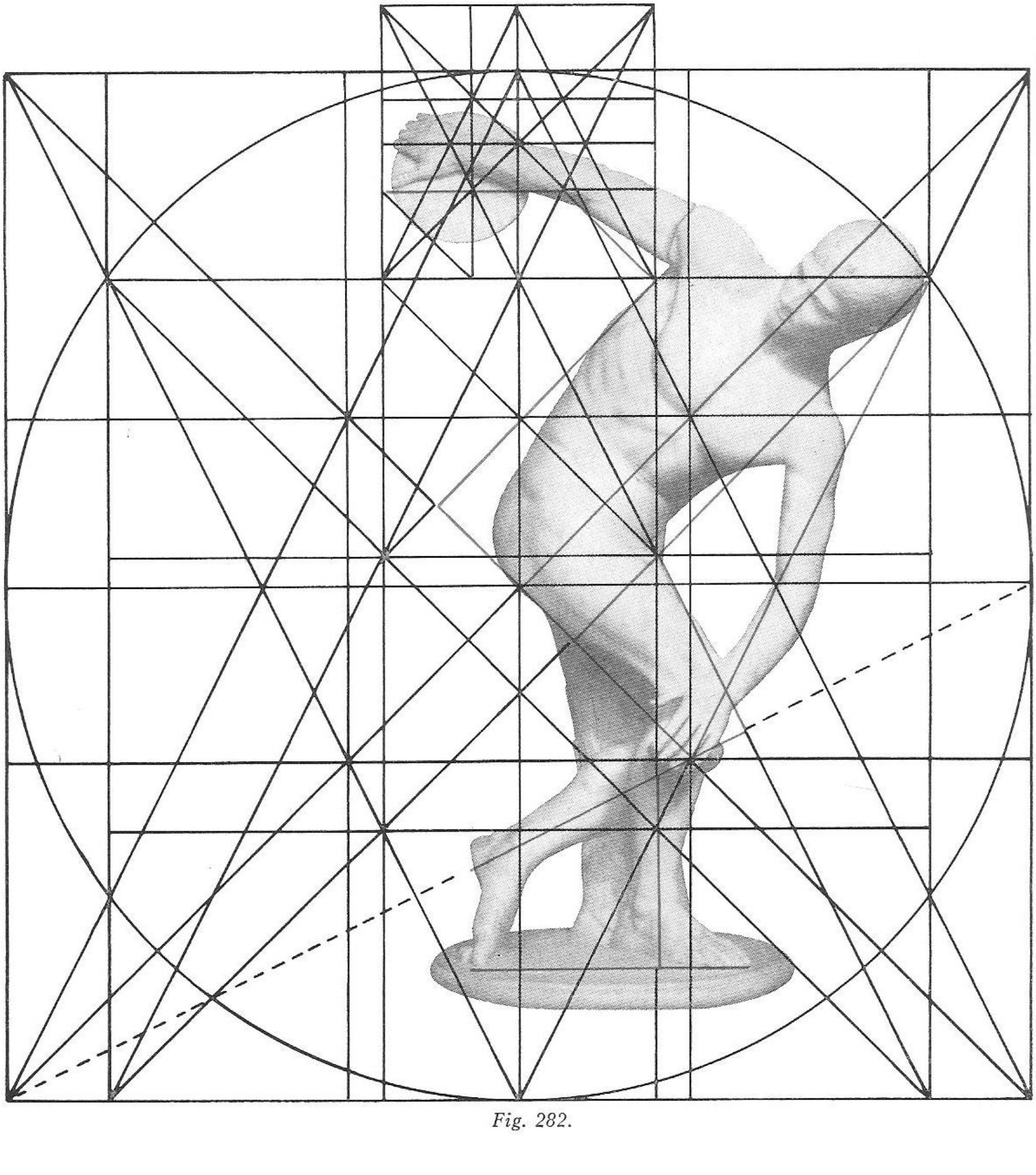

Fig. 282.

The chin (shown by arc IK) was placed ⅙ part of the square above the base-line instead of the more usual ¼.

The respective guide-lines woven throughout the symbol acted as a kind of support or source of inspiration for the artist. From these he selected the lines of the discus-thrower's skeleton.

In *Fig. 282* we see the symbol and its lines printed in black while the lines se-

lected by the Greek sculptor as the basis for his discus-thrower are in red.

We can observe plainly how they form the figure's basic composition and indicate all the main angles of the body, arms, legs, etc. They moreover mark several exact lengths and widths.

The details of the discus-thrower's body are indefinable. All we can hope to trace in an analysis of this sort are the "bones". In the realm of art it is particularly difficult to uncover the stage at which geometry ends and the creative artist branches off on his own.

For our final look at Greek sculpture and its associations with ancient geometry we move forward another century and select a sculptured figure of the Aphrodite from Arles, made about 330 B.C., and displayed in the Louvre in Paris.

We conduct our analysis in *Fig. 283* in the normal manner by entering the vertical axis from feet to the roof of the head (1-2). The circle, basic square, diagonals, acute-angled triangles and lines of 3-part division are added.

This figure boasts more in the way of clothing than those inspected earlier, and her toga is draped over the arm, round the waist and down to the feet. The figure therefore is wider than normal—but it is interesting to see how, with the exception of the right hand which appears to grip the circumference of the circle, the whole figure including toga is placed within the three central squares.

The preliminary guide-lines mentioned above (circle, basic square, etc.) have been entered in black. In red we have the more important part of the symbol: the square on the circle's rectangle, 17-18-19-20. The 3-part dividing lines of this square are (vertically) 21-22 and 23-24, and (horizontally) 25-26 and 27-28. We have also entered the small square required to indicate head and neck proportions.

The goddess has bowed her head slightly to one side. The angle of the head is shown by diagonal 32-29.

Line 33-34 coincides with the root of the statue's nose when the appropriate distance is measured in an arc from the centre of the symbol, similar to the procedure adopted with the discus-thrower. The same is true of line 35-36 and the figure's chin.

Square 21-23-30-29 is divided 3×3, and the necessary action taken to enter the 6-part dividing lines. Two of these, 37-38 and 39-40, mark the width of the body as they did in earlier Greek statutes.

The figure's basic proportions are plainly based on the same principles as guided artists both in Greece and Egypt for hundreds, thousands of years previously.

We have preserved an unending stream of sculpture from the period of Greek glory. It is spread throughout the world. To reproduce an extensive and detailed representation of the entire field of Greek sculpture in a book of this character would be impossible for space reasons alone.

But the examples studied illustrate that Greek sculpture was not the free, unfettered subject we consider it today. Its tightly bound links with Egypt are evident. The slackening process is traceable. But it took centuries before art threw off the reins of geometry completely. Because of the very nature of the subject, with the freedom of the individual artist in mind, it would be unwise even to hazard a guess at the date of art's genuine freedom. It may well have remained a geometric (and religious) prisoner until the Middle Ages —when the building and architectural field also tore or wore away from geometric guidance. It is impossible to tell.

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Searching for the rules of proportion that governed art and architecture through the ancient centuries is not an

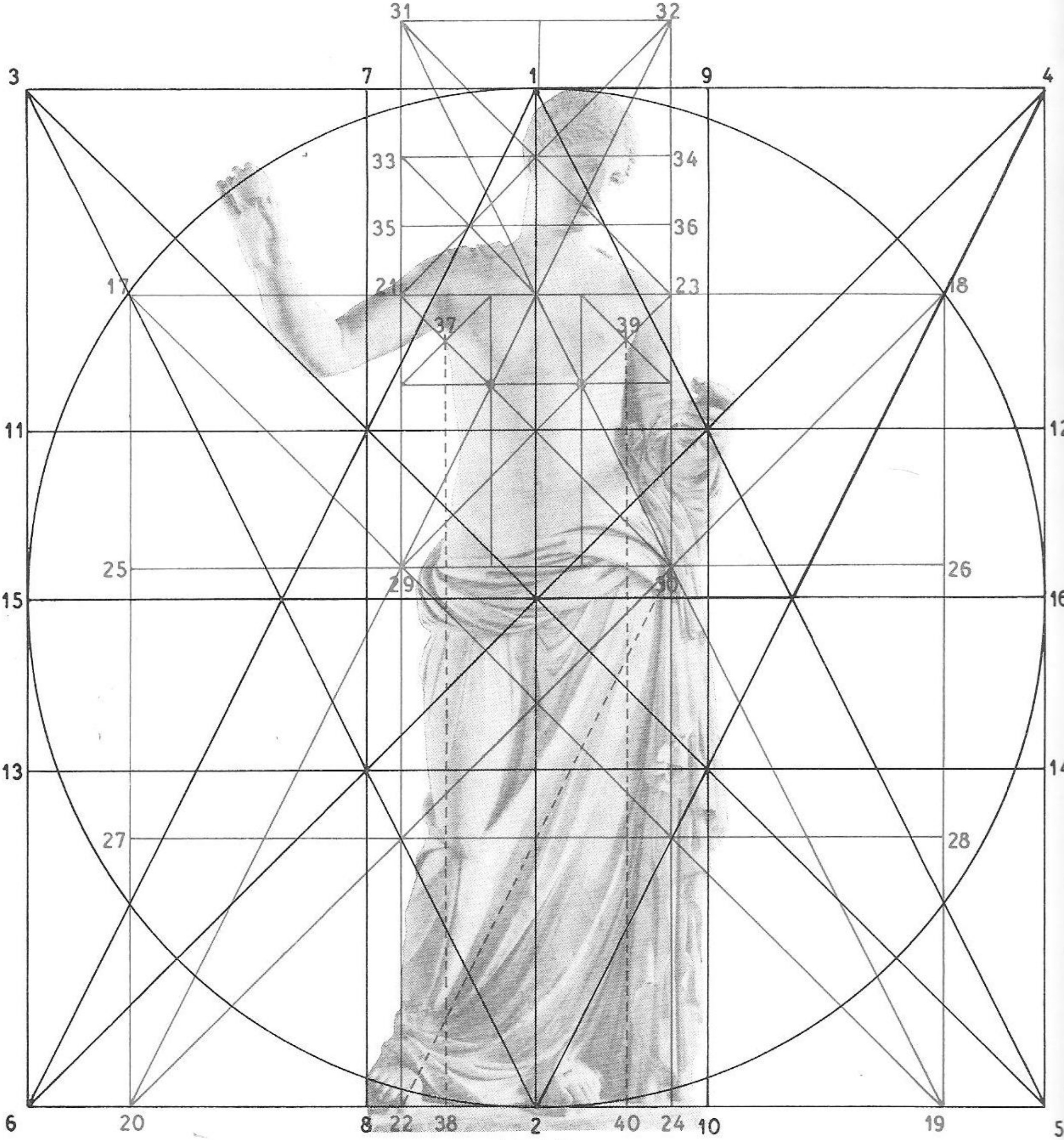


Fig. 283.

occupation restricted to our generation. For hundreds (probably thousands) of years "outsiders" have tried to locate the esoteric system, knowing that it existed and eager to bring it to light.

And simultaneously we have the para-

dox of brethren initiated in the system being familiar with it and applying it quietly in their work.

The certainty with which outsiders maintained that a mystical, secret system of proportion was (or had been) in use by designers, artists, architects and geometers of the past was based partly on ancient literary hints of some harmonious concept. In the search this concept has been termed "Ad Quadratum", "Ad Triangulanum", etc.

The rumours persisted as faithfully as in many other subjects involving a little imagination.

We can call to mind many areas of the world—usually remote islands—in which it is believed (or rumoured) than Blackbeard or some other colourful pirate buried a huge treasure trove—and never managed to collect it again.

Even though several centuries may have passed since the mystical treasure was buried, the legend lives gloriously on. The local population have no doubt about the matter. And periodically strangers with picks, shovels and (in more recent years) mine detectors have tried their luck in the area.

We read earlier of extremely likely evidence that Euclid and his contemporaries did their utmost to discover the treasures of ancient geometry. They were aware of its existence, and had something concrete to concentrate upon.

They were presumably also fully aware that initiated circles around them were even at that time making use of the geometric rules. It was not a question of delving with a mental shovel back into the past. The search area for Euclid was his own period.

Despite their strong hand, they did not succeed. It was impossible for them to breach the initiates' wall of silence, and without someone to guide them it was equally impossible to know whether the factors they eventually assembled on their own were the secrets of the Temple, or something else altogether.

Euclid's research of ancient geometry was dated at around 300 B.C. and his efforts were devoted to pure geometry. He did not look into its application in practice.

Another hopeful treasure-hunter who on the other hand concentrated on geometry's practical side was the Roman architect, engineer and author, Marcus Vitruvius Pollio, who lived in the 1st century B.C.

His main duties, as far as we can make out, were to erect buildings for and in honour of Julius Caesar and Augustus. But Vitruvius was also interested in the history of architecture.

His best-known treatise *De architectura* has now been translated into many modern languages, and concerns a number of practical problems of building of his own period. In addition he conducted some research into similar problems of earlier eras: art and building of the ancient Romans and ancient Greeks.

Today we regard Vitruvius almost as an abstract idea, and his book as an architectural bible. Yet his theories on proportion were insufficiently documented and justified to silence further discussion on the subject.

If *De architectura* had been able clearly and concisely to lay down the rules by which ancient proportion was established *and to prove the truth of Vitruvius's theories,* then there would have been neither need nor room for future discussion. But unfortunately this was not the case.

He put forward many unfounded, unsupported assertions, and thus robbed his theories of the structural strength one might expect of a technical report.

What was Vitruvius's personal status in the Italy of his day? Was he likely to be familiar with the esoteric rules taught

and operated behind the Temple's firmly closed doors?

The answer, on reflection, must be negative.

If Vitruvius had been trained within initiated circles, it is completely unthinkable that he would proceed to jot his experiences down in a book intended for the world to read. It would have been such a break with established tradition and such a blasphemous attack on Temple property that even the mighty Caesar would have been helpless to protect his favourite architect.

In Vitruvius's day ancient geometry was in full use in those spheres which lent themselves to it.

Euclid's new form of geometry had had 200—300 years in which to establish itself and develop, and was doubtless used widely by those outside initiated circles. But we saw earlier in this book an example of ancient geometric proportion applied at least 1300 years after Vitruvius, namely in the construction of Cologne Cathedral.

Ancient geometry existed in Vitruvius's time in the same way as it existed in the era of Euclid, and we may reasonably assume that Vitruvius was unfamiliar with its secrets. (It is in fact not impossible that he was entirely ignorant of such a thing as a secret form of geometry. For if as an outsider he had known of it slightly, he would most likely have mentioned it in his book.)

Vitruvius was not at any stage a man of the Church, and as far as I have been able to discover he was never implicated in the construction of any temple.

He may, however, have known a little of the rules of ancient geometry, but considered that it would be impossible to identify them in his own era.

On Greek art, Vitruvius wrote that according to surviving legend proportions were allotted as follows:

"... if a man lies on his back with hands and feet outspread, and the centre of a circle is placed on his navel, his fingers and toes will be touched by the circumference. Also a square will be found described within the figure, in the same way as a round figure is produced."

We see in these directions a grain of justification for the methods by which we ourselves have been working in the past chapter.

We have a circle in which the figure is placed, and we have a square (on the circle's rectangle) which determines the figure's height since this square has throughout our study of Greek art indicated the height of the figure's shoulders.

Thus Vitruvius was on the right track and held part of the truth in his hands, but did not possess the full facts.

Like Euclid, he had been able to establish certain geometric conditions in words, but in commiting them to paper he approached the problem from the wrong direction and produced an incorrect construction.

Unfortunately we have no surviving example of Vitruvius's own attempts to construct his theory in practice, but Leonardi da Vinci made a full study of Vitruvius and as late as 1490 tried to reconstruct the geometric proportions discussed by Vitruvius and reproduced above.

In *Fig. 284* we see the result. He has followed the instructions given by Vitruvius.

Much can be said of this construction —and a lot has already been recorded. The first objection we could raise to da Vinci's construction is: How is the square related to the surrounding circle, for there must be some constructive link?

The illustration does not reveal the relationship, and a test on the basis of ancient geometry shows that da Vinci's square-circle combination does not belong to the ancient system.

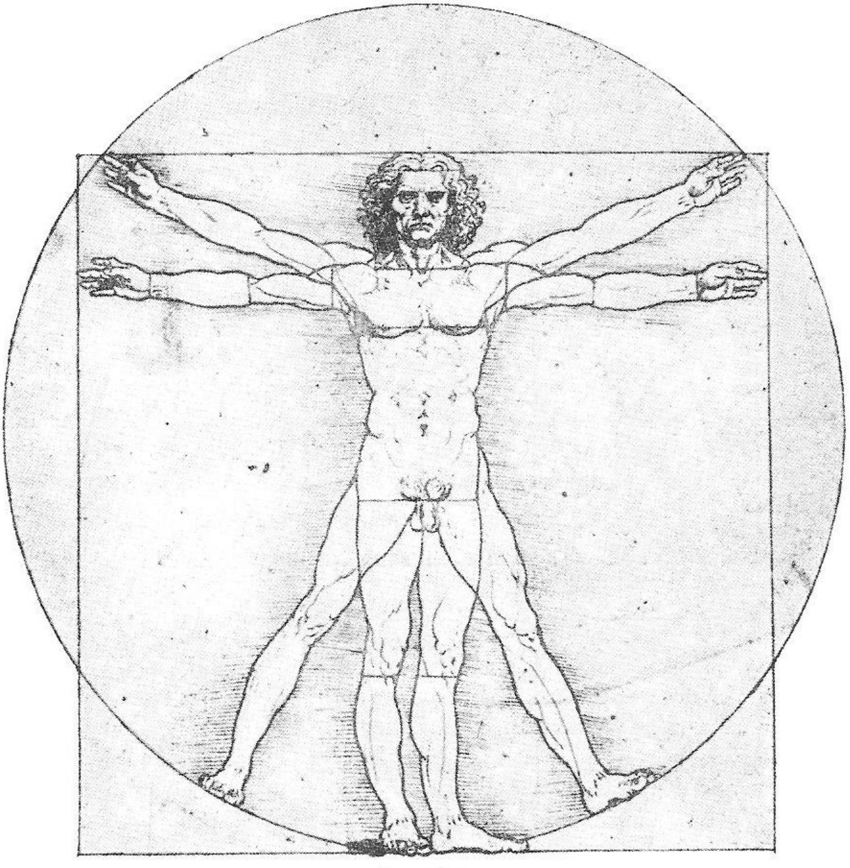


Fig. 284.

From a strictly anatomical standpoint the construction is incorrect because the man's pelvis line does not lie at the centre of the circle but lower than the centre. In other words the legs drawn on the lowest point of the circle's perimeter (one might almost call it the base-line) are shorter than the legs that follow the circumference. And the further up the circumference they move, the longer they become.

Structurally speaking, one might be entitled to ask: What geometric factor places the feet on the circumference of the circle? As indicated, the length of the legs alters according to the position on the circle's circumference. Where then is the factor that marks the true position of the foot?

Geometrically this construction provides no guide-lines that one could employ in practice, and as far as we can judge from da Vinci's drawing of Vitruvius's theory, the latter was able in some way to scrape a few facts or guesses together—and came up with a wrong interpretation.

Vitruvius was an architect and therefore accustomed to discussing and describing areas in proportion or relation to each other. The fact that he did not reveal any information on the relationship of this particular circle and square can in my opinion be caused only by his ignorance of the ratio.

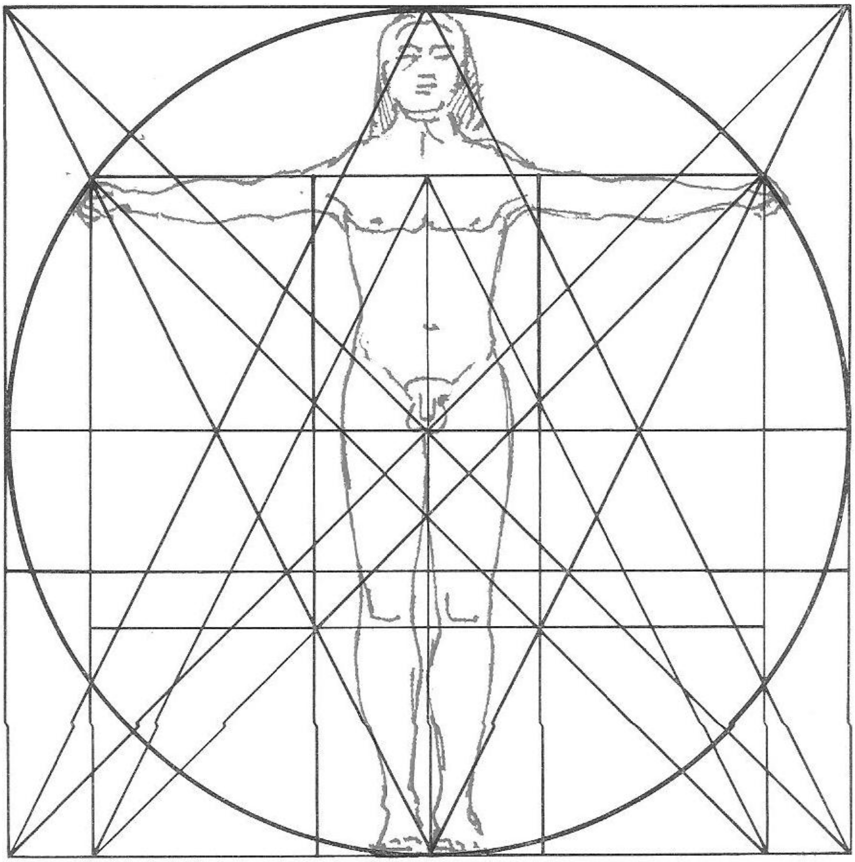


Fig. 285.

Leonardi da Vinci tried to reconstruct the situation discussed by Vitruvius, and apparently was successful. But from this evidence it would appear that what Vitruvius described and what he was told or guessed at were two different things.

Let us consider the construction in the light of our study and analysis of Greek art.

First, it is true that the human body is constructed within a circle. It is also true that the hands and toes can touch the circle's perimeter. But in the Greek system the hands and toes can touch virtually any part of the circumference without detracting from the figure's proportions—whereas Vitruvius's construction alters the length of the legs if these are moved from the vertical.

In Greek dimensions there is a square within the circle that indicates the figure's height. This is the square on the circle's rectangle, and it marks the line of the figure's shoulders.

Vitruvius apparently was given to understand (or guessed) that the square should mark the figure's total height, and not the shoulder height.

We realise in this comparison of the two "systems" that there are a number of points of agreement between the information obtained by Vitruvius and our experiences of Greek figurative art. The difference between the two is that the geo-

metric construction built up by the Greeks (and Egyptians) in their art really did provide many proportions of the human body which we can trace and check from figure to figure. Da Vinci's impression of Vitruvius's theory, on the other hand, tells us nothing of these proportions.

If Vitruvius had got his facts straight and if da Vinci had been able to follow them faithfully, the result would have resembled the figure in *Fig. 285* in which the human body is placed within the circle inside the basic square and appropriately positioned in relation to the square on the circle's rectangle.

We have vindicated here the theories with which Vitruvius toyed 2,000 years ago. Not that he was right in his thinking. He had quite simply lost the track and was unable to find his way home.

Thus we see that the theory that the human form is proportioned within a circle is not a new one. That more profound work has not been done previously on the theory seems to be an admission that the Vitruvius/da Vinci profile lacks sufficient information to allow experiment. The method is rejected and the theory that Man is created within a circle follows it into the historical trash-bin. But in fact the theory was sound enough. It was its presentation that was faulty.

Vases, jars .... and geometry

YET ANOTHER intriguing aspect of art of which we have thousands of surviving samples from antiquity is that of making ceramics and pottery. Fired clay pots, jars and vases have been recovered in astonishing quantity from archaeological diggings throughout many of the ancient areas of civilisation.

The collections, added to continuously as more excavation work is set in motion, have found their way all over the world, and are tangible proof of a highly developed sense of design and creative ability among early people.

Most of these ceramic collections comprise large vases, amphorae and bowls, and have been the subject of hundreds of books by experts and amateurs in the art of ceramics.

The Englishman, Flinders Petrie, whom we met in Chapter Six during our look at the Great Pyramid, was one of the first researchers to devise a method of dating ceramics and pottery with relative certainty. It was possible, he suggested, by a close study of the structure of the clay (broken jars and fragments were unfortunately more common than flawless pieces of pottery). Petrie moreover examined the layer of sand or soil above and below the piece of pottery and on comparing their estimated age within the position of the pottery he was able to place it fairly accurately in the archaeological calendar.

On completion of his work and surveys in Egypt, Petrie moved in 1890 to Palestine where he recovered large quantities of, among other things, pottery and ceramics.

There has been immense progress in dating excavated objects compared with Petrie's methods from the turn of the century. Today we can in many instances, for example, apply the well-known *carbon 14* test to discover the amount of radioactivity in, let us say, an ancient clay cooking pot. From this information, ex-

perts calculate the age of the pot to within a reasonably few number of years.

Although every item of old ceramics may be of interest historically, it need not be of much interest to us geometrically, and as we did in our study of figurative art we shall also here concentrate on items in which we can trace the influence and use of ancient geometry.

In addition to dating a piece of pottery according to the condition of the clay and by means of carbon 14, a system has also been built up in certain areas of classifying pottery according to the surface decoration. Greek pottery, for instance, has been divided into several periods of decoration. One such period, about 1000 B.C., has actually been termed the Geometric Period on the grounds that vases, jars, etc., were decorated not with pictures or illustrations but with geometric friezes and borders made up basically of, for example, a small square repeated again and again to form a pattern.

The most prolific collections of *objets d'art* belong to the Greek era, stemming both from the mainland and from surrounding provinces and islands such as Rhodos. This island exported large quantities of ceramics to central Europe. The isle of Samos, too, was famed for the artistry of its potters. Etruscan potters in northern Italy were another well-known group.

If we examined central and southern Europe as a whole about a thousand years before Christ, we would discover a network (with little or no interconnection between the respective areas) of local industries intent on the production of ceramics. This being so, we might expect thousands of different shapes of vases to be produced by the individual factories or centres, just as the different manufacturers of pottery and earthenware have their own characteristic shapes and models today.

But this is where we come across a surprising fact: despite the distances over which these ancient manufacturers were spread, despite the passage of the centuries, the shapes of the vases and pots which have survived to the present time bear an amazing mutual similarity.

They seem to possess a general harmony of design which cannot alone be explained by their function, i.e. they were all used to contain wine, water, oil, grain and other everyday materials. The fact that they all served the same purpose does not mean that they should resemble each other to the extent that their curved lines can be identified with those of other vessels from a different era.

In her book, *A Handbook of Greek Art*, Gisela M. A. Richter, writes:

"According to the locality in which the vases were found (Altrea, Corinth, Boetia, Argas, Crete, the Ceylades, Cyprus, Samos, Rhodos, Italy, etc.) they differ somewhat in appearance. On the whole, however, the style is remarkable uniform."

This confirms that in spite of the widespread localities and dates of manufacture vases and pots shared a certain uniformity. I believe the idea of conformity would better be expressed by "harmony", for although the family resemblance is evident there is sufficient variation in decoration, size and colour to dismiss the concept of direct conformity.

The scene is similar to another with which we are familiar: the art of building. Researchers through the ages have sought in vain for the secret thread that wove a harmonious link through the buildings of antiquity. Obviously a common system had been applied—but what? And how?

This book has shown that the missing link was ancient geometry and its principles of proportion.

We find something of the same harmony in ancient pottery, and the clues

point here, too, to a link with the proportions of ancient geometry. Support for the theory comes from the fact that the actual number of shapes has been restricted although the possibilities were unlimited.

Before starting our geometric examination of a few pieces of ancient pottery, we shall try to discover how a trade such as the potter's obtained early knowledge of ancient geometry. The latter was the province of the religious initiate, and for the pottery trade to be familiar with its principles there had to be a close association with the Temple.

As mentioned earlier, one period of Greek pottery has been labelled the Geometric Period and was roughly 1000 B.C. With this in mind and with our knowledge of Greek building and sculpture, we see that (ancient?) geometry was applied to the potter's trade centuries before the same rules were transferred to building and sculpture.

Surviving ceramics give no indication of who actually manufactured the ancient vases and pots, and history gives no assistance. Was it the work of a special group of the population or certain individuals?

A considerable number of pieces of pottery were discovered on excavation to have a form of geometric symbol imprinted in the clay at the base of the object, an ancient trade-mark of sorts. One of the early attempts to decode these symbols was a book by Flinders Petrie in which, among other things, he put forward a theory that these geometric signatures formed the basis of later civilisations' written languages.

Petrie's book was severely criticised by linguistic experts. This, they considered, was a field of which the famed archaeologist should steer clear.

The signatures have never in fact been linked with any concrete period, area, or people. Until the solution to the mystery is uncovered at some future date, they will continue to be regarded either as a manufacturer's (family?) trade-mark, or the signature of the individual artist who actually created the pot in question. As we shall shortly see, the former theory is likely to have been nearer the truth.

We have no idea who, in the period in which we are raking (ca. 1000 B.C.), might have been responsible for production of pottery. To obtain some clue perhaps it would be useful to examine a similar situation in an era nearer our own time.

Throughout central and southern Europe during the busy period of cathedral and church construction, roughly 1000—1500 A.D., we find that the main materials used in building were timber and bricks. The latter were manufactured by the million.

Here we have the same raw material (clay) as used by the potter. Although there may be a big difference in form and function between bricks and other clay objects, the raw material and treatment of clay must warrant classification of brick-maker and potter under the same general heading.

We know that the bricks of the Middle Ages were almost invariably manufactured by monks in their own brickworks. Even at this late date it was the men of the Church who took on the major share of brickmaking, not private firms or individuals.

Brick-making, although the process seems a simple and straight-forward one, is in fact complicated and requires a fair degree of experience. The training of a real brick-maker and the assembly of experience takes time. It would thus be natural for us to assume that this skill displayed by the inmates of monasteries and abbeys in brick-making was something they had inherited from the distant

past—like so much more of their abilities, wisdom and tradition.

So it was with pottery. Before the learner could hold aloft his first successful vase or jar he had to undergo strict training. Turning, smoothing, shaping, thinning and generally modelling clay was not an art to be adopted lightly—as anyone who has tried will verify! And apart from the shaping process, there was the business of colouring, decorating and glazing.

Although not of a mechanical nature the potter's years of experience represented a hefty investment nevertheless. First he had to find and develop an area where the clay was suitable for his surface. This possibly included purchase of the plot of land. He then had to construct a firing kiln, which required expert knowledge. The intending potter went on to excavate the clay and wash it to remove foreign particles and pieces of chalk which might otherwise explode during firing and ruin the bowl or vase. Not until the clay had been brought up to a high standard and faultless consistency could the design and shaping begin.

Decoration and glazing of the fired items was a science of its own. It must have required a far greater knowledge and ability than the same job demands today. There were no shops down the street from which to buy the necessary paints and glazing materials. The ancient potter could not buy his chemicals from a convenient centre. He probably manufactured them himself from whatever raw materials were to be found in (and under) the district.

These processes demanded experience and knowledge. The necessary know-how to open a pottery or brickworks was assembled and practised by several generations.

What a perfect set-up for Temple organisation! It would have been a natural sphere for one of the orders of monks to develop. Experiment, practice and research. The monks had both the time and the ability for these things.

Why should the monasteries turn their hand to the making and design of pots, vases, jars and bowls? There must have been a need. I believe the need arose the day the Temple brothers harvested their first grain crop, pressed the juice from their first grapes, and the oil from their first olives. Where could they store the fruits of their harvest?

As the Temple owned rich areas of land (far larger than the plots of the common man) their production and harvests were presumably proportionately greater than normal, and many mouths had to be fed from the Temple fields. Not only on a basis of share and share alike, but because Temple brethren appreciated the pleasures of food—and drink.

Storage of large quantities of solids and liquids demanded the use of hundreds of containers and jugs. Unable to acquire a suitable quantity from the outside world, and following the tradition of being self-supporting as far as possible, the Temple set up its own pot-making factory and plunged into the world of ceramics and clay-throwing.

Having eventually, perhaps after many years, met their own requirements and needing only to produce replacement jars from time to time to make up for breakages, the Temple potters began selling their products to the outside population. All the while they experimented with a range of glazes, firing methods, clay mixture, shapes, etc.

In designing new shapes of vases, the brethren naturally applied their experience and knowledge of ancient geometry. Geometry was so much a part of proportion and design within the Temple that it was unthinkable that any other standard should be applied to pottery. Al-

Fig. 286.

though variations of shape thus arose, they were limited by the rules of geometry.

If my pot-making theory holds water, it should not surprise the observer to find ancient geometry used both in the design and decoration of many vases and jars.

Of course, it is possible that the pottery trade somehow obtained the secrets of ancient geometry by some other means, particularly if the geometric link could be shown to apply only in one or two areas. But as far as I have been able to judge, it was spread throughout central and southern Europe over a long period, and

D	A	G
E	B	H
F	C	J

Fig. 287.

I consider the only logical explanation is that pot-making was from earliest times a speciality of Temple craftsmen, and was later adopted by orders of monks.

Gradually, as monasteries in recent times abandoned the industry (perhaps in the face of outside competition?), the art of pottery was taken over by private manufacturers who based their business and designs on the existing work of the monasteries. A number of the traditional vase outlines and shapes were retained unaltered, but new ones were introduced, their dimensions, proportions and shapes having no connection whatever with ancient tradition and bearing no resemblance to the old vessels.

We learned earlier that in spite of the wide range of possible shapes of vases and amphorae, a relatively small number of main shapes kept reappearing, obviously governed by a strict principle of design.

In *Fig. 286* we see 17 of the most common shapes of vases found more or less all over central and southern Europe during the long period from about 1000 B.C. to 1200—1500 A.D.

We see immediately that these vases are made up of curved and rounded lines, i.e. we shall be unable to discover their silhouettes hidden directly in the lines of ancient geometric symbols since, apart from the circle inscribed in the basic square, all the symbolic lines are straight.

But notwithstanding this fact we shall shortly see how the vases' silhouettes were in fact designed by means of ancient geometric symbols: in effect their curves are made up of partial circle-arcs. The centres of the circles from which these arcs were extracted are all to be found as intersections of lines within our symbols.

Apart from the task of tracing these special circle-arcs, the procedure in setting up our symbol proves simpler than in past analyses. Most vases are designed on the basic square, its guide-lines, and the 3 × 3 division.

The nine squares are sub-divided again for different profiles into familiar patterns, and in one or two isolated instances we require to add lines such as the sacred cut.

To simplify (and standardise where possible) the coming analyses we shall each time split the basic square into nine smaller squares and give each an identifying letter.

The three central squares from top to bottom will be labelled A, B and C. The three on the left will be D, E and F, and those on the right G, H and J. We see this explained in *Fig. 287*.

Before trying out our geometric system on a few photographic reproductions of antique vases, we shall—as in the preliminary study of temple construction—plan a simple example ourselves to illustrate the procedure.

In *Fig. 288* we see a diagram within the basic square, and our vase will occupy the full height of the square.

The basic square has been split by the vertical cross, the diagonal cross, the acute-angled triangles, and the lines of 3-part division.

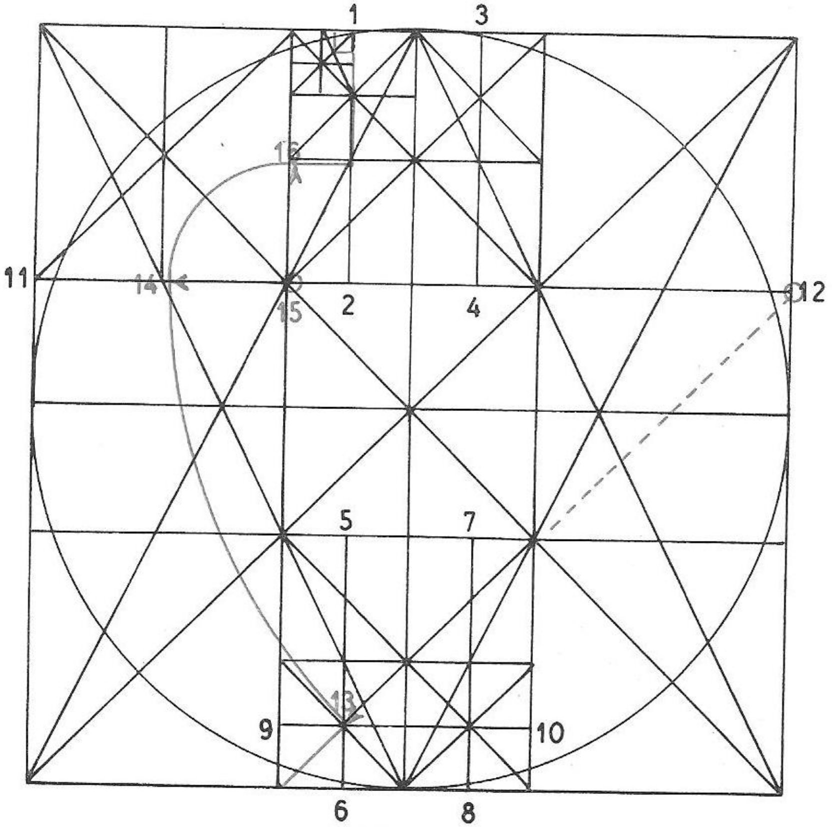


Fig. 288.

The small square A has then been divided 4 × 4 by its diagonal cross and half-size version. Two of the vertical lines of 4-part division (1-2 and 3-4) were selected as the width of the neck of the vase.

Moving to square C, we prepare the profile of the base. Again we apply the 4 × 4 division, selecting for further use the two verticals (5-6 and 7-8) and the lowest horizontal (9-10).

As the base of the vase we choose the whole length of square C's base-line, i.e. one-third of the base-line of the basic square.

The waist of the base is selected as half the length of line 9-10, i.e. the distance between lines 5-6 and 7-8. The angle of the foot is made to coincide with square C's diagonal.

The problem now is to find a suitable curve to join the neck and the base of the vase. In fact we combine two curves.

As the centre of the main curve, we decide on the right-hand end of the upper horizontal line of 3-part division in the main square (11-12). With the compasses at point 12 we draw arc 13-14.

For the second curve we remain on the same horizontal but move across to its intersection with the left-hand vertical dividing line, point 15. The new arc is 14-16. At point 16 we coincide with the horizontal axis of square A, which marks the junction of neck and body.

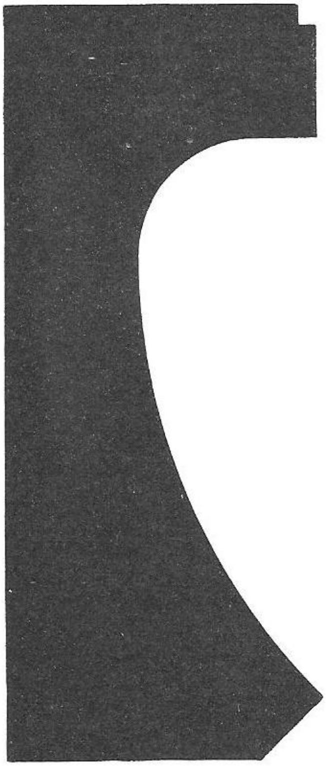


Fig. 289.

Earlier we divided square A 4×4. In the upper left corner square (adjacent to point 1) we enter the 3×3 division. The upper horizontal line of this division is taken as the thickness of the vase's surrounding lip.

This completes the profile of the vase, and we have built it exactly as the ancient potter did nearly two thousand years ago. Our choice of lines, curves, etc., was the simplest possible in order to show the procedure clearly. The same theme permits numerous variations of height, width, curve, etc., but not the infinity afforded by free creation.

Many surviving vases, as we shall discover, are built around three arcs, and have been planned in a more sophisticated manner than our primitive effort above. But despite the sophistication, despite the variety, the system remains the same: based on ancient geometry.

In practice, the ancient potter probably designed the silhouette of his vase on a small piece of clay tablet or papyrus (or whatever drawing materials were at hand). He then decided on the required height of the finished vase and reproduced a fresh drawing to scale, but contented himself with only one half of the vase as shown in Fig. 288.

Once done, he clipped or cut away the part of drawing shown in *Fig. 289* and stuck it or placed it on a thin piece of wood or similar stiff material. The profile was cut out of the wood—and there he had the perfect template, an exact copy of the vase planned in his initial small sketch.

Sitting at his wheel, the potter simply placed the template against the side of the vase whenever he wanted to check his progress. In this way he was able—everything of course depending on his manual ability and experience—to reproduce any shape he wanted and to repeat the same shape as often as required. Little wonder later researchers recognised a harmony or uniformity about ancient pots and vases.

So far we have made minimum reference to the ornamentation of ancient vases, but as we shall presently observe, decoration was also governed by the lines of the diagram.

In making the template the potter let one or two sharp edges jut out from the side of the pattern. The position and height were decided by the diagram's lines. As he turned his vase and placed the template against the spinning mass of clay, the sharp projection made light grooves in the side of the vase to indicate where the painted friezes and borders should be placed.

To test our geometric theories and system on surviving vases it is best to use photographs in order to come as close to

reality as possible. On the other hand with photography we again meet the problem of angle and perspective as well as the object's third dimension.

The material finally selected, however, was considered the finest available. In cases where the camera lens was slightly above or below the object's horizontal axis appropriate compensation has been made in placing the basic square so that it represents the height of the vase.

The most stable line of a vase photographed from other than straight onto the horizontal axis is its vertical axis. The latter line is least affected by the third dimension.

*

The first vase we shall examine under the ancient geometric microscope is one manufactured around 800 B.C. It is now in the National Museum in Athens.

The period during which our vase was moulded is a century or two later than that recognised as the Geometric Period of ceramics, but the latter era stretched over a number of years and greatly influenced subsequent workmanship and design. The vase we are to examine is termed an Attic geometric crater, and was used as a wine container.

In order to compensate for the photographic angle our basic square required to be raised slightly in relation to the vase: the base-line is rather higher than the foot of the vessel, and the top of the square runs correspondingly above the upper limit of the vase.

This has had the effect of shifting the vase's decoration in relation to the diagram, but the purpose of our study is more to examine the shape (silhouette) of the vessel than its decoration. And yet it is possible to trace the origin of the decoration within our diagram, which is seen superimposed on a photograph of the vase in *Fig. 290.*

The basic square is 1-2-3-4, and has been completed with vertical and diagonal crosses and the 3×3 lines of division, as seen in our earlier examination of Greek art. The vertical lines of division are 5-6 and 7-8, while the horizontal are 9-10 and 11-12.

We start our study in square C (see Fig. 287) which is the lower central square. This square is supplied with guide-lines in the same way as the basic square: vertical and diagonal crosses, but instead of 3-part dividing square C we execute the same operation in the two upper squares (of four) produced by the vertical cross. Only the verticals 13-14 and 15-16 are required.

We see how these two lines indicate the width of the high pedestal on which the bowl proper is placed, and in numerical terms we note that the pedestal occupies $\frac{2}{9}$ of the width of the basic square. We observe too that it extends from the base-line to the lower line of 3-part division in the basic square (line 11-12). The height of the pedestal is thus $\frac{1}{3}$ of the basic square, while the bowl itself occupies $\frac{2}{3}$.

In Fig. 288 we saw how the ancient potters employed a special technique to design the curved lines of their handiwork, taking as the centre of a circle various familiar points within the geometric diagram.

This particular vase we are examining was designed from two circle arcs, one larger than the other.

The large arc has as its centre the middle of square G, the radius being from point 17 to point 13. The sweep of the radius goes to point 18, which is the centre of square D. This is a fine geometric construction, whereby the length of the radius is $\frac{2}{3}$ the width of the basic square —which gives the vase its width.

Square D is split by a similar arrangement to square C: first into four smaller squares by the vertical cross. Point 19 is

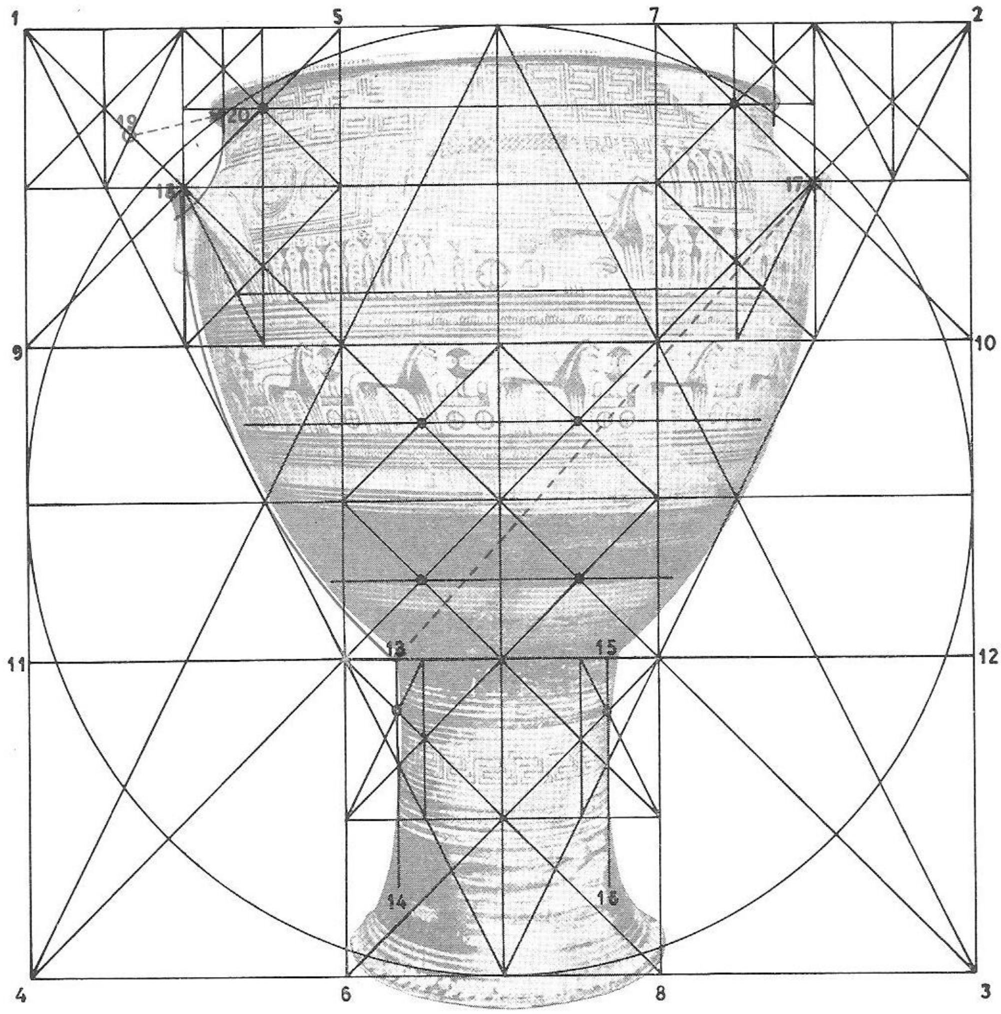


Fig. 290.

that at which the vertical line of 3-part division would pass in the upper left of the four small squares. Point 20 lies on the vertical line of 4-part division in the upper right square. Radius 19-20, when swept down to meet the vertical axis of square D, provides the small upper curve of the vase.

On account of the photographic angle it is not possible to pinpoint the height and slope of the vase's base, but it is not unlikely that it lay (on the original designer's drawing tablet) between the base-line and the lower horizontal line of 3-part division in the two small, lower squares in square C.

Thus we have completed the vase's profile and if we were to construct a template based on our geometric drawing, we should be able to mould and shape a vase identical in curve and angle to the one just examined.

Regarding the decoration of the vase we can plainly see how the horizontal

lines are marked by the horizontals of the diagram; the various friezes match fairly well. But to state with certainty that the decoration stems from the geometric diagram would require a more "squared-up" photograph than the one with which we have been working, i.e. more at right angles to the subject. Moreover, decoration undoubtedly meant a further sub-division of the diagram, a sub-division which would confuse our view of the dimensions we in reality seek: the vase's profile.

It is naturally easy to criticise an analysis of this type, and it would have been greatly improved if the photographic material had been of a better quality, but one should bear in mind here that a vase is seldom photographed with a view to conducting a mathematical analysis; it is normally angled to make the vessel appear attractive to the eye.

Another point one should recall before launching into criticism is that the smaller the item the more difficult it is to photograph from the correct angle. And finally it should be borne in mind that there is often a gap between the item originally planned by the designer—and the item finished on the potter's wheel. The manufacturing process may produce minor (though decisive) alterations.

The object of our study is not to try to manufacture *exact* analytical copies of the individual vases, but to illustrate the probable process that preceded the actual physical construction of a vase, and to show how ancient geometry appears to have been the factor which permitted designers to reproduce the same (or related) shape of vase over and over again and in whatever size was required. The diagram remained unchanged, it was a matter for the designer to select the initial size of the basic square.

The next vase we shall examine geometrically is one from the first half of the 7th century B.C., i.e. from the same period as the preceding but about half a century later.

It too is exhibited in Athens' National Museum, and although it in no way resembles the previous vase, its design lies within the same symbol with one or two small amendments.

We see the vase and analysis in *Fig. 291* in which the basic square is 1-2-3-4, the vertical lines of 3-part division 5-6 and 7-8, and the horizontal lines 9-10 and 11-12.

The foot of the base lies in square C, in which has been entered the vertical and diagonal crosses and the two acute-angled triangles—which provide the marking points for 3-part and 4-part division.

Square C's upper horizontal 3-part dividing line is 13-14, and this line we see as the junction between the vase's relatively high base and the vessel proper.

The two vertical lines of 4-part division are 15-16 and 17-18, and we observe how these mark the width of the base at its junction with the bottom of the vase proper. The total width of the base is marked by the whole length of the base-line of square C, line 6-8.

The curves of this vase appear also to have been produced by the use of two arcs. One has its centre in square H.

The latter square, we see, has been split by the vertical cross into four smaller squares, one of these being again split by the vertical lines of 4-part division. One of these latter lines has been numbered 19-20, and we observe that the larger of the two arcs has the radius 19-21. The arc is drawn from point 21 to 22 (the basic square's horizontal axis).

At this point (22) the arc is succeeded by another, this time with its centre in the middle of the basic square.

In practice the junction of these two arcs would form a slight "knee" or angle but it is very easy to compensate for the

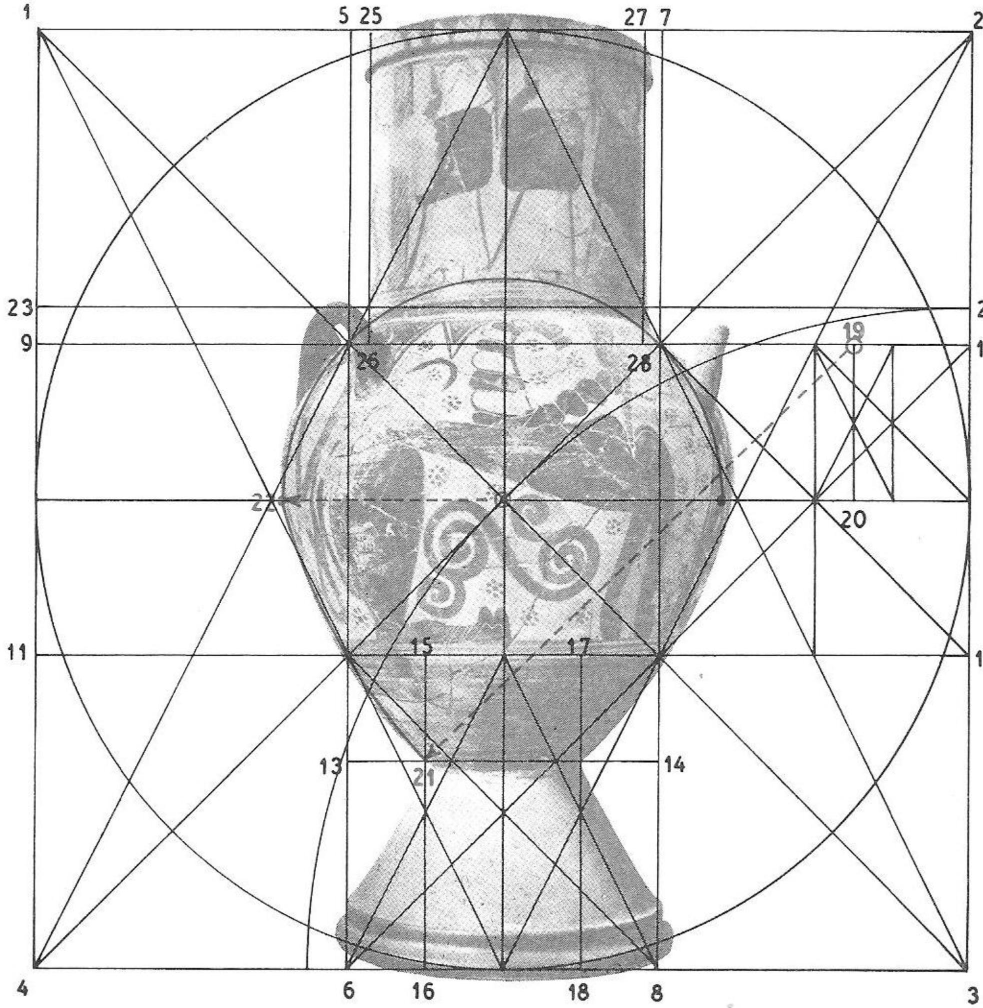


Fig. 291.

angle when constructing the template for the finished profile.

We now enter the upper horizontal sacred cut in the basic square, which we call 23-24.

The sacred cut intersects the acute-angled triangle at two points within square A, and vertical lines are drawn through the points (25-26 and 27-28), which indicate the width of the high vertical neck of the vase.

Thus the sacred cut was employed as a variation in planning, providing both height and width of the vase's long neck.

With regard to the vase's decoration there is in reality only one horizontal frieze. It is marked by line 11-12, the line of 3-part division in the basic square.

Apart from one or two details at the base (which commences vertically before narrowing towards the bottom of the vessel proper) we have reconstructed the lines necessary to produce a template for this vase, and the decoration, we find, is

not of a geometric nature with lots of friezes and borders.

The decoration consists of silhouettes and symbols drawn from the imagination (?), the whole assembly being split into two sections: that on the vase itself and that on the neck.

Decoration on the body of the vase extends from the lower line of 3-part division in the basic square (11-12) to the sacred cut (23-24) which marks the junction of neck and body. The ornamental area of the neck extends from line 23-24 to the upper edge of the basic square 1-2.

The vase pictured in *Fig. 292* is also from the 7th century. It is to be found in the Louvre, Paris. I selected it because its shape differs so distinctly from the two others just examined, and one can see at a glance that its curves are based on centres outside the basic square. One part of the curve in any event is so "flat" that its radius must lie outside the square.

This in fact proves to be the case but it does not mean that the rules of design are cast aside. It is simply the introduction of yet another variation on the same general theme.

The master craftsman who designed this vase added at each side of his basic square another small square from which he selected the centre of one of the curves, thus giving the body of the vase a long slim appearance.

We see the basic square at 1-2-3-4, and the vertical lines of 3-part division 5-6 and 7-8. The same lines horizontally are 9-10 and 11-12, produced to points 13 and 14 respectively to provide for the extra square.

The finished vase occupies in width $\frac{1}{3}$ of the basic square, the vessel's width:height ratio being thus 1:3.

As usual we begin our search in square C. Here we have entered both the 3-part and the 4-part lines of division, and we see how the latter set of lines (15-16 and 17-8) indicate the width of the base at the base-line.

In the lower right corner one of these small squares has also been split by the vertical cross and we see how its centre (marked by line 19-20) marks the height of the vase's low base.

In order to find the vase's curves we must turn our attention to the extra square, 10-13-14-12, which has been split into four smaller squares, the upper right-hand of which has also been split by the vertical lines of 3-part division.

At point 21, which is the bottom of one of the 3-part dividing lines, we have the centre of the arc which forms the lower part of the vase. Radius 21-22 is swung upward to point 23, which lies on line 11-12.

From point 23 the centre of the curve alters: moving to square B. It is found by entering the horizontal lines of 4-part division in the lower right corner of square B, the radius (24-23) sweeping upward to point 25, where it meets the upper line of 3-part division in square B (line 26-27). At this point (25) the vase switches from a curved shape to its long—relatively wide—neck.

In the upper left corner of square A, preparations have been made for the 6×6 division of the square, one of the small squares created by 3×3 division having been split by its diagonal cross. The point of division is 28.

When we join points 28 and 25 we achieve precisely the angle of ascent of the neck.

Many of the horizontal lines in the geometric diagram are to be found repeated in the vase's decoration, which is a combination of a geometric decoration and a pictorial scene, but a further, finer subdivision of squares A, B and C locates almost all the friezes. It is a matter merely of continuing the process of division.

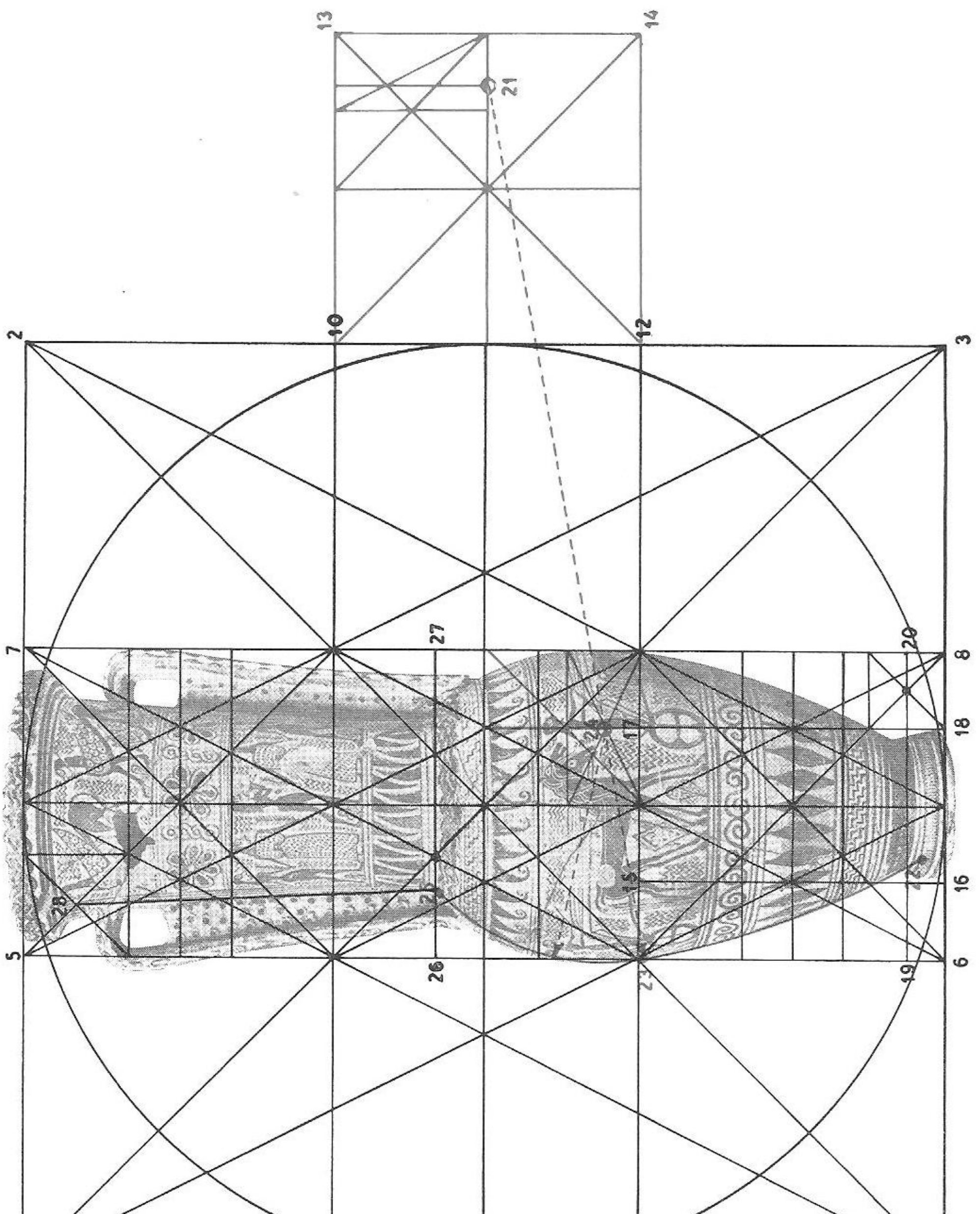


Fig. 292.

One or two details still defy our examination of the vase's profile: for example, location in the diagram of the vase's handle which is place on the side of the neck like a long fin.

But such details must remain unsolved

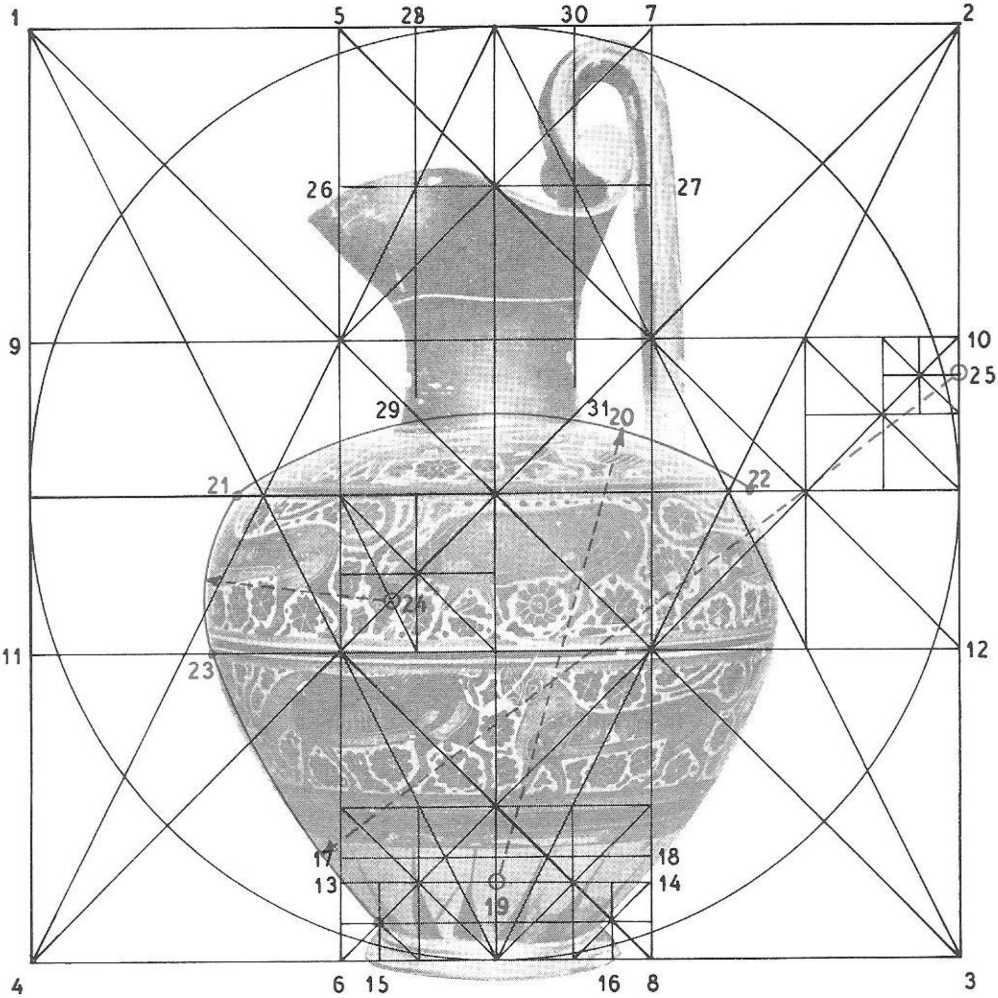


Fig. 293.

on account of the quality of the photographic material at our disposal, including the angle of the picture.

We are nevertheless able to produce a satisfactory template which could be used to reconstruct the profile of the vase.

The next vase on which we shall test the strength of ancient geometry is one from Corinth. It is from the period roughly 600 B.C., and is today held by the British Museum in London.

We see the vase in *Fig. 293* and note that it resembles a jug in form more than a vase. It has been turned unfortunately in such a way that the handle is not perfectly clearly in view, but the jug's other dimensions are plainly seen.

The vessel's stubby body comprises three curves, as we shall shortly see. The jug occupies a basic square whose height is equal to the full length of the vessel including the high handle.

The basic square is as usual numbered 1-2-3-4, the vertical lines of 3-part division

5-6 and 7-8, and the horizontal 9-10 and 11-12.

We look in square C for the basis of the dimensions on which the foot of the jug was planned.

Square C is first split by the vertical and diagonal crosses, the two lower component squares also being divided in this way, which indicates the lowest horizontal line of 4-part division in square C: line 13-14.

This line extends across four small squares, the two outer of which are split in half by vertical axes, and we see here that the jug is not in fact completely symmetrical: one of the axes (marked by point 15) runs flush with the bottom of the jug proper, the other marked by point 16 lies out to one side.

I believe however that the intention of the designer was that these two lines should mark the width of the base of the jug, which numerically means that the base equals $\frac{6}{8}$ of the base-line in square C.

In the same square the lower of the two horizontal lines of 3-part division is 17-18; this is the level at which the jug's decoration stops.

We find the first of the three composite arcs (responsible for the jug's ample curves) on the vertical axis of the diagram at its intersection with the lowest line of 4-part division (13-14) in square C. The point in question is 19, and the radius is 19-20. The curve resulting is 21-22: the top sweep of the jug's shoulder.

The next arc-centre is to be found in square B, which is divided first into four smaller square. The lower left of these is again prepared for 3-part division. The upper point of division (24) in this square is the centre of the circle which provides arc 21-23.

The last of the three arcs required is found in square H. The square is divided first into four smaller squares, the upper right of which is again divided in the same way. The upper right corner of *this* square, too, is divided into four, and we find that the designer selected point 25 as the centre of the third arc—which extends from point 23 to the base of the jug.

In square A the height of the jug (without the handle) is indicated by the square's horizontal axis, 26-27, and the width of the neck is shown by two vertical lines of 4-part division in the same square, lines 28-29 and 30-31.

The decorative area of the jug is divided into four horizontal sections. The uppermost of the four is marked by the horizontal axis in the basic square. The second and largest section extends from the horizontal axis to the line of 3-part division (11-12), and as already noted the third section is contained by line 17-18 in square C.

With the foregoing details of the jug's construction we are able to reconstruct and produce a copy of the vessel in any proportion we desire. The only main detail we lack is the handle.

But I believe that if the jug had been turned exactly side on to the camera, we should have found the handle indicated by the basic square's vertical line of 4-part division—which has not however been included in this diagram.

Apart from the standard 3×3 division of the basic square, the main dimensions were taken from a 4×4 division of the various squares.

Our next vase is seen in *Fig. 294.* It is an amphora from the 6th century B.C., and is owned by the Vatican Museum in Rome. In form the vessel is traditonal, and a common item in Etruscan pottery.

As with the earlier samples of vases, this one is placed in its basic square, which has been divided 3×3.

The diagram has been raised slightly in relation to the vase in order to com-

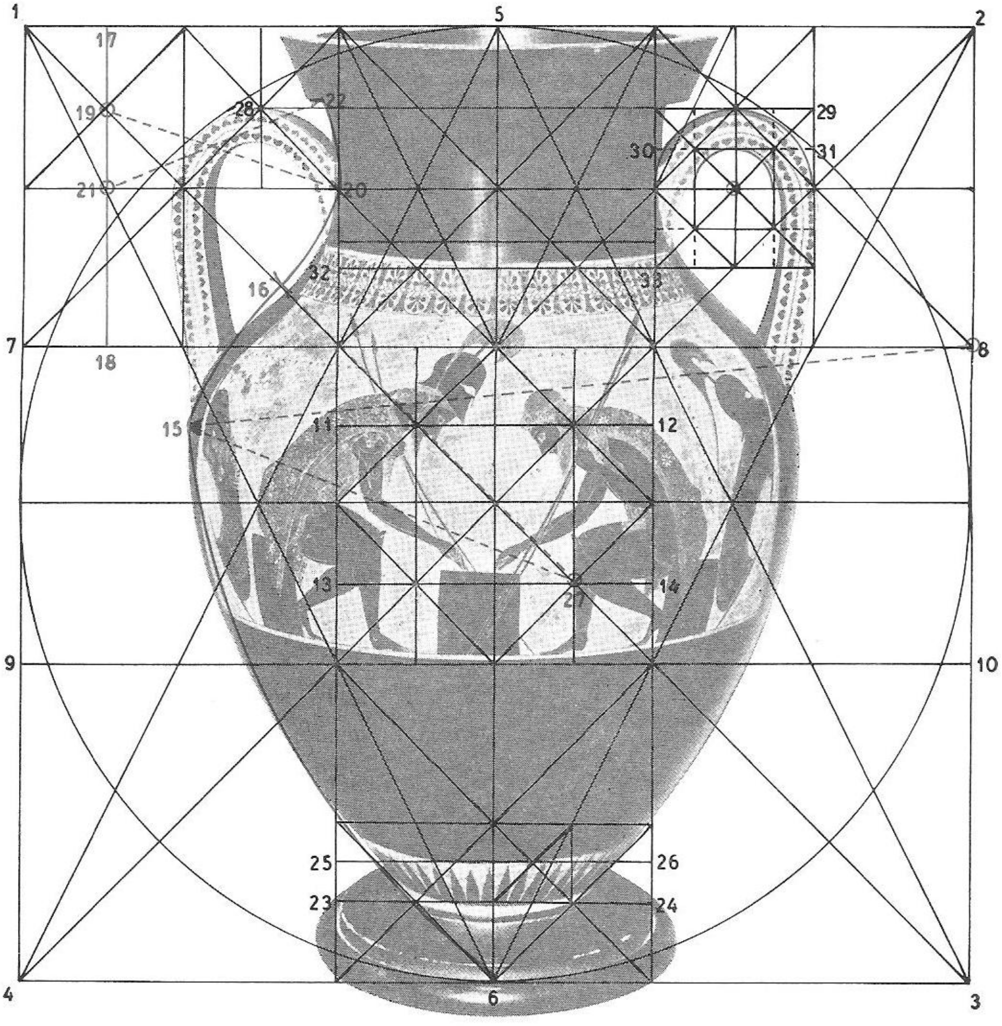


Fig. 294.

pensate for the photographic angle from which the picture was taken.

If we can imagine the vase viewed from the proper angle, we can in square C study the construction of the base.

The foot was made slightly wider than the lines of 3-part division in the basic square indicate, but a recess is included on the foot to mark these lines.

It would appear that the height of the base was governed by line 23-24, which is the lowest line of 3-part division in square C, and we also see how the strip of decoration (acute-angled triangles) fits the diagram when one of the small squares is further reduced in size. The line is shown as 25-26.

The main curve of the vase's body was produced in a very simple manner. The centre of the curve is to be found at the junction of line 7-8 (one of the lines of 3-part division in the basic square) and line 2-3 (the side of the basic square).

The radius 8-15 is swung in an arc to

point 6 at the base of the diagram. Thus the curve of the vase in reality extends to the base, meeting the diagram's vertical axis, but is intersected by line 23-24 which is the junction of base and body.

The next arc was taken from square B, which has been equipped with its lines of 4-part division. The centre of the arc is point 27, the arc itself being from point 15 to point 16.

At this latter point the arcs reverse to shape the neck of the vase, and we find the centres of the two arcs in square D.

It has been prepared for a 4-part division, and we see one centre at point 19 (the arc being drawn from 16 to 20), and the other centre at point 21 (which provides arc 20-21).

Thus we have traced almost the whole of the vase's outline.

In square G we see the possible origin of the design of the handles. The square is split into four smaller squares, the upper left square being further divided in four, and preparations are made for a 4×4 division.

The horizontal axis of this small square marks the height of the handle, and when the line is produced across to squares A and D it indicates clearly the bottom of the lip which surrounds the top of the vase. This line is 28-29.

The lower half of the same small square was apparently the source on which the upper part of the handle was based. This part of the handle, we see, comprises quite simply a pure arc of the circle that can be drawn within the half-square. If another half-circle is entered, based on the 4×4 division (line 30-31), we find that the thickness of the handle is indicated by the space between the two arcs.

When the lowest line of 4-part division is also entered in square A (line 32-33) we see how it marks the upper limit of the vase's decoration, while the lower line of 3-part division in the basic square (line 9-10) marks the bottom of the decorative panel.

Finally, a further detail from square G: In the upper left corner we see the acute-angled triangle entered in the square from which the curve of the top of the handle was taken. Notice how the side of that triangle is exactly parallel with the edge of the vase's lip. The angle is precisely the same and it is tempting to believe that it was intended originally that these two lines should coincide.

Several more details in this vase could be made the subject of geometric analysis.

Consider, for example, the two stooping figures and observe how the movement of their bodies follows the lines of the diagram. Further study in this direction would no doubt provide more information about the vessel's decoration.

But our purpose with the vase has been fulfilled: to illustrate its association with the rules of proportion based on ancient geometry.

★

The last vase we shall examine in relation to ancient geometry is a beautifully ornate item from about 570 B.C. It is housed in the Museo Archeologico in Florence.

The vase (or more correctly crater, or bowl) and the geometric diagram are seen in *Fig. 295*.

As in preceding analyses the item is placed in its basic square, which has been divided 3×3.

This analysis, too, starts in square C, at the base of the vase.

The first point we discover is that the base of the vessel appears to out of alignment in relation to the body of the vase; it sticks out more to the right than to the left of the vertical axis. Since the axis is correctly positioned on the vase proper, it must be the base that is squint. We will refrain therefore from examining its width.

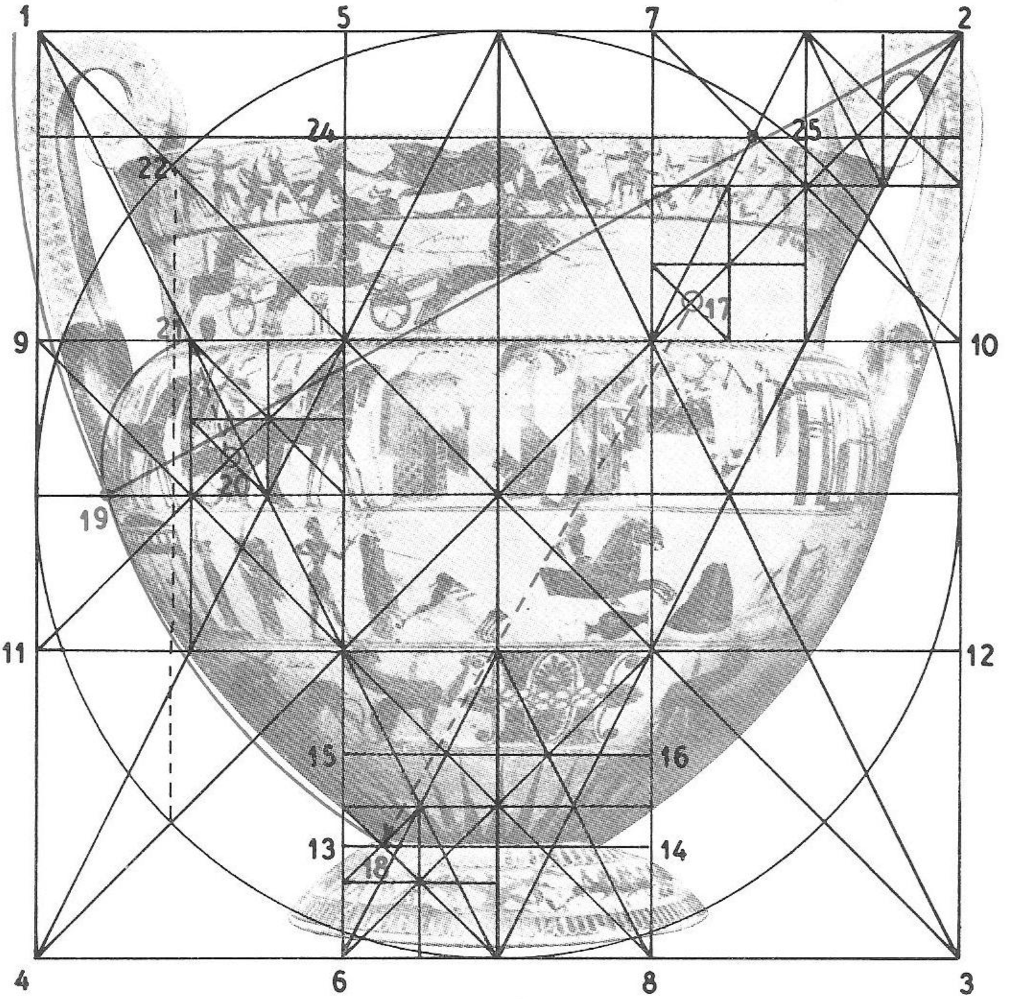


Fig. 295.

The upper half of square C has been equipped with one of the horizontal lines of 3-part division, while the lower half has been further sub-divided: by the vertical and diagonal crosses, and the lower left square which resulted has also been supplied with a vertical and diagonal cross, again providing two upper and two lower squares.

Line 13-14 is entered in the two upper squares, and appears to have been the factor which decided the height of the base—which we see to be ⅜ of the height of square C.

The vase's curves appear to have been formed by two arcs, the larger of which has its centre in square G. The latter is split in four by the vertical cross, and the lower left corner square is divided 4 × 4.

The centre of the large arc is at point 17, the radius running to point 18 in square C. The arc sweeps from point 18 to point 19 in square E, where the arc continues, but alters its centre. The new

centre is point 20—which resulted from a 4×4 division of square E and one of its smaller squares. The new extended arc lies between points 19 and 21.

For the vase's vertical neck the designer employed a geometric factor with which we are extremely familiar but which we have not previously encountered in the world of ceramics

This is the half-size square, in this case half of the basic square. It is indicated by the intersections of the diagonal cross with the basic square's inside circle. Only one side of it has been entered: a broken line downward from point 22, marking the side of the vase's neck.

The curved portion of the vase, including the base, occupies in height ⅔ of the basic square. The vertical part rises upward from line 9-10.

As mentioned earlier, square G was split into four smaller squares. The square in the upper left corner has been prepared for the 3×3 division, and we see how the lower horizontal line of 3-part division was responsible for marking the height of the vase proper (line 24-25).

Just under this square, in the bottom left corner of square G, we saw the small square divided 4×4 in order to provide the centre of the large arc. Another examination of this square shows that a horizontal line of 4-part division marks the junction of the vase's vertical neck and the angled lip.

The height of the handles is indicated by the top of the basic square, but these have a curve which passes beyond the sides of the basic square—a rather abnormal diversion.

If we take the corner (2) of the basic square as the centre of a circle, and start an arc at point 19 where the handle is joined to the vase, we obtain a curve which extends beyond the basic square—but follows exactly the line of the handle.

The tighter curve which forms the top of the handle was undoubtedly inspired by a circle, but since this as far as I can see, bearing in mind the path of the curve outside the basic square, was a construction not founded according to the accepted rules of proportion, I suspect it to have been a piece of free creation by the artist and will not therefore attempt to trace its origin.

The decoration of the vase consists of six friezes, the heights of all of which are marked by horizontal lines within the diagram.

This concludes our study of this attractive vase, and rounds off our look at the ceramics industry of nearly 3000 years ago.

If the choice of subjects in our earlier chapters was difficult, the selection of suitable and representative vases in the present chapter was no easier. The wealth of available material was overwhelming.

The remark at the outset of the chapter that the vases exhibited a certain uniformity, whether from one part of southern Europe or another, has been proved correct—and yet subtle differences in design technique have been uncovered.

The opportunities presented to the designer for variety in planning, are as numerous as the moves open to a skilful chess player.

There were no doubt a number of standard vase types which could be manufactured quickly and simply by every potter acquaint with geometry. He could select whatever size he required—depending on purpose or perhaps kiln capacity.

But apart from the standard "mass-production" types, there were endless variations on the same theme—although the alterations were not perhaps striking.

The system was readily applicable to the art of pottery—and yet left unlimited scope to the practised ceramics designer to create freely within the framework of geometry.

CHAPTER SEVENTEEN

Cuneiform and Numerals: The shape of things to count

LET US LOOK back over the development we have traced since we first began our study of ancient geometry with Man's early mathematical thoughts and speculation.

Esoteric geometry, although it did not begin as such, was founded in the initial reflections on the Sun and the Moon as the source of all inspiration, and from this base wise men, witch-doctors, priests, monks, philosophers and their initiated brethren gradually built up a geometric system which influenced wide aspects of Man's cultural development.

We saw how the system was for thousands of years one of the guiding lights in Egyptian culture. Ancient geometry was applied in numerous fields, from the planning of the Great Pyramid at Gizeh to the structure of contemporary Egyptian systems of length and capacity measurement. We also saw geometry used in the world of art in several forms.

Through many natural and some devious channels the knowledge and wisdom contained in ancient geometry spread to other countries. Moses, we discovered, was one of the Egyptian message-bearers. And much later Pythagoras was another.

Not only was the system and its application in setting out dimensions and proportions brought to other countries. It was also conveyed by successive Temple generations to considerably later periods of time. As far advanced as the early and later Middle Ages we managed to trace samples of ancient geometric planning in monumental buildings of religious association.

It was presumably the case that the further back we search through time, the greater significance the system enjoyed in speculative development. The nearer we approach our own time the less its importance to philosophy. It became more and more a system strictly of building rules, a secret of the craftsmen's guild with no real religious background. But in Chapter Ten we saw how ancient geometric speculation as late as Plato's era was taken as illustrative material for Timaeus's account of the image of the world.

This latter image must not be compared directly with our application of numbers and numerals when we, for example, give an account of astronomical relations between the Earth and the stars. In order to produce any satisfactory explanation of these relations we are obliged to make extensive use of both numbers and ratios if we are to illustrate distances, orbital times, etc., but our use of numbers is of secondary importance to the conditions they describe. By this I mean that numbers and geometry constitute an explanatory link between the astronomical observer, the object he is examining, and

the fellow human (you and me) to whom he reports his findings. Timaeus on the other hand actually identified the link (geometry) as synonymous with the subject of discussion, the world and its place in the universe. Geometry through the eyes of Timaeus and his contemporaries did not describe an image of the world; it *was* an image of the world. And played therefore a much more significant speculative role than numbers and geometry to today as illuminative and explanatory factors.

This honourable and exalted position held by geometry was more pronounced the earlier one searches back into history. Numbers and geometry were distinctly the sacred province of the Temple. For outsiders to tamper with these subjects was out of the question.

The actual symbols used to denote particular quantities (numbers) were probably passed out by the Temple to the people at large simply because necessity ruled it so.

Early civilisations were eager beavers when it came to trading, both domestically and internationally. Familiarity with numbers is essential in order to trade, if a nation is to progress beyond the my-two-cows-for-your-horse stage of barter. Goods have to be counted and checked—and so has the money paid for them.

Prior to Temple authorisation of an official set of numerals there were undoubtedly many forms of tallying and counting, from notched sticks to scratching lines on stones. Uniformity was called for.

But of course releasing a system of numbers to a community is not necessarily synonymous with telling the people how the numerals originated or of their association with .... ancient geometry. The numerals were produced by the Temple as a finished tool, polished for use.

I think it highly unlikely that the Temple volunteered the information willingly, nor do I believe that it issued a numeral system for the good of the community. It did so in order to preserve its own strength.

Trade had reached the level where numbers and symbols for numbers were an undeniable necessity. The Temple had to face the fact that either it put forward its own symbols as the only appropriate, or the chances were the people would eventually find another system independently of the Temple. Since the latter was accustomed to giving the people a lead in all matters of an abstract or philosphical nature, the Temple hierarchy regarded it in their own interests to fill the numerical gap. And of course the Temple itself was vigorously engaged in commerce on its own account.

Numbers and counting developed ages before the need arose to *write down* numbers, quantities and measurements. Speculation on numbers began so early in development that there was neither need nor desire to commit these thoughts to writing.

The first numerical symbols developed by each civilisation differed in looks from those of other civilisations—but their significance was the same.

The Maya Indians of South America, for example, developed a sophisticated system of numerals based on dots and strokes, and were able to express large quantities quite simply.

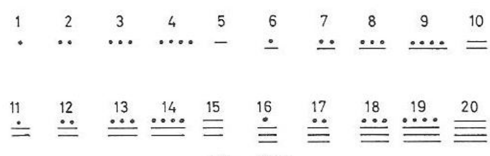


Fig. 297.

Their system was vigesimal (based on a unit of 20), and the first 20 symbols are seen in *Fig. 297.*

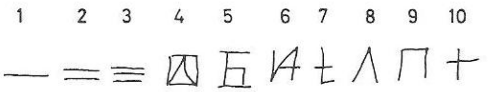


Fig. 298.

At about the same time the Chinese operated a decimal system and in *Fig. 298* we see the first 10 symbols in their system. As opposed to the progressive dot/dash system of the Mayas, the first three symbols of the Chinese system are strokes after which each symbol is constructed independently of the preceding one. Although the two systems are different in appearance, they cover the same concept: numbers.

The Sumerians, who flourished in Mesopotamia around 3000 B.C. and who are generally recognised by historians and archaeologists to have developed the oldest of the ancient Oriental systems of writing, used cuneiform characters for both numerals and script.

Cuneiform is the name given to ancient dart- or arrow-shaped characters first used by the people of the Near East several millenia B.C. The name has been coined from the Latin *cuneus,* a wedge, and *forma,* a shape.

In essence the Sumerian system of numerals, although of another appearance, is built up identically to the Maya system. The symbols can be understood perfectly clearly in any language. It is simply a question of which form to select.

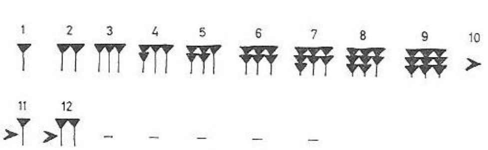


Fig. 299.

In *Fig. 299* we see the Sumerian symbols representing the numbers 1 to 12.

In *The World of Mathematics,* published 1956, James R. Newman includes a chapter on the Sumerians, and relates that they could count, write and read. They wrote part of their history on black clay tablets, which would indicate a high cultural standard at an early stage in Man's mental development.

Newman puts forward the interesting theory that the letter N originated when the numeral 2 (two vertical strokes) was written hurriedly; likewise that the letter Z owes its origin to the speedy scratching of two horizontal strokes.

His conclusion is that since clay tablets have been discovered to contain a historical account (the tablets are written in cuneiform), it must follow that the Sumerians were a cultural nation who could both read and write. And that a degeneration of the numerals in their language provided the original basis for one or two letters in their subsequent alphabet.

I believe that both assertions are bold—but inaccurate. They ignore the period of history involved.

The fact that archaeologists have found numerous clay tablets covered with script does not necessarily mean that the people as a whole could write—but that *individual groups or persons* within the society (probably the Temple) were able to write and read the written text.

Even today there are many areas or countries in which the ability to read and write is restricted to a narrow section of the elite. Considerable effort is spent in making good such national faults, but always the spur is *need;* the people *need* the ability to read and write.

Politicians and governments cannot model a modern society from a population which cannot read and write. The people, for example, are unable to read the continuous stream of proclamations, rules and regulations required in a progressive society.

Secondly, it is not sufficient that the working class are satisfied with the use of arms and legs; they must be able to use

their brains and absorb instructions in order to operate industry smoothly.

In the same way, agricultural workers must be able to read of the experiences of others in order to appreciate the route chosen by the country's leaders in the development of society, and the improved use of the country's agricultural resources.

And there is the factor that written publicity (perhaps in the form of advertising) cannot be put across to illiterates.

This is the motivating background to the present urge to teach underdeveloped countries the three "Rs". The development and structure are based not so much on a humane desire to provide a nation with the ability to read and write as on a straightforward *need*.

The basic need has not always made itself universally felt. Only a few short centuries ago the populations of many countries were from a literary standpoint suppressed by their respective intelligentsia. The cultural leaders had no desire to provide the common mass with abilities which might provoke spontaneous demands and claims.

Consider the publication of the Bible and its distribution to a wider public.

When the Latin version was compiled and prepared for publication around the year 400 A.D. a council of Catholic priests assembled to examine, sort and reject the collection of scriptural passages, which was to form the basis of the new book.

Particularly revealing texts were eliminated, as well all passages that varied from the message understood and distributed by the Catholic Church.

The result was not intended—although "purified" of all undesirable theories and passages—for the common man for it was printed in Latin. The language was by all means widespread but it was generally only men of the Church and of imperial courts who had been encouraged to master Caesar's tongue.

On the publication of the Bible, however, it was discovered contrary to earlier belief that more people than the Church reckoned could speak Latin, and that in spite of the severe standard of censorship a mass of material had slipped through the net which ought never to have been let loose. Shortly after the book's release it was withdrawn from circulation. Reading of the Bible was forbidden until a new edition was prepared. And it is reported by history that the ban was accompanied by a threat of death if anyone was found to possess the forbidden volume.

Thus we see how the initiated, leading class were able to control the quantity and content of information put before the people—and how much stronger was the Temple in even earlier days when it occupied the central position in people's lives? It was the only permanent point round which the nation could revolve, the only constant source of inspiration.

The Temple's influence, the period, the people: all point to the fact that cultural impulse was dictated by the Temple towards the people and not vice versa.

It is therefore to be expected that whether we examine a numerical system or a written language the origin for each must be sought within the Temple's walls. Not until the religious hierarchy regarded the moment as suitable did they release the information to the people or to groups of people, from whom it filtered downwards to the masses.

With regard to the theory that the letters N and Z resulted from a degeneration of two vertical and two horizontal strokes respectively, the fact is apparently ignored that these letters as sound-symbols originated much later than the inscriptions of the ancient Sumerians. Whether of Greek or Latin origin, they first appeared long, long after the Sumerians as a people had vanished.

The disappearance of a nation through time or geographical development need not mean, of course, that the experiences of the people are lost in the sand; they live on in the minds of the nation's successors, and continue to develop. But (bearing in mind the possibility that certain numerals degenerated into letters) Sumerian numerals and letters were formed from cuneiform characters and contained no independent vertical or horizontal strokes which might be transformed to other characters.

Furthermore, we ought to bear in mind that in the early days of writing ancient scripts were divided into units which represented complete syllables, words and even ideas (ideograms), whereas our alphabet—developed as it is from Greek and Latin—has a composite symbol for each *sound* the tongue can produce.

This basic difference would therefore to some extent prevent a comparison between old scripts and later ones. Recent alphabets were constructed on different principles. It was discovered that individual sound-symbols could be used to form innumerable syllables—thus permitting a simplification of the alphabetic make-up. The older the language, the more symbols it possesses. Witness the development, for example, from the Egyptians with their 600 symbols (approx.), to the Sumerians and Chaldeans with 350 cuneiform symbols, to the more rational 100 symbols of the Old Persian script, again cuneiform.

Slowly development progressed, ending (?) at the present day with the basic 26 letters of the English alphabet. To this figure, however, we are still obliged to add various European peculiarities such as æ, å and ø (Danish), ä (Swedish), ü, ö (German), ð (Icelandic), etc. With this relative handful of symbols we are able to express an infinitely greater variety (and shade) of thought than the Egyptians with their 600 symbols—and our vocabulary increases every day.

We shall not in this book record the chronological development of written language from the Egyptian hieroglyphs onward. In any event the task would be extremely difficult. Where does one script end and another begin, and when did the need for rationalisation register.

The only thing we can do here is to establish when the script in question was employed, and where. It has never been satisfactorily explained why a particular script takes the form it does, or whence it stems. The object of this chapter is to throw some light on the problem.

*

As recently as 1600 A.D. European nations had no knowledge of the dominating script of ancient times: cuneiform.

Pliny mentions that the ancient Babylonians kept records on black clay tablets, but does not specifically say that the script differed from that which existed in Pliny's Greece.

Europe had to wait until 1621 for Pietra della Vaila in his report from the ruins of Persepolis to mention and describe the remarkable inscriptions. He included some sketches of the symbols but these were not sufficiently explanatory to permit a start to the decoding process.

In 1761 King Frederik V of Denmark commissioned Karsten Niebuhr to visit Arabia and Persia on a journey of research. Niebuhr made accurate copies of all the inscriptions at Persepolis, and published the results in his excellent book on the trip, in Copenhagen, 1778.

These cuneiform characters were discovered carved into stone faces, but later research showed that stone was not in fact cuneiform's original element. It began on clay tablets.

Cuneiform inscriptions have since been

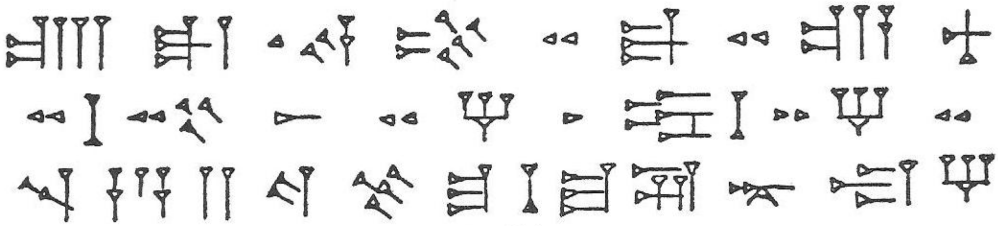


Fig. 300.

found in nearly every country in the Euphrates-Tigris region: Armenia, Asia Minor, Persia, Babylonia, Assyria, Midian, Egypt and Mesopotamia.

A sample is seen in *Fig. 300.*

It is interesting to discover that all the above-mentioned nations, although speaking different languages, employed the same symbols to express themselves in writing—with of course their own individual values allotted to the symbols. It is the same today. We in western Europe and in many other parts of the world use the same ABC . . . . alphabet with one or two minor additions mentioned earlier. But our languages differ from each other.

The structural form of cuneiform characters (which represented phonetic syllables) presented ample opportunity for combination, and the civilisations which made use of cuneiform script were not bound to stick to the same symbols for the same sounds as those employed by the people who taught them how to write cuneiform. In transferring the symbols from one language to another, the bearer's imagination was given a slack rein.

Purely from a practical viewpoint it would have proved impossible to transplant a Babylonian symbol in the Egyptian language and to retain the Babylonian phonetic value. Pronunciation of the two languages was different.

But how then can it be that a long line of nations from the Red Sea to the Black Sea, from the Mediterranean to the Persian Gulf and beyond at various periods shared the same system of script although their languages differed; a group of countries whose civilisations were on different levels, and whose mutual relations were, to say the least, mixed. Wars were more common than trade agreements. Babylonia conquered Egypt, the Egyptians fought the Hyskos Shepherd Kings, the Persians battled against the Greeks.

But despite international disputes it was possible to spread over a wide area such a basic thing as a common form of writing: cuneiform. One would expect that peaceful and friendly relations were needed as a background to this form of intercourse.

The explanation to this phenomenon is not at all mystical if one accepts in principle that writing originated initially within the Temple's walls.

The respective Temple societies did not declare war on each other. Their civilian populations might well be belligerents but inter-temple strife was unknown. Not only would a temple welcome as guests and brothers priests from the temple of an enemy nation, but the incomers would also be admitted to whichever section of study they wished to devote their time.

The normal custom was for a Temple brother, after a number of years in his mother-temple and after progressing through a series of particular degrees of status, to be equipped with documents to prove his identify and to show the degree held. With this document of reference he was dispatched to foreign temples to further his training, or to temples with (traditionally) special educational fa-

cilities. For example, Babylonian temples specialised in astronomy and advanced well beyond the knowledge possessed by neighbouring temples.

We can trace the custom over a long period. We saw how Pythagoras, for instance, journeyed from his native Samos with letters of introduction which gained him admittance to the Egyptian Temple. Plato, we recall, also travelled widely to supplement his knowledge.

And the esoteric system of travel for the sake of learning can be traced as late as the 19th century when it was quite the custom in Europe for apprentices who had completed their training and become *journey*men to travel abroad in order to round off and widen their knowledge. Often they carried letters of introduction from the head of the local guild to his counterpart in some foreign country, with the request that the new journeyman be welcomed as a guest and given further training. Frequently, despite the distance between their two countries, the two guild leaders were old personal friends—from the days they themselves set out and met as journeymen.

It is not then surprising that such a significant factor as the invention of cuneiform script could be spread throughout an extensive geographical area. The movement was gradual, but in relation to the development of civilisation time was unimportant.

An immeasurable slice of time passed before cuneiform characters, born in the brain of a Temple brother, occupied their final status as the written language of several civilisations.

In our day-to-day business and life we are so accustomed to using a written language that few of us spare the process a thought. But it is interesting to analyse the operation.

What in effect do we mean by the process of writing?

The answer must undoubtedly be that to write is to set down a number of drawn symbols in such a combination as to express a united thought or idea. It is essential that the writer himself understands the value of the symbols, whether these represent phrases, objects, or ideas, or whether they stand for individual sounds. And it is also a condition that the symbols exist in order that the writer may use them.

Reading the written symbols is "merely" a process of recognition. It demands that the reader is familiar with the symbols and knows what they represent.

In consequence this means that the writer cannot—out of the blue—introduce new symbols. If he does this, if he invents new written characters, then no one apart from himself will be able to read them.

Similarly he cannot alter the existing symbols beyond the point of recognition, for then again no one else will be able to read them. (Indeed, if the alterations take the form of carelessly shaped characters, the writer himself may not be able to read them again. One can think of one's own handwriting scribbled perhaps in haste!)

The correct symbol must always be used in the appropriate place and be given its accepted meaning if other people are to be able to read the script.

The fact that the writer may not alter the shape or significance of a symbol means that a written language cannot originate gradually, each new writer adding a symbol or two. No one in that case would be able to read the finished script.

Use of writing necessitates the existence of a complete set of symbols. Perhaps initially the basic core of symbols, signs, characters, words—call them what one will—is a meagre one. But it contains sufficient flexibility to cope with most written subjects. And additions may only be made by influential authorities for

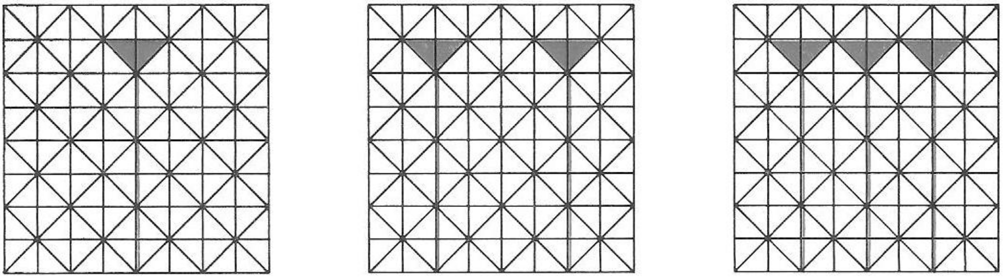


Fig. 301.

without leadership the system would collapse in chaos.

The reader must agree that a written language cannot develop in a society without some body or group to lead development. Otherwise there would be few illiterate people in our world.

Societies in which illiteracy is rife are not, compared with western Europe, new societies. They are extremely old and long-established. If time and need alone were the deciding factors it is incredible that such ancient nations have never mastered *en masse* the elementary arts of reading and writing.

A further point of amazement is that many of these societies possessed, in their leaders, a written language several thousand years before those of western Europe —but the so-called "underdeveloped" nations of the world still cannot on the whole read and write.

It is thus evident that these accomplishments were withheld from the masses for a longer period than was the case in Europe.

It may in consequence be assumed that the inspiration to write, to form written characters and to communicate in writing must come from an influential source—in the form of a recognised and recognisable series of symbols.

From the central point of origin the system of radiates outwards in ever-increasing circles, followed by additions and amendments (issued from the source) to be learned and practised by all adepts.

By its very existence a script cannot bear the stamp of the individual. Basically it *must* be uniform in order that the symbols may be recognised by other writers. If individualists were in a position to influence a particular style, script would finish up as a battlefield with hundreds of "systems" and adaptations fighting for survival.

This is a logical view of the origin of script, and we may assume that the individual written language had its own source, from which guidance could be given to its users within the basic framework.

Such a source had to represent a strong authority in society, otherwise the script would never have gained acceptance. A new script intended as a replacement for a system of pictographs or hieroglyphs (almost the forerunner of the strip cartoon) would undoubtedly have met tremendous opposition, mainly from those circles practising the old system. Why, the human mind is apt to query, should we alter to something new if the old is serving our purpose well enough?

Who were powerful, influential enough to force a new system on society? Our friends, the brethren of the Temple.

Let us have a look at the means at the disposal of the Temple to start a new form of script. We can drop in on any

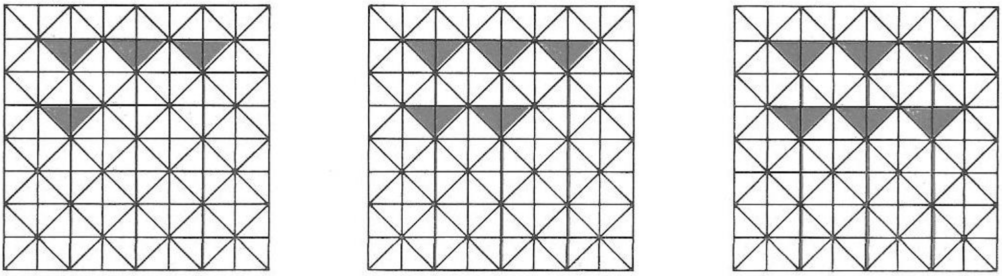


Fig. 302.

ancient temple in the area in which cuneiform began—along the banks of the Tigris or Euphrates.

We discover that the Temple is familiar with geometry and numbers to the extent that we learned earlier in the book, and we have just seen how the actual formation of numerals differed from society to society but that the significance of the symbols remained unchanged.

In our "chosen" temple we find one of the brethren musing over numbers and geometry, working perhaps with a primitive and awkward collection of symbols representing the numbers 1 to 10. Perhaps he has not invented any symbols at all yet. Numbers in his experience are simply abstract ideas, they have no material or pictorial image. How can he rectify the situation?

Instead of building up a system of dots and/or strokes, our geometric brother decides that he will somehow take his inspiration from his god-given system of geometry.

We recall from an earlier chapter how the ancients visualised areas (and squares) divided into triangles, and not as we today into small squares.

Two triangles certainly form a square, but if one has always thought in terms of triangles, then this is the basic unit that springs to mind. Our view is exactly the same today—in reverse. We "see" a square always, and merely record the fact subconsciously that the square is made up of triangles. Simply a question of attitude and habit.

Plato's declaration that "all rectilinear surfaces are composed of triangles", rings through from the past as an evergreen truth.

So our contemplative temple brother, determined to produce a rational system of numeric symbols, turns to his geometric diagrams sketched perhaps in sand or wet clay.

Before him he has the usual basic square, split by the vertical and diagonal crosses, and sub-divided into a number of smaller triangles. I would imagine that the square was divided into (or made up of) 128 triangles, i.e. $8 \times 8 = 64$ small squares each divided into two triangular halves by its diagonal. This particular division was commonly used by the ancients, and we saw in Chapter Nine how Egyptian mathematics was based on this factor of division.

In *Fig. 301* we see the first three numerals, and observe how the triangle was a natural source of inspiration to the man intent on deriving a series of symbols from the triangulated square. His first three selections are made from the most honoured position in the diagram: the centre. And since there is ample space, he draws the leg of the wedge down as far as the bottom of the diagram.

But as soon as he starts on numerals 4,

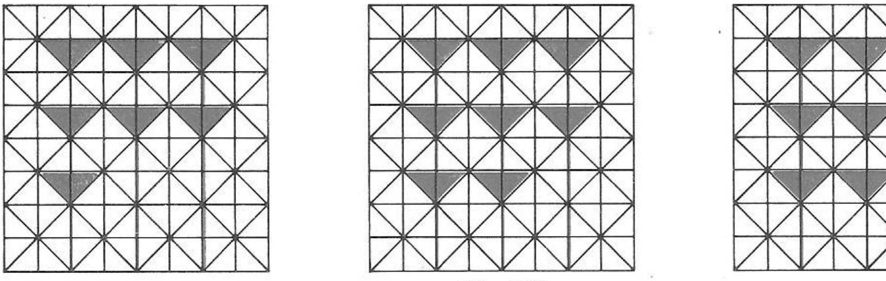


Fig. 303.

5 and 6 he must make adjustments in the spacing in order to fit them in.

In *Fig. 302* we see the triangular wedge retained as the head of the symbol, but one of the legs has been shortened slightly in order to make room for the new part of the symbol.

Note, too, that in moving from 3 to 4 the priest has had to start a second layer of symbols instead of continuing horizontally. Apparently—especially if he worked on the diagram I have attributed to him —there was no more room on either side of the diagram without extending beyond the side of the square.

Moving on to the symbols for 7, 8 and 9, we see in *Fig. 303* that the problem of space is encountered again if we are to keep within the diagram. Once more the legs of the symbols are shortened, and a third layer is introduced.

The possibilities appear to be exhausted, and for the numeral 10 a new wedge-shape is produced. We see it, together with the numeral for 11 in *Fig. 304*.

Placing the 10-wedge alongside our earlier numerals we find we can construct symbols for values 1 to 19. To depict larger values the Temple brother added the 10-wedge as often as required.

Once the first collection of symbols was completed, development probably halted for a time. The immediate need had been fulfilled: to produce convenient symbols for numerical values.

The geometric diagrams from which the symbols were designed were probably fairly large. The problem now was to transfer the shape of the symbols in the most convenient (and reduced) form to existing writing materials, principally clay tablets or slabs.

The general supposition regarding the practical shaping of cuneiform characters is that the writer employed a tool shaped like a narrow chisel. He held it more or less as we today hold a pen, and sketched his cuneiform symbols, twisting the chisel through 90° in order to produce the wedge shape.

This commonly accepted theory is both obvious and apparently correct. The only thing is that it cannot be executed in practice.

From personal experience and experiment in the formation of cuneiform characters I discovered that, to reproduce the symbols in clay, the material must have a special consistency, rather drier than the clay used by a potter as he models a vase or bowl. If the clay is too wet, the symbols run together.

Having produced the appropriate consistency of clay and rolled or pressed it into a flat surface, one would imagine the difficulties to be over. In fact, they are about to begin.

Theory hitherto has suggested that the above-described method of writing was responsible for cuneiform characters. But

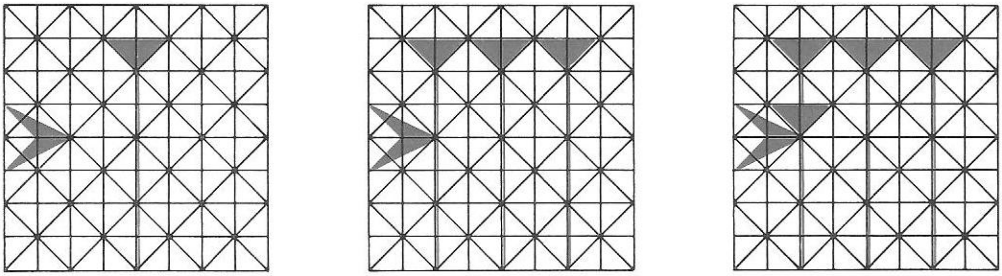


Fig. 304.

what happens if the theory is carried out in practice?

The chisel is placed in the soft clay with the "cutting" edge horizontal to the field of vision, and the writer carefully draws it towards him through the clay, turning it through 90° in order to produce a cuneiform "wedge".

As the chisel or sharpened stick is turned, the clay removed from the shallow hole gathers behind the blade of the chisel and comes to rest against the edge of the character—destroying its shape. Regardless what type or form of stick/chisel one uses a little clay will always be scratched from the surface of the writing tablet, and as cuneiform characters in many cases are only a millimeter or two deep, the difficulty in removing the discarded clay would be considerable.

Of course, it is natural and simple to persuade oneself that although we cannot strike the appropriate routine, it should nevertheless be *possible* to construct some kind of cuneiform character in clay.

Finally, after innumerable fruitless attempts and with patience at low ebb, we sit with the smoothed clay in front of us, contemplating, dabbing at the clay with our stick, poking .... poking! That's it! The process suddenly reveals itself. Cuneiform characters were not *drawn* into the clay, they were *pressed* in. Poking into the clay, we discover that the material hole produced by the chisel is sharp all the way round, has no excess clay, and is an exact reproduction of the chisel's profile.

We follow up our initial success with a series of similar tests, pressing different shapes into the clay. Exactly as a seal is impressed in hot, soft wax.

What happens in this process is that some of the material is displaced to make way for the object being pressed into the clay, but instead of the excess clay collecting round the edge of the impression it is pressed partly downward, partly to the side. It does not spoil the impression of the seal. A close examination of ancient clay tablets reveals that the clay between the various cuneiform characters has a slight tendency to bulge.

The technique is simple and can be executed by any careful person after a little practice. This must have been the technique of the Sumerians, Hittites, Chaldeans and other writers of cuneiform script. It probably began with a single wedge-shaped seal, the individual parts of the characters being pricked into the clay. Later it was decidedly easier and time-saving to manufacture 10 sticks, each with its own numerical value. Wishing to write the value 9, the Sumerian scribe then had to press only one stick into the clay once instead of nine time as required previously.

Simplicity of application and speed in execution support the "press" theory as opposed to the "scratch". Once the skill

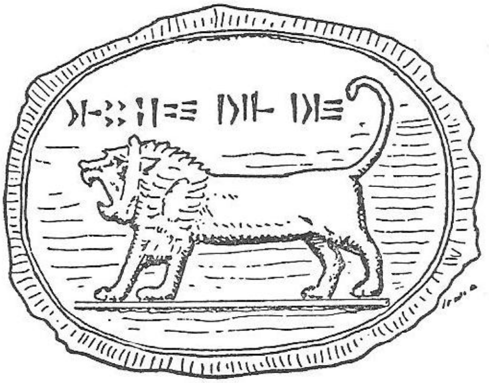


Fig. 305.

has been mastered it is remarkable how easily a clay tablet may be filled with rows of neat, strange cuneiform characters; and when a selection of different value sticks have been carved it speeds the process immensely.

Another factor in favour of the above theory is that such a process of impression in clay appears to have been in general use. In *Fig. 305* we see a royal seal reputed to have belonged to Darius the Great of Persia (ca. 500 B.C.). It bears several cuneiform characters and the picture of a lion.

The seal was apparently part of a signet ring (it is reproduced somewhat larger than actual size), and was used to impress the king's personal signature on letters, declarations, etc., in clay tablets.

Several other seals from the same period have been recovered by archaeological expeditions and serve to substantiate the theory that clay impressions were a recognised means of "writing".

There is even some evidence that the technique developed into a printing process. In *Fig. 306* we see three signet cylinders. In the form of small columns, they were evidently rolled across a surface of soft clay reproducing with one action the entire content of the column. A speedy

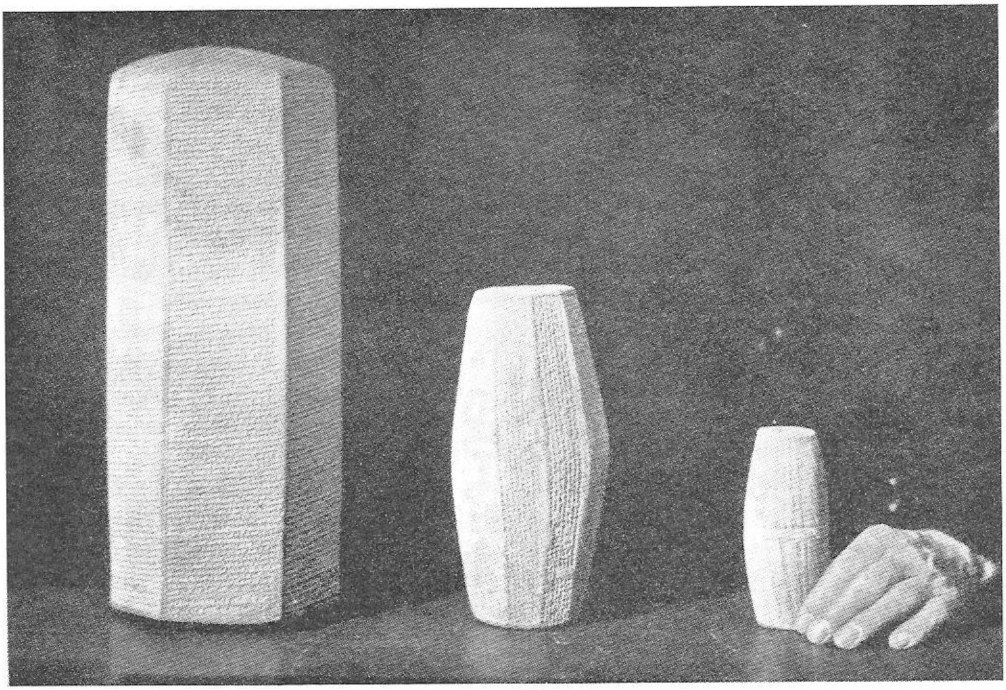


Fig. 306.

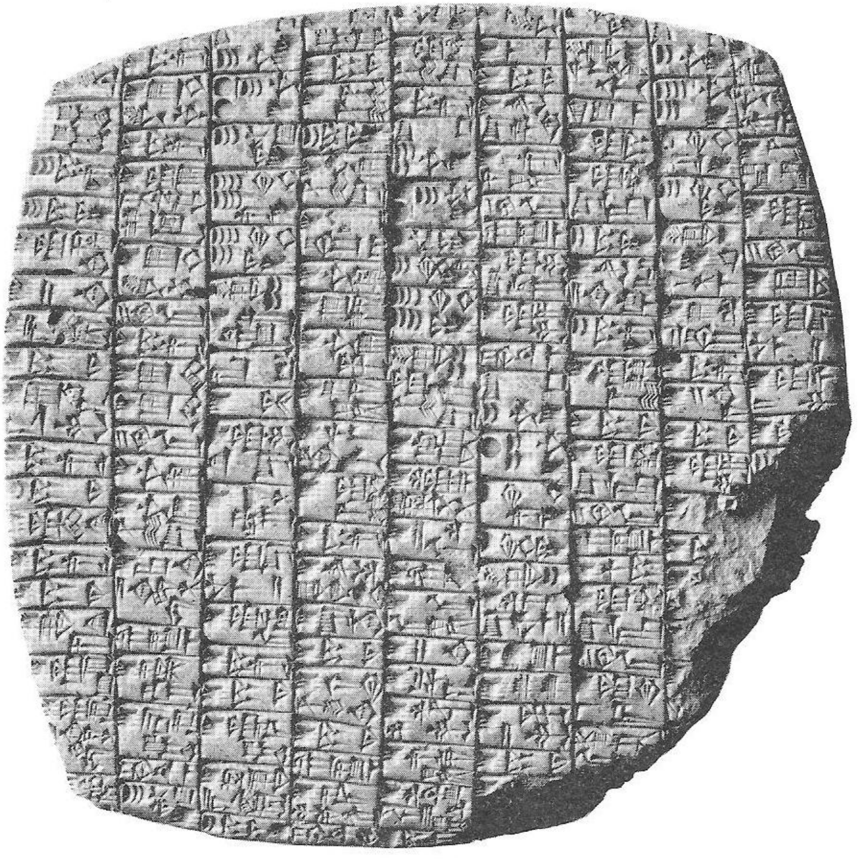


Fig. 307.

means of producing several copies of, for example, a royal proclamation.

In *Fig. 307* we find a clay tablet containing Sumerian cuneiform script. The tablet represents a list of objects with details of dimensions and quantities.

The actual content is of minor importance, but note the surface of the tablet generally. It is divided into nine vertical columns, which are in turn sub-divided by horizontal strokes.

We see clearly how the symbols, syllables or words apparently all had the same width, but varied in height according to the composition of the individual cuneiform characters.

It appears obvious that the script was written and intended to be read from top to bottom or from bottom to top of the tablet, and not horizontally.

This photograph of the clay tablet, showing a shadow effect, reveals perhaps better than an examination of the tablet itself that the work was done by means of stamp-sticks. The shadow is thrown uniformly to the left of the vertical columns, which indicates that the scribe held the stick slightly at an angle, pressing the left side of the stick further into the clay than the right side.

The shadows also indicate that the pressure on the individual sticks differed slightly. Some of the shadows are wider than the others. This, too, emphasises that

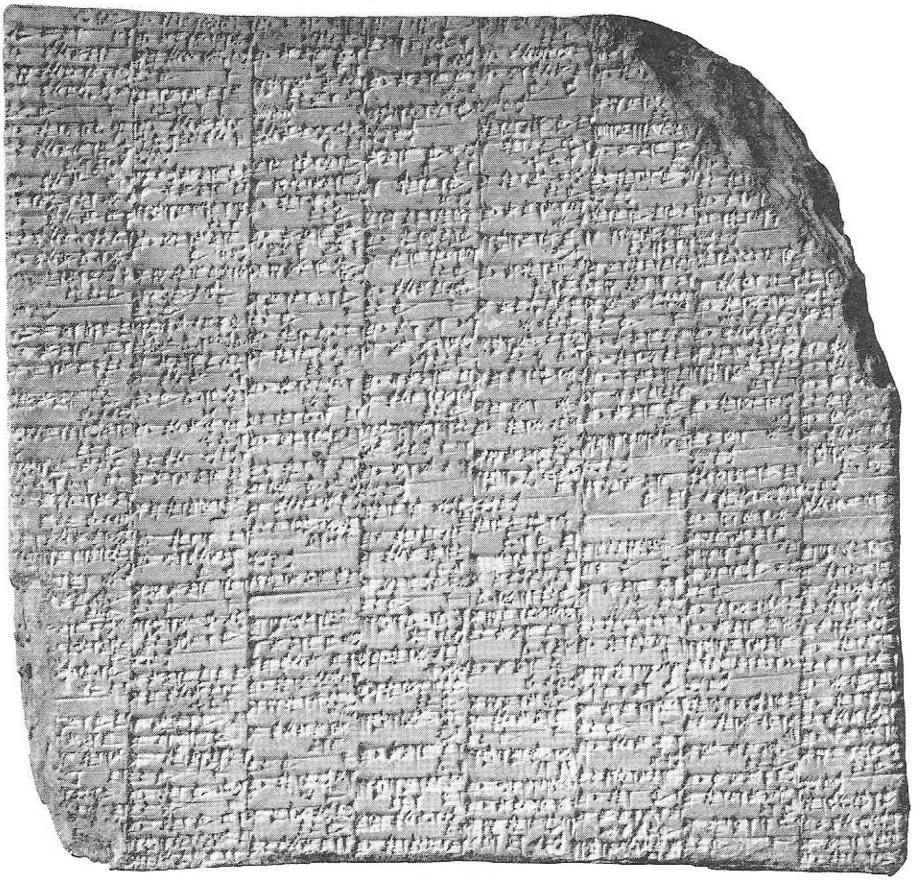


Fig. 308.

the work was done by hand by means of the stamp-sticks described earlier.

The illustrated tablet is reproduced at about half its actual size. Notice how the tiny symbols are quite sharp at the edges —in spite of the small format and the fact that the tablet is several thousand years old.

The formation of the cuneiform characters (including the length of the wedge "legs"), and the uniformity of the symbols, exclude—at any rate in this tablet—any possibility of *hand*-writing. The characters must have been impressed.

Fig. 308 shows another clay tablet, originally Babylonian and covered with a mass of Old Babylonian cuneiform script. This tablet stems from quite a different society, country and language from the preceding one—yet there is a distinct likeness.

As with the Sumerian tablet, this one is divided into columns of equal width, the width equalling that of the stamp-sticks used to press the symbols into the clay. Each column is split into a number of irregular rectangles, the height of each rectangle depending on the number of cuneiform wedges it contains.

The shadow here again reveals that not all the sticks were applied with the same pressure. The symbols are even smaller than on the Sumerian tablet.

This Babylonian tablet has been reproduced actual size, and I think we can

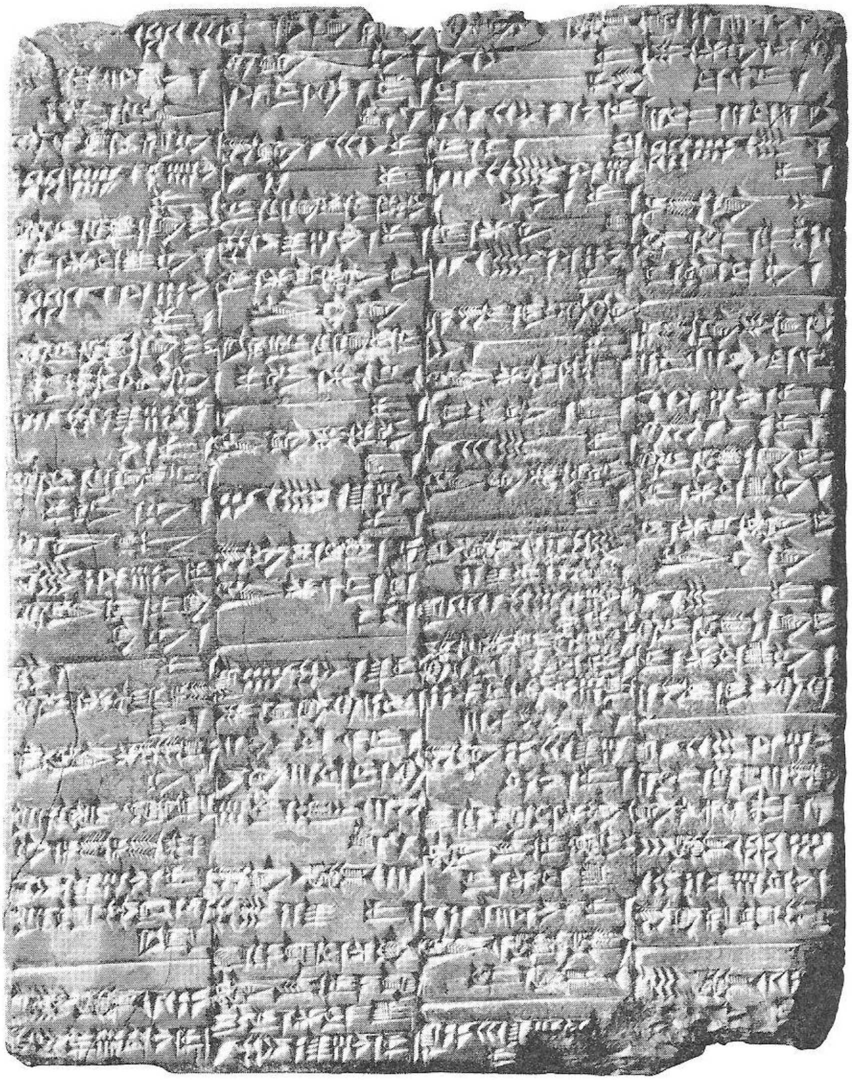


Fig. 309.

agree that the work was not carried out free hand, i.e. each symbol drawn or scratched individually into the clay. Not unless the scribe had the fine sight and steady hand of an expert chaser or carver. Since the tablet contains only a list of everyday objects it is unlikely that a highly specialised engraver would be employed to produce the work.

Fig. 309 shows another Sumerian clay tablet. Like the two others, this tablet is divided vertically and horizontally. The horizontal division, however, seems to be into larger word (?) units, and vertically the symbols show a tendency to run together with those of the adjacent column. I believe the evidence in this case indicates that this tablet was written with a single-wedge stick, instead of with a series of composite stamp-sticks.

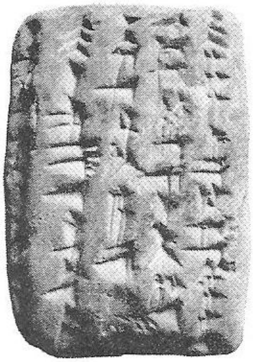


Fig. 310.

Fig. 310 shows a tiny clay tablet, dated around 600 B.C., bearing cuneiform script apparently "written" with a single-wedge stick. The vertical columns are clearly distinguishable, but at the same time we observe that the symbols are not so uniformly or tidily executed as in the earlier examples.

This is also true of the clay cylinder in *Fig. 311*. The items in the latter two illustrations were both produced by the Babylonians, were found in Babylon, and date from about 600 B.C.

If we accept that cuneiform script originated as described in the past few pages, with the construction of numerals based on ancient geometric diagrams, we appreciate that it would be perfectly natural—when the need arose to write words (as opposed to numbers) to turn to the same diagrams and develop them according to requirements.

In the first handful of numerals we find the basis for our syllabic script, namely the long-shafted used in the numerals 1, 2 and 3. The progressive combination was to arrange the symbols in two layers ( ). This meant shortening the shaft slightly to keep the symbol within the basic geometric square. The third combination, arranging the symbols in three layers ( ) with correspondingly shorter shafts, emphasises that the script planner did his best to remain within the geometric diagram.

When the need registered later to produce a syllabic script, the planner(s) turned to the horizontal wedge in order to extend their possibilities for variety ( ). This was used in the same manner as the vertical wedge, and an arrangement of horizontal and vertical strokes ( ), as well as shortened shafts provided ample material with which to meet the language's early needs.

Later, as alternatives were gradually used up and new words or phrases required a place in the language the 45° wedge ( ) sloping either to left or right was introduced, also with a variety of positions and lengths.

In fact, after years of examination and test I have discovered that all known cuneiform scripts fit into the lines of the geometric diagram. And the system was elastic enough to meet the demand for the 300 symbols that eventually developed.

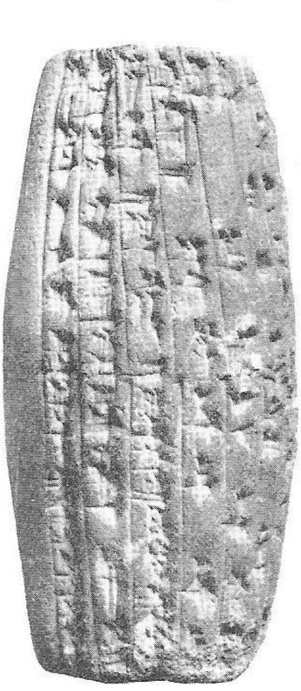


Fig. 311.

A system of writing based on the diagrams of ancient geometry, whether fully developed with both numerals and syllables, or whether meeting only a small proportion of a language's requirements, would have been readily understood by a Temple brother once he was instructed in its structure. And if he himself was familiar with the system's original source, i.e. ancient geometry, it would be a simple exercise to recall how the symbols were formed.

The Temple brother who, thanks to his degree of initiation, was allowed to share the secrets of script, writing and reading would have no difficulty in transferring the basis of the system home to his mother-country. In so doing he would be in no way whatever obliged to retain the symbols' original sound (phonetic) value. He or his brother-priests could allot to the cuneiform characters values that suited better their own tongue. But the geometric background and system would still form the basis of the script's appearance.

This would explain why, although sharing an obvious similarity of appearance, surviving examples of ancient cuneiform script represent totally differing languages.

Furthermore, there is the factor of how nimble-fingered the craftsmen were who carved the hundreds of stamp-sticks, how worn the sticks became, and how carefully they were applied.

It is also possible that in spite of the "mass-production" technique able craftsmen and scribes existed who could actually write the characters directly into the clay by hand. There is every likelihood that many experimented with the possibilities.

Fig. 312 shows a clay tablet dating from "Abraham's time", roughly 1500 B.C. Despite its large dimensions the tablet is said to be of little historical value since it merely records some accounting information. But note the script, how evenly it has been applied, how uniformly long the wedge-shafts are.

This tablet is quite ordinary, with nothing to attract the historian, and was not executed artistically or personally (as perhaps in the case of an official letter or communication). It is a lay-out of objects and figures, and presumably was not printed by the "cylinder" technique nor by signet-ring. It is therefore likely that it was written individually symbol by symbol, and the general sense of uniformity indicates that the symbols were formed by a stampstick.

Egyptian writing consisted of 600 different picture-signs or hieroglyphs, each picture being a symbol that represented a complete idea, a phrase or—in a few instances—a single word.

As far as cuneiform writing was concerned Chaldean script, for example, consisted of about 350 characters, but as opposed to the Egyptian system cuneiform symbols represented syllables which by appropriate composition could cover the same field of concepts and phrases met by the Egyptians with their 600 hieroglyphs.

To illustrate the cuneiform system we may compose a somewhat corresponding example from the English language. We can use numerals as symbols representing these five words:

1	2	3	4	5
head	first	hand	sign	post

From these five words (symbols) we can make up five new words, e.g.

1-2	2-3	3-4
head/first	first/hand	hand/sign
4-5	1-5	
sign/post	head/post	

and we understand immediately how five symbols made it possible for cuneiform writers to compose as many as 10 words (sometimes more, sometimes less).

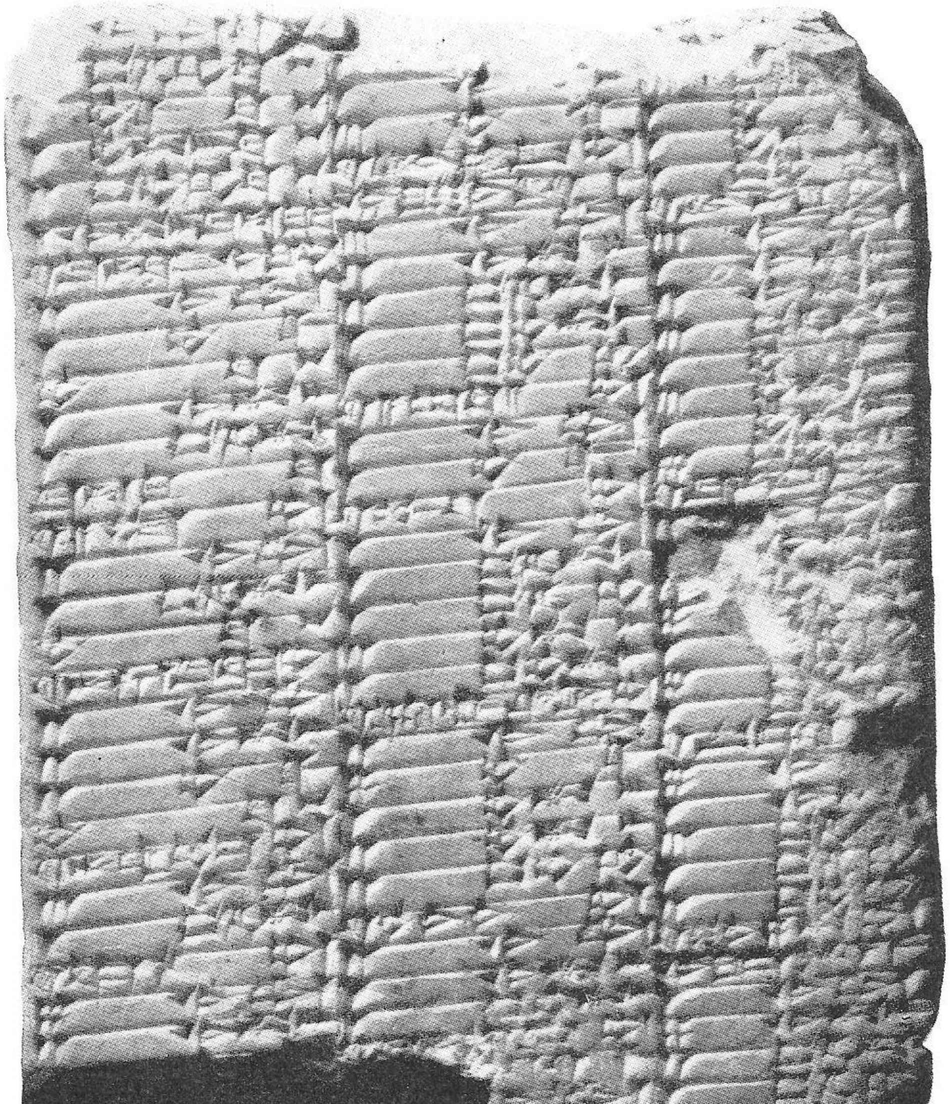


Fig. 312.

It is nothing short of amazing that philologists, with no native ability in the ancient spoken languages, have been able nevertheless to translate long-dead cuneiform scripts. Their research and detection into the subject is one of the most exciting and dramatic chapters in historical exploration.

The oldest known form of writing was the hieroglyphic system evolved by the Egyptians. It was followed chronologically by cuneiform, which developed in countries surrounding Egypt from about 3000 B.C. onwards. The Egyptians were also familiar with cuneiform, or at any rate with its existence and use. Assyrian and

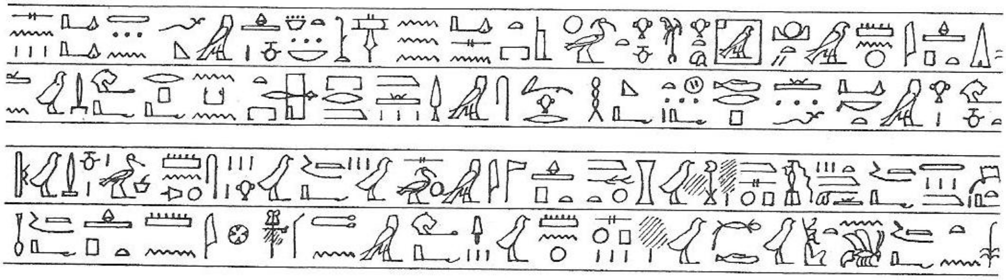


Fig. 313.

Babylonian clay tablets, containing cuneiform inscriptions, have been excavated from Egyptian temple sites strengthening the belief that certain groups within the Egyptian Temple were able to read cuneiform.

There is a long jump from hieroglyphs to cuneiform, if in fact any link exists between the two at all.

Hieroglyphs, as stated, portray in picture form ideas, feelings, sentences. An example of hieroglyphic writing is seen in *Fig. 313* and is reproduced from Lepsius's *Denkmaler aus Aegypten und Aethiopien.*

It is most likely that hieroglyphs began as a handful of illustrations, and gradually developed into a sophisticated system of cartoon-like symbols (although without the humour of the 20th century cartoon!).

A similar situation arose (and still exists) in China. Standard Chinese symbols represent ideas and phrases, not phonetic groups. And the language has produced so many characters that few but specialists have mastered Chinese in its entirety.

Cuneiform was the first language to break with old tradition, and to be based on a geometric system with a number of abstract symbols representing individual sounds which could be placed together to express words, names and sentences.

In *Fig. 314* we have 12 different phonetic sounds in Chaldean cuneiform. We observe that a new, wider wedge has been introduced, which placed strategically and in sufficient numbers helped to produce the (approx.) 350 symbols used by the Chaldeans.

In *Fig. 315* we see the geometric origin of these phonetic symbols. The 12 symbols occupy the same square as used in our earlier study of numerals. The phonetic value is indicated underneath.

Note how the signs are kept within the

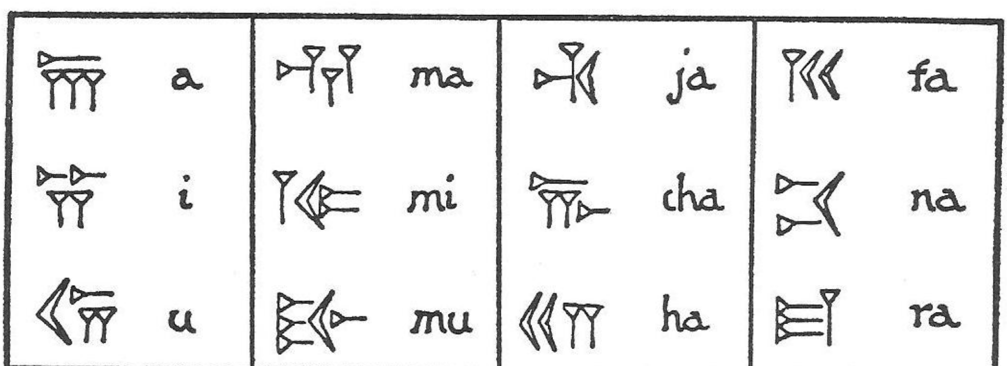


Fig. 314.

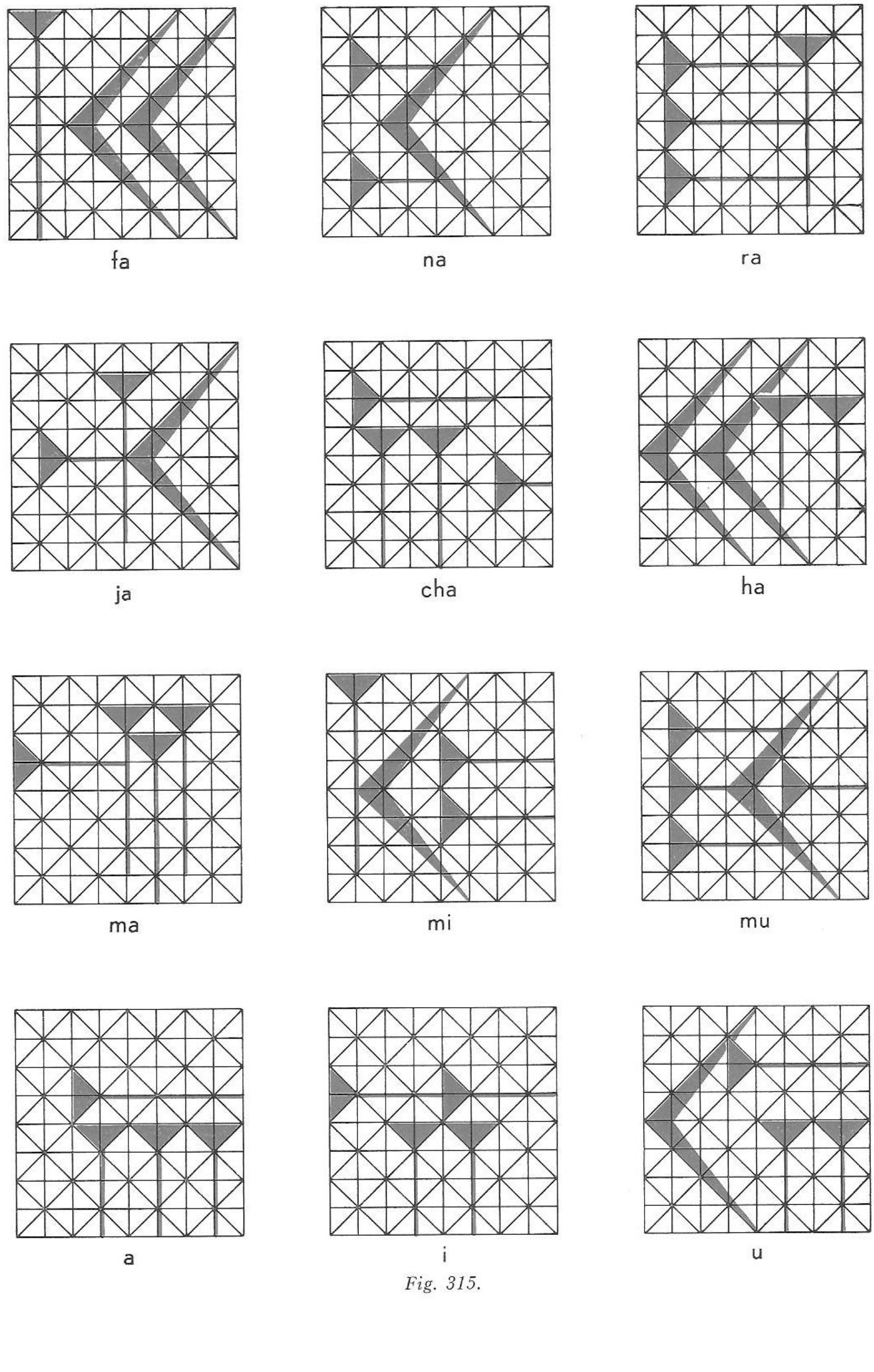


Fig. 315.

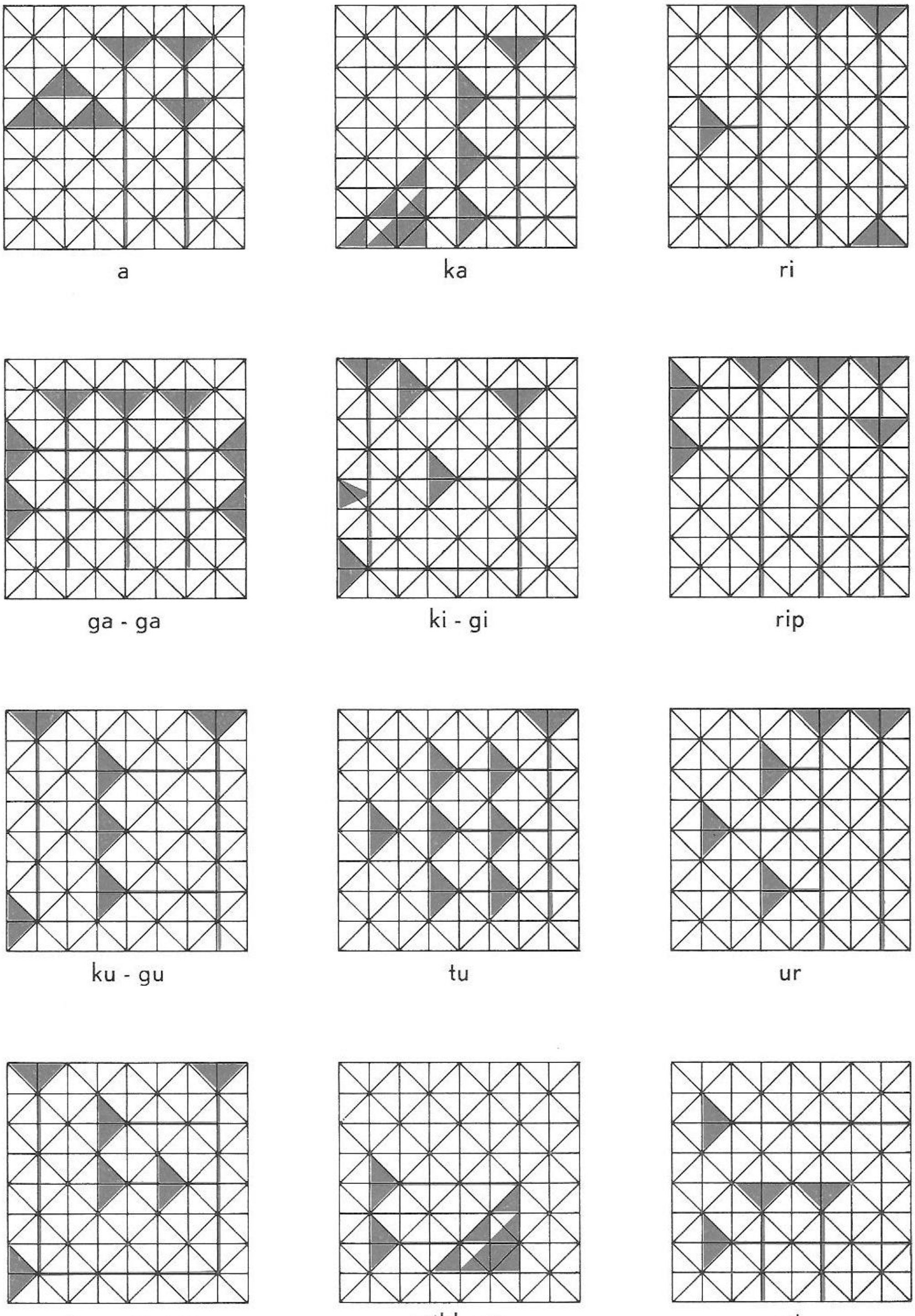


Fig. 316.

basic square. Almost all of them make full use of the square's area, either horizontally or vertically or both.

Similarly in *Fig. 316* we find 12 cuneiform symbols of Assyrian origin. The phonetic value is given underneath each symbol, which is placed in the background from which I belief the symbol's formation was originally chosen.

We find another variation of wedge, or rather two variations on the same theme: a large wedge covering twice the area of the standard size, and a small wedge covering half the area.

These samples are copies of extracts from an original inscription, and it is curious to record that not only do the new wedges lie perfectly naturally in the diagram, but they are also properly positioned in relation to the other wedges in the symbol. Such clear evidence can only be an indication that the script was originally "invented" on the basis of geometric diagrams.

The first three syllables are *a - ka - ri*. The word *akari,* according to linguistic specialists, meant Amurru or Amorite, the geographic term to designate the land of the west (in this instance Syria).

Fig. 317 illustrates 12 characters of the last cuneiform language to develop: Old Persian.

Old Persian was the cuneiform language that most resembled present-day theories of writing, since the individual symbols were streamlined to the point that their phonetic values were reduced in number to about 100 characters.

These 12 characters, too, fit easily and naturally into the constructive basic square without varying greatly from our established means of design. It is worth bearing in mind that these symbols represent more than 10 % of the entire written vocabulary of the Old Persian scribe, and were selected at random.

So much for an examination of 36 Chaldean, Assyrian and Old Persian cuneiform characters. We saw that each fitted exactly into our geometric diagram, into a diagram divided according to the custom of ancient times: by triangulation.

The symbols were normally of extremely small dimensions, and it must have been a fantastic piece of work, requiring the incisive care of the surgeon, to carve or gouge the tiny stamp-sticks. Was it not for the proved existence of cuneiform-covered clay tablets, it would perhaps be difficult for the modern mind to believe that ancient craftsmen could work to such a fine degree of accuracy.

The placing of vertical and horizontal wedges in relation to each other was vital to the legibility of the script, and it would have been (to say the least) an intricate task judging the position and length of the wedges by hand. Therefore I believe that the source of inspiration was geometry, *and that the Temple's trained craftsmen* were responsible for producing the required stamp-sticks. At some stage the hierarchy were obliged to release the script from the confines of the Temple to the common people, or at least to those who could be trained to write.

It is probable that the Temple remained responsible for the manufacture of stamp-sticks, and that these were bought by scribes. It is also possible that the sticks were smuggled or stolen from the Temple, or were placed in the hands of outsiders by some other means, and that they were copied and used. But the Temple never gave up responsibility for invention of new symbols to cover new or developing sounds in the language. One of the things that never seeped through to the outside, however, was the source from which the characters were formed.

Cuneiform characters succeeded—at any rate chronologically—the hieroglyph system of the Egyptians. In *Fig. 318* we see one theory of how hieroglyphic

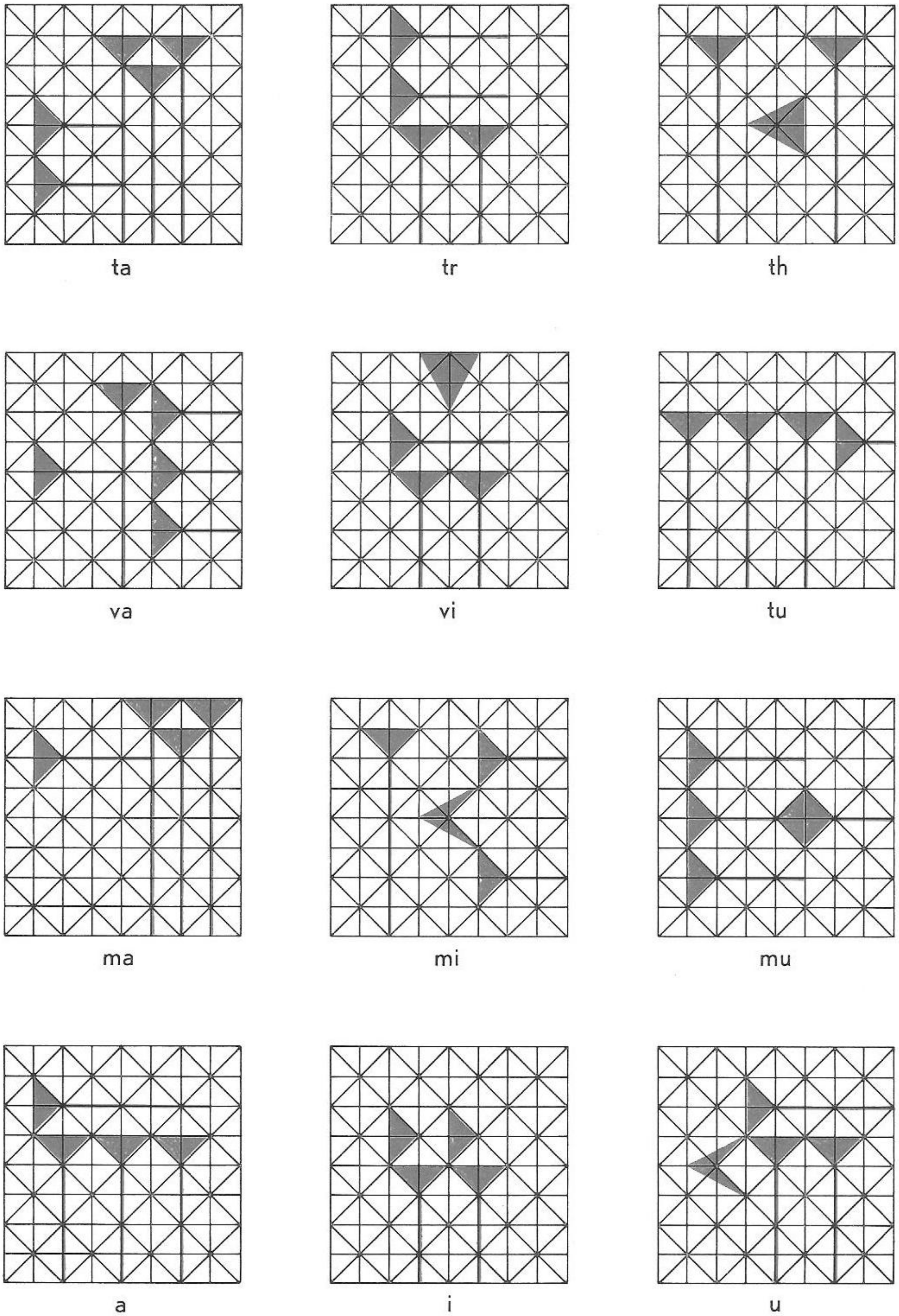


Fig. 317.

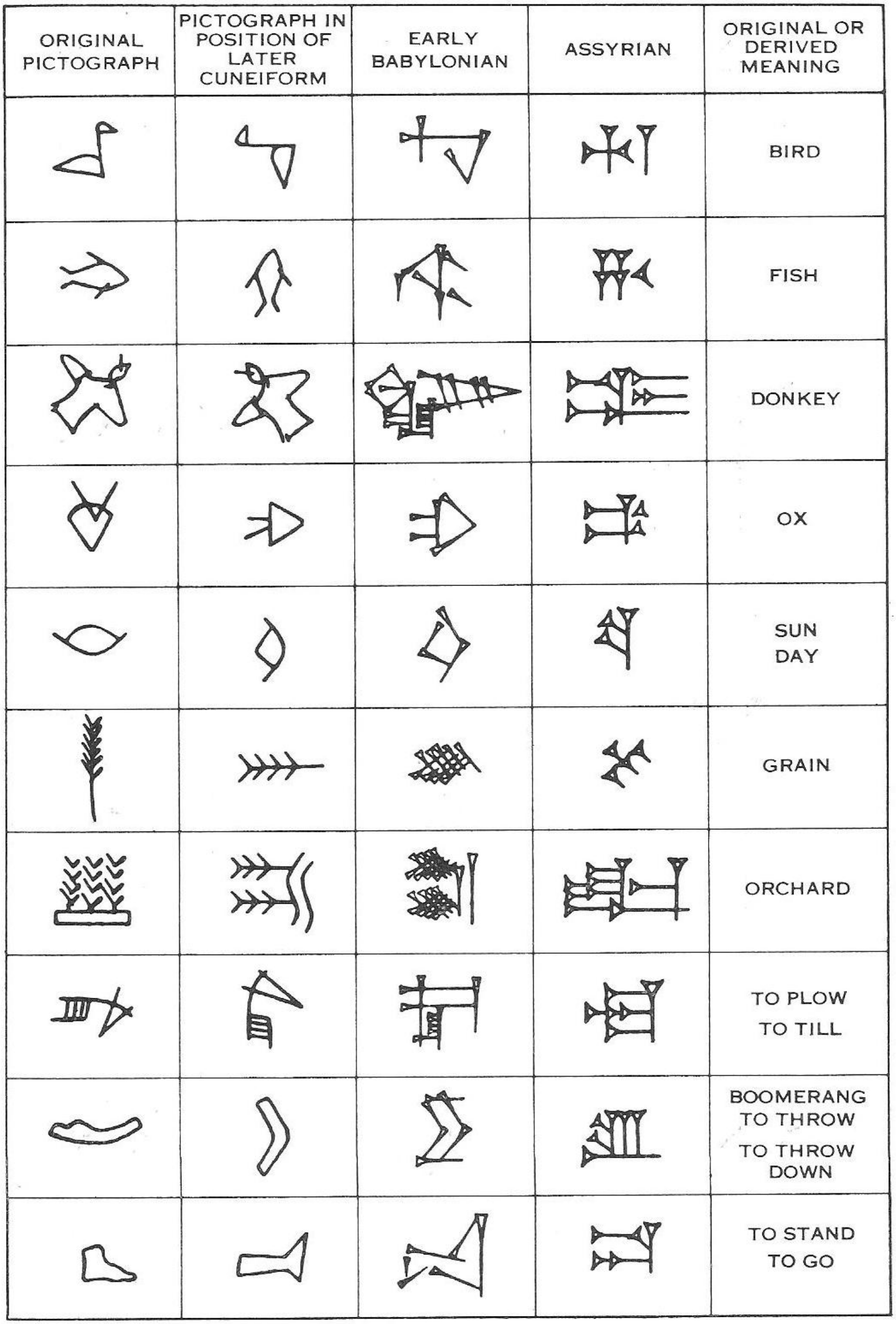

ORIGINAL PICTOGRAPH
PICTOGRAPH IN POSITION OF LATER CUNEIFORM
EARLY BABYLONIAN
ASSYRIAN
ORIGINAL OR DERIVED MEANING
BIRD
FISH
DONKEY
OX
SUN DAY
GRAIN
ORCHARD
TO PLOW TO TILL
BOOMERANG TO THROW TO THROW DOWN
TO STAND TO GO

Fig. 318.

images *gradually transformed to* cuneiform wedges.

It is suggested that the original hieroglyph was rotated quarter of a turn anticlockwise, and that the result was written in the form of cuneiform characters. The examples shown are Early Babylonian and Assyrian.

If the theory is studied realistically, it must be admitted that the resemblance between hieroglyph and cuneiform is so slim (in some cases the two are totally different) that in normal circumstances comparison would not be attempted. The only reason for this effort was apparently that the researcher was determined to trace a common source for the two types of writing—and was persuaded to compare them physically.

Notwithstanding the hollowness of the above theory, however, hieroglyphs and cuneiform characters undoubtedly enjoyed some bond, if only on the basis of their proximity in time and geography.

Clay tablets have been found in Egypt with Babylonian cuneiform script, and Egyptian excavation has produced a form of native cuneiform, indicating that certain circles in Egypt were familiar with and presumably used cuneiform. But in Egypt's case it was impossible to introduce cuneiform generally: her pictorial hieroglyphs were too firmly established to allow a complete breakthrough into cuneiform, even though certain circles in the Egyptian Temple may have been aware that the latter system of writing had the advantage over hieroglyphs in the way of rationalisation.

The Temple brother (or group of priests) who drew up the plans for introducing cuneiform writing to the known world was naturally fully conversant with Egyptian writing, and by admitting the shortcomings of pictograph images he was able to advance a step in the direction of rationalisation. He prepared to launch *symbols representing syllables instead of* words, sentences and concepts.

In this respect (inasmuch as he had studied the faults of hieroglyphs) it is correct to say that Egyptian hieroglyphic provided the inspiration for cuneiform. But I am afraid it is an unacceptable theory that hieroglyphs and cuneiform characters in their appearance stem from the same source. The very make-up of the two forms of writing belies the theory. Each hieroglyph represented in many cases not only a word or an object, but in addition a series of words or a whole idea.

Cuneiform consisted almost exclusively of syllabic symbols, which would make it virtually impossible to stylise the hieroglyphs and transport them to other languages.

The origin of the hieroglyph lay in the simple, primitive ability to draw a picture of the idea one wished to convey. The art was to evolve as simple a picture as possible.

The origin of the cuneiform character must have been a geometric diagram. The constructive theme in the signs themselves indicates this.

I regard it as equally obvious that the newly invented form of writing was controlled by certain groups of society who fashioned the characters, supplemented them when required with new words and syllables, and made sure that the script did not fall into disuse or degeneration. Without the guiding factor of educational leaders any writing would develop into chaos.

Compare, for example, the development of cuneiform with our own way of writing. Everyone who attends school is taught how to form and write the same standard script. But no two children leave school with the same handwriting. Every individual gives the standard script his or her own flourish of personality—but it is not vitally important to the survival of our

writing system since all copy the standard script, which is taught and retaught by generations of teachers. The script itself cannot degenerate.

Just as schools take care that hand-written script does not degenerate into a maze of millions of different versions, so the Temple teachers of Babylon, Chaldea, Persia, etc., were careful that a standard form of cuneiform writing was maintained.

Egyptian hieroglyphics remained unchanged in form and appearance for thousands of years. Why then should cuneiform characters have altered so much that in comparing them with hieroglyphs we can find no point of likeness? The answer can only be that the theory that they were ever linked must be erroneous.

I submit that the theory set out in this chapter regarding the origin of cuneiform characters fits more readily into the historical background, normal development of Man's speculation, and esoteric tradition.

*

An idea, an object, a custom lasts only so long. Cuneiform, too, had a limited life. It was however a few thousand years before fresh impulses throbbed through society and forced the old method of writing somewhat into the background.

The new inspirations came in the guise of modern writing materials. Man learned how to cure leather so finely that it could be used for writing upon. Moreover the Egyptians had developed the papyrus and paper, which were sold to surrounding countries, who had also learned from the Egyptians how to paint and draw on finely woven linens.

A selection of writing materials thus arose, quite different from the material normally used for writing purposes: the clay tablet. And attempts to transfer cuneiform to these new materials were on the whole unsuccessful. The characters had to be drawn, and were thus much larger than the scribe was accustomed to.

Dyes were still far from perfect and were apt to be sucked down into the fibres of the paper or linen, leaving a hazy edge on the cuneiform wedges. The technique of writing became slower and less legible than by stamp-stick and clay tablet. The stamp process could not be transferred to the new materials since the equipment had not been adapted for printing. The problem was how to employ linen, silks, papyrus, paper and parchment in writing.

These materials of course did not appear on the scene overnight. Papyrus was used in Egypt for hundreds of years for hieroglyphs, to which it was well suited. It became more and more in demand as the number of Temple scribes increased.

Paper was known within the Temple, where it was used for writing experiments. One of the early needs that registered was for new numeric symbols to take the place of cuneiform characters which could not be written on paper, etc.

The new numerals did not however come to life in the temples of the Euphrates or Tigris. Nor were they developed in Egypt. Their inventors were the priests of India whence they were brought via Persia and Arabia to the Mediterranean. They were the Hindu-Arabic (Gobar) numerals which European scripts use today.

We talk of them as Arabic numerals because they came north to the then known world from Arabia, but in *The Romance of Writing* by Keith Gordon Irvine we are told that the Arabians called the symbols Persian, the Persians referred to them as Indian, and further research has failed to bring the trail further than India. The assumption must therefore be that they originated among the temples of that country. Irvine does not indicate where he obtained his in-

formation. He does however print a row of numerals supposed to resemble closely the original symbols, but constructed and based admittedly upon guesswork since no contemporary material exists to prove the appearance of the ancient symbols.

When the Arabians handed the numerical symbols over to the Mediterranean-Euphrates-Tigris people the symbols looked more or less as we know them today.

We are now at a stage when it is, if possible, even more difficult than previously to trace the original symbol since we are on the threshold of comparative freedom in script. The symbols are reproduced by the individual, not by the official Temple carver. It must be admitted that if we compared the numerals that appear on a broad cross-section of the handwritten material we come in contact with every day with the original standard school numerals it would often be difficult to recognise the "matured" script.

The alteration from standard numeral to the widely varied versions executed by each of us later in life is due to personal idiosyncracies. And it was most likely the same with Man's first attempt at handwriting (as opposed to cuneiform stamping, and hieroglyph drawing).

The further back we look, the more carefully we find our own personal handwriting was produced. A definite effort was made in the early days to reproduce an exact likeness of the original standard symbols. Later, as writing becomes simply a tool with which to express oneself and as long as others can understand the written text, there is a tendency towards carelessness.

But let us have a look at the Hindu-Arabic numerals and compare them with our geometric diagrams.

Although thousands of years had passed since cuneiform script was introduced, the Temple/People relationship was largely unchanged. It was still the Temple hierarchy who took the initiative in spheres of general interest, and whether the numerals originated in India or Arabia, the Temple influence and position would be the same as in Egypt or Babylon.

The Temple discovered that the new materials were unsuited to current methods of writing, i.e. cuneiform. But they were admirable for drawing upon. They could accept curved and of course straight lines, and as the first requirement within the Temple was probably paper for accounting purposes the leaders decided that new numerical symbols were needed.

Faithful to the tradition of thousands of years the priest or committee of priests turned to geometric diagrams for inspiration. Their search centered on the basic square, the inside circle, the vertical and diagonal crosses, i.e. symbol "D", with one acute-angled triangle.

As with earlier formation of script, the most suitable shapes were then selected from the diagram as numerical symbols.

We see the result of their work in *Fig. 319* which illustrates how the Temple script experts followed the lines of the diagram in order to produce the new symbols.

The symbol for 7, we notice, was originally V-shaped, the form being taken from the acute-angled triangle in the geometric basic square.

This group of symbols had the advantage that they could be written either from left to right as is most common today, or from right to left as was the practice in many scripts in bygone days (and on occasions today).

In *Fig. 320* we see the reverse version of the numerals in use today, as they would be written from right to left.

These symbols, both in their appearance and practical use, were so successful and popular that they overcame all opposition

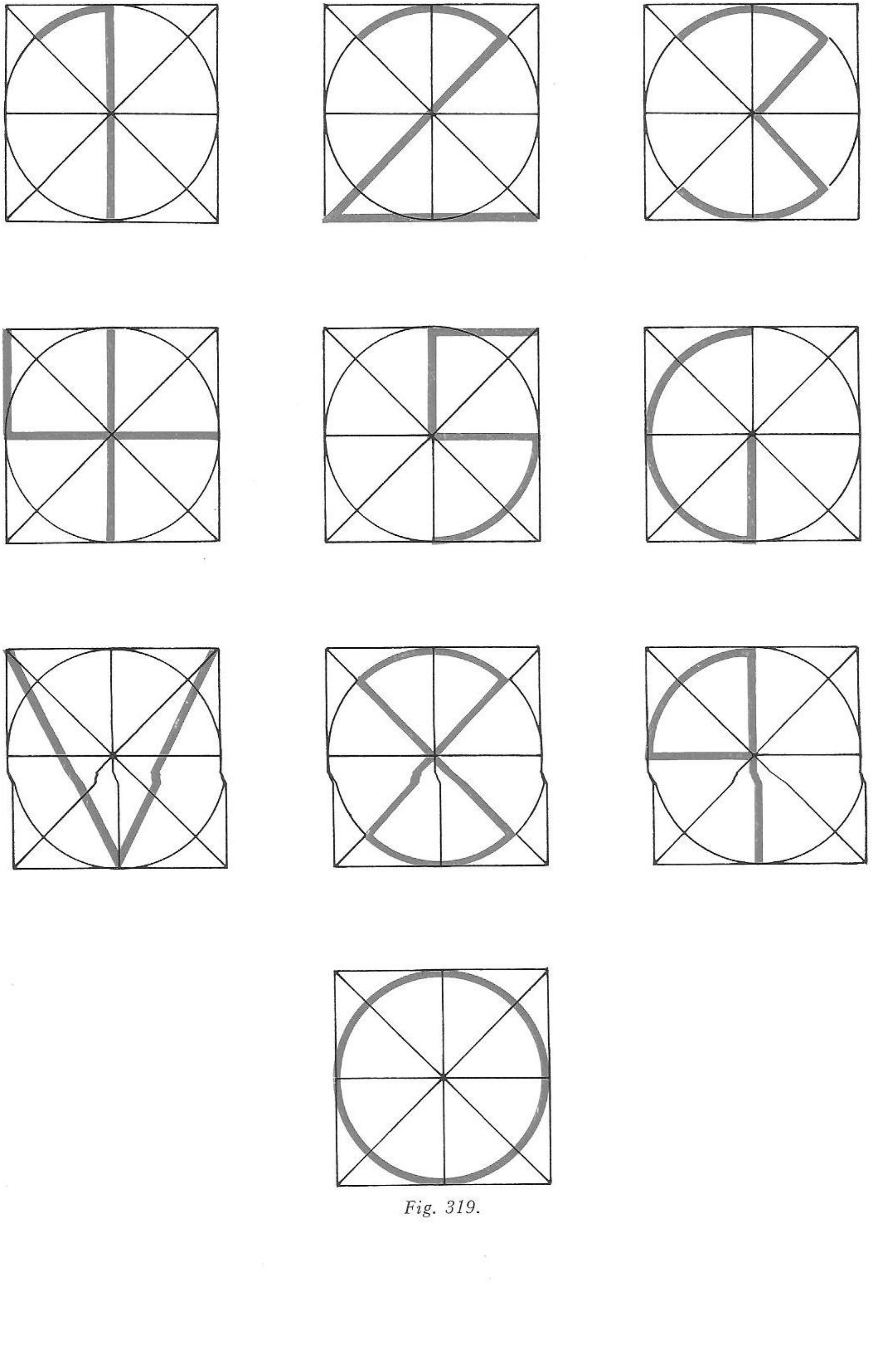


Fig. 319.

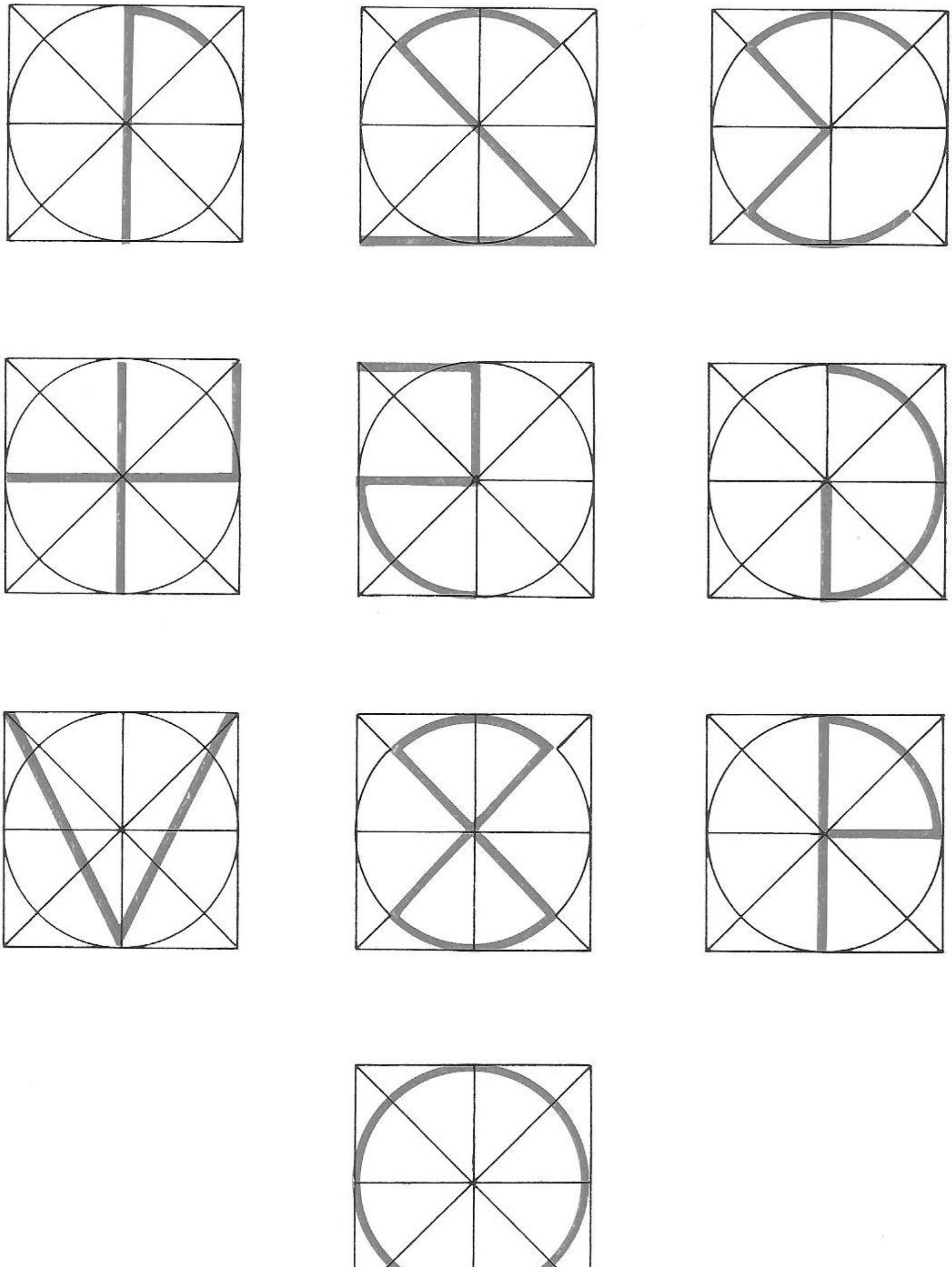


Fig. 320.

from existing forms of numerical symbols, such as cuneiform, etc. They spread from society to society until they now dominate most of the world's script.

The process of distribution was unchanged. The symbols passed from temple to temple, after which the priests in each respective area released them for general use. Minor alterations were incorporated here and there, but the symbols remained basically the same.

It was presumably thanks to these minor changes that the numeral 7 took on its present form. Since it originally was the only symbol to make use of the acute-angled triangle, the desire may have registered to rationalise this numeral and to fit it into the same basic symbol "D" as the other nine symbols occupied. It is easy to recognise in Fig. 319 or 320 the place in the symbol from which the present numeral 7 was selected.

It naturally took many centuries for the Hindu-Arabic numerals to conquer Europe—particularly in face of Roman competition. The two doubtless lived side by side, as they do to a certain extent today, until the invader from Arabia claimed victory.

It is not impossible that the numerals were passed on to certain societies with no knowledge whatever of the original geometric basis for their formation. They were simply copied from the numerals of other civilisations. Variations would have crept in—especially if the numerals copied were already written in an untrained (geometrically) hand.

But despite the many possibilities that exist for degeneration of our numerals it is quite astounding to observe how they have retained their basic form down through the centuries. This could only have been feasible when an authority like the Temple bore responsibility for guidance and instruction of contemporary writers.

Our Hindu-Arabic numerals would not fit so neatly into the geometric diagram if in fact they did not belong there initially. The Temple kept their origin secret—but, knowing the geometric birthplace, was able to guard against degeneration and disuse.

CHAPTER EIGHTEEN

ABC of the Modern Alphabet

THE PRECEDING chapter closed with the discovery that a product of the Temple in India, i.e. Hindu-Arabic numerals, arrived in the Euphrates-Tigris region to *displace cuneiform numerals. One of their* outstanding qualities was their suitability for inscription on the numerous writing materials evolved to take the place of clay tablets. The Hindu-Arabic symbols (also known in modern Europe as Gobar numerals) were spread from India to Persia, from there to Babylon and thence throughout the whole of the cultural world.

But though the numerals were well-known and approved by leaders of northern civilisations, it took some time to persuade the relevant Temple authorities that cuneiform ought to be replaced entirely. The latter after all was well-established and highly suitable for writing in soft clay. And moreover cuneiform possessed both numerals and letters/syllables whereas the new system was one of numerals only.

The new writing materials (principally papyrus) were recognised as more convenient than clay. They could not be used for cuneiform—so a new form of writing had to be invented to complement the numerals produced by the Indians.

Invent a new form of writing. Rather easily said. But what a task! Not only did the appearance of the actual symbols have to be decided, but Temple authorities had to be persuaded that the move was beneficial, and that it was time cuneiform script (used for thousands of years) gave way to a more progressive and rational *system of writing.*

Launching a new written language in those days was a feat of persuasion, brainwork and daring. Imagine proposing to the peoples of Britain and America that the English language is outdated and should be replaced altogether by another *which has not yet even been invented!* Such was the boldness of the plan to switch from cuneiform writing to a new system.

It succeeded—but it took time. There were no mass communications, no television, newspapers or advertisements, to assist publicity of the new script. Admittedly the circle of scribes affected was relatively small by present-day standards—but distance and communications were a hindrance.

For a long time cuneiform and the new script(s) rode side by side, the former dominating. But gradually progress won ahead.

The first recognised system of writing following (and interlapping with) hieroglyphics and cuneiform was that of the Pheonicians, who flourished around 1000 B.C.

The oldest surviving trace of their writing was discovered carved on a stone

Fig. 321.

at Dhiban, a village in central Jordan. The Moabite Stone, as it has been called, contains a record of the successes gained by the Moabite king Mesha against Israel.

The inscription is also referred to as the Mesha or Mesa inscription.

The Moabite Stone was found in 1868 by beduins who smashed it in several pieces in the hope of increasing the value of their find. It was obtained and assembled as well as possible by the French researcher M. Clermont-Ganneau, and is now on display in the Louvre. Its approximate dating is 850 B.C. and the inscription has been described as the oldest documentary example of Hebrew language and script.

The Stone in its present state is illustrated in *Fig. 321.* The smooth portions of the surface represent the parts that Clermont-Ganneau was unable to trace. The text, reconstructed as far as possible, is seen in *Fig. 322* and is reproduced from *Die Inschrift des König Mesa von Maab* by Rudolf Smend and Albert Socin, Freiburg, 1886.

The inscription is made up of 22 letters, which are to be found assembled in *Geschichte der griechisch-römischen Schrift* by Arthur Mentz, Leipzig, 1920. To avoid any error in reproducing the Mesha alphabet the letters were photographed and appear in *Fig. 323.*

Mentz himself wrote of the script:

"Diese Schriftzeichen sind im grossen und ganzen gleichförmig. Gewiss zeigen selbst die Buchstaben der Mesainschrift, die als Prunkschrift recht sorgfältig geschrieben ist, kleine Abweichungen in der Richtung unter der Formung der Linien, aber die Einheitlichkeit ist doch gross."

Thus in Mentz's opinion, although the 22 letters are different they nevertheless have a strong family resemblance. They possess a plain, uncluttered, simple look.

Despite wide variation in the slope of some of the symbols, the mutual resemblance is obvious although Mentz was unable to put a finger on the factor that bonds the letters into a family.

His observation was sharp, particularly since he had no idea that in fact such a clear link existed. The same link as bound together the cuneiform wedges: ancient geometry.

The script of the Phoenicians (shared by King Mesha and his Moabite people, who were of the same Semitic branch as the Phoenicians) apparently came to life in the Temple of these sea-traders. Familiar with the system of triangulation from which cuneiform characters were derived, Phoenician priests examined the new Hindu-Arabic numerals and realised that they were produced from a different geometric diagram. They thus in their search

Fig. 322.

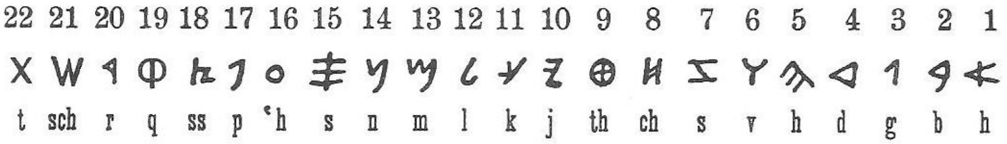


Fig. 323.

for a fresh script to accompany the new numerals selected a similar geometric diagram from which to choose their rational system of 22 symbols.

We see the diagram in *Fig. 324*. It comprises a circle surrounded by a square and with a second square inscribed within the circle. We recall that the outer and inner squares are in the ratio 2:1. Guide-lines are entered (vertical and diagonal crosses) together with two acute-angled triangles. This would indicate that the Phoenicians did not have sufficient variety with the square-circle-square combination to produce 22 differing symbols.

It is also possible that the acute-angled triangles were included to confuse any attempt to trace the basis of the alphabet's formation.

Another factor worth noting is the number of letters in the alphabet: 22.

Esoteric tradition demanded that Temple initiation lasted 22 years if the candidate was to obtain the full "course" of knowledge. The two figures tally. Intention or coincidence?

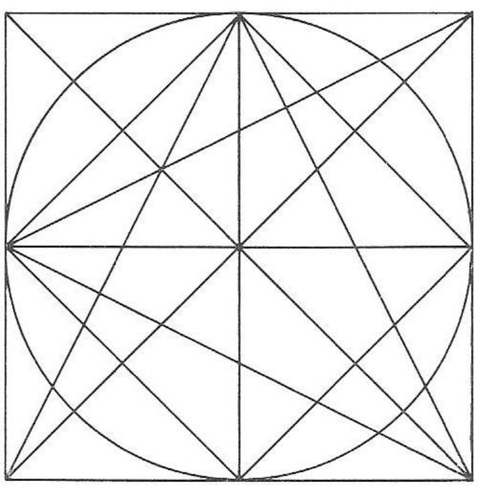


Fig. 324.

Before we have a look at the most likely source from which these letters originated, we ought to bear in mind once again that whether numerals or letters such symbols must have had a recognised source from which they could be reconstructed. Particularly when printing as we know it did not exist and when writing was executed by hand.

It would be impossible to manufacture, write and *maintain* 22 shapes unless they were based on a diagram. If drawn entirely from the imagination of a Temple priest, no matter how powerful or authoritative his status, the symbols would rapidly degenerate and disappear. And there is no doubt that the Phoenician script did not disappear.

As the *Encyclopaedia Britannica* points out:

"The Phoenicians ..... rendered one great service to literature: they took a large share in the development and diffusion of the alphabet which forms the foundation of Greek and of all European writing. The Phoenician letters in their earlier types are practically identical with those used by the Hebrews, the Moabites, and the Aramaeans of north Syria."

Without the geometric diagram to guide Temple scribes and to regulate the appearance of the script the letters would quickly have become unrecognisable. The Greeks increased the number of symbols to nearly 100—all within the framework of the ancient geometric diagram.

Letters and handwriting today are a

natural part of our education and culture. We all know, having been taught in school, how letters *should* look although our handwritten reproduction of the letters may leave much to be desired.

(It is interesting in this connection to read of tests conducted with word-blind children who have difficulty in recalling the shape of individual letters. In a recent test in Denmark one young girl wrote: "Dear father, I am writing 'father' with a small f because I cannot remember how to make a capital f . . . . .")

Our school-teachers taught us how to write the model standard letters. They in turn had been instructed by higher authorities.

It is perhaps feasible in a tiny society to invent a written language based on symbols drawn entirely from the imagination—if the community is self-supporting and cut off from the outside world. But as soon as the script is taken outside the community it is impossible for anyone else to understand it. When the symbols are repeated outside the community, who can check their resemblance to the original imaginative shapes? And how much will the individual's personal handwriting affect the symbols' formation, etc.? When these in turn are copied time and time again by strangers perhaps learning the alphabet there is little doubt that the original shapes will soon be forgotten, and the result will be degeneration and death to the bastard alphabet.

But if a system or diagram or fixed image exists on which the alphabet was originally based, then degeneration of the symbols is much less likely.

It is not within the scope of our research to establish the date (even approximately) of the invention of Phoenician script. The only date we have is that of the Moabite Stone. Archaeologists have studied its content, surroundings, background, etc., and have reached the conclusion that it was carved about 850 B.C.

Before the engraving process, the symbols of King Mesha were presumably sketched on the stone in chalk, so although they exist today as symbols gouged into a slab of stone the carving work is really a kind of "printed copy" of handwritten signs.

There is no reason whatever to suppose that stone was the most popular means of writing at that period. It is more a question of what is likely to have survived a span of nearly 3000 years. A carved stone can readily survive that length of time, so can a clay tablet if it is stored in favourable conditions. Less robust materials are likely to crumble and decay.

If some catastrophe hit our planet at this very moment and the bulk of the world's population were wiped out, throwing Man back into primitive life, our culture (including literature, paintings, manuscripts, etc.) would be buried under a heap of ruins.

After several thousand years a new, developing Mankind would emerge, and at some point in their lives the Earth's new inhabitants would perhaps begin digging in the ruins and sand to learn about the past.

They would probably uncover our gravestones, memorials, and perhaps a few signs manufactured in metal—but all fabrics, paper, leather, etc., would have rotted and decayed into non-existence. Their inscriptions would be lost to searching archaeologists.

In those circumstances it would be erroneous indeed for the searchers to assume that our writing materials in 1967 consisted only of stone and metal simply because they had found those materials.

When we in our turn assess the surviving relics of Egyptian culture, we discover wall surfaces decorated with hundreds of hieroglyphic symbols and figures. These inscriptions have survived time.

And we thus know how Egyptian script looked.

We also know from illustrations on temple walls that the Egyptians were experts in manufacturing fine linen materials, and that they are recorded as the inventors of papyrus.

It is thus both natural and reasonable to assume that the Egyptians (and similar nations of their era, including the Phoenicians) possessed more convenient writing materials than slabs of stone. A rock pillar may be an imposing sight covered in hieroglyphic symbols but it makes a poor subsistute for paper or other easily handled materials when a message has to be transported over long distances or when notes have to be taken simply in order to remember a few facts (such as household accounts).

Solon's father is reported by Plato as telling of a discussion he had with an old Egyptian priest about 700 B.C. during which the priest mentioned that scribes of the Egyptian Temple had made written records of every occurrence of major interest both within and outside Egypt. This diary had been maintained for so long that the Egyptians were able to tell the Greeks of events that took place in Greece *before the Flood*. The tale is recounted in Plato's *Critias,* and even with a pinch of salt on the veracity of the story there can be little doubt that the Egyptians had at their disposal more convenient writing materials than stone.

The Phoenicians—although not recorded by history as having erected anything like the Great Pyramid—were also extremely advanced in their culture. They were also merchants of the finest calibre. It is equally certain that they, too, possessed other writing materials than stone, and that these influenced the creation and popularity of new scripts, forcing cuneiform into the background so successfully that it was a completely forgotten factor until its rediscovery in the 17th century A.D.

The new method or style of writing had its good and bad points. It could be executed by hand on suitable materials, but precisely because it was written by hand it presented considerably opportunity to depart from the original symbol formations. Perhaps so far from the original that the latter threatened to be lost completely. This was the position when the Phoenicians wrote their symbols, and later passed these on to the Greeks.

Much later, when printing was developed, the original symbols were brought to light. It was much easier in print to retain a standard form of letter than when the letters were written by free hand. Just as it had been easy for Temple carvers to produce a standard cuneiform wedge for "printing" on clay tablets.

The exact shape and slope of some of the Mesha Stone symbols (and of symbols subsequently found of the same type) vary somewhat depending on the researcher. It is difficult perhaps for a scholar to reproduce a script exactly in every detail if he is not aware that the length, height, slope and proportions of the letters in reality fit into a strictly accurate geometric diagram.

I have chosen to analyse the symbols reproduced in Mentz's book not because they fit more readily into my analytical diagrams than the symbols of other scholars, but because his work is of a more serious and comprehensive nature than many of his colleagues without at the same time delving too deeply into the origin of the script in relation, for example, to hieroglyphs and other systems of writing.

In the coming analysis I shall use photographic reproductions of Mentz's symbols. The Phoenician alphabet was written from right to left. The number above each symbol, therefore, in Fig. 323 refers not to the

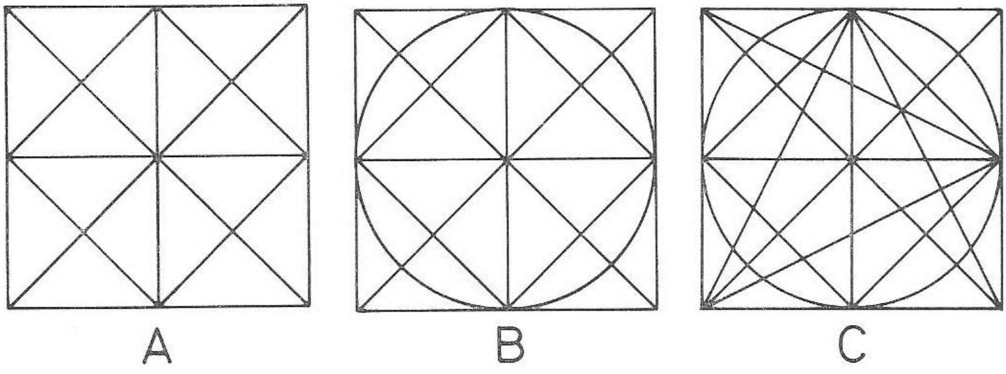


Fig. 325.

symbol's phonetic or other value but to its position in the alphabet.

Confronting the Phoenician-Moabite letters with their original parent symbol, we shall find that three letters (6, 8 and 21) out of 22 do not fit exactly into the symbol—or rather the alternatives leave a suspicion of doubt about the correct answer. But on reflection it must be agreed that it is really incredible that 19 out of 22 symbols retained an unmistakable resemblance to their original geometric parent—in spite of the fact that they were written in hand (and carved) by a human being, with all the idiosyncracies of the human individual.

When a number of symbols are constructed from a geometric diagram they cannot *all* touch or use *all* of the diagram's lines. We shall therefore for the sake of clarity break our latest analytical diagram into three recognisable categories or stages.

In *Fig. 325 A* we find the basic diagram with two squares, vertical and diagonal crosses. In this diagram we shall construct all those symbols whose lines coincide with those of the diagram without affecting or touching the diagram's other geometric shapes, the circle and acute-angled triangle.

Fig. 325 B shows the diagram with the addition of the circle, and here we shall draw those symbols which require the use of the circle.

In *Fig. 325 C* we have the full diagram, showing the acute-angled triangles.

In practice, when the alphabet was being designed the letters had likely been constructed within the complete diagram, i.e. Fig. 325 C.

In *Fig. 326* we find those symbols we can call group A, nine in number: 10, 7, 5, 3, 22, 20, 18, 15 and 11.

No discussion is necessary. All nine fit so neatly and naturally into the geometric diagram that the result is almost breathtaking. It is perhaps worth emphasising symbols 5 and 11. How else could these have been invented if not from precisely this type of geometric diagram?

Theories have been aired in the past that the Phoenician script derived from symbols for well-known objects such as cow, door, fishing-rod, etc. The letters, according to the theorists, were an abstract image of the idea they expressed. But these theories do not account for several letters, e.g. no. 5, which lack all visible connection with common objects or shapes.

There can be little basis to such theories since the Phoenicians had long been familiar with Egyptian hieroglyphs and Assyrian-Babylonian-Sumerian cuneiform, both forms of language having a wealth

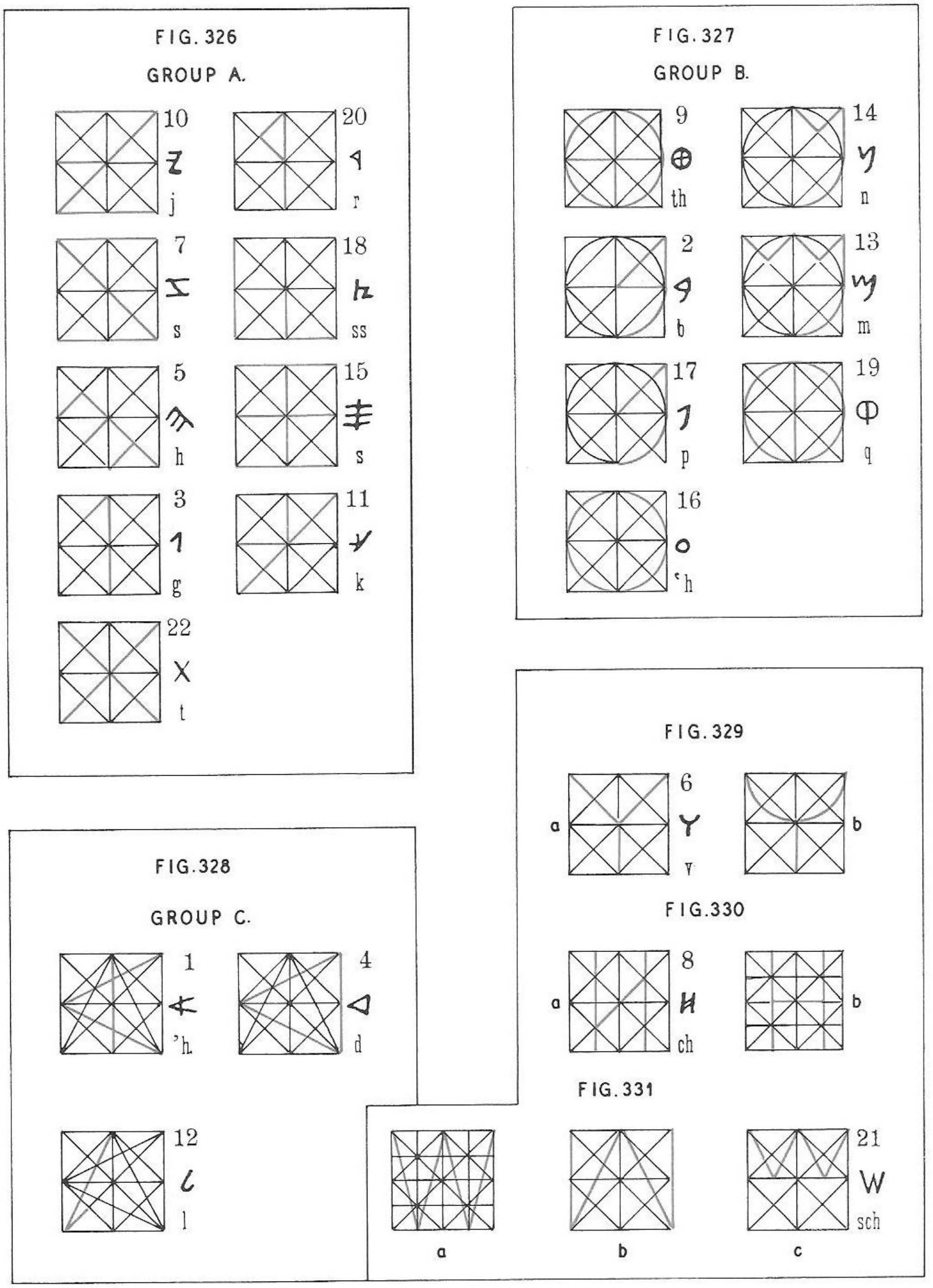


FIG. 326
GROUP A.
10
j
20
r
7
s
18
ss
5
h
15
s
3
g
11
k
22
X
t
FIG. 327
GROUP B.
9
th
14
n
2
b
13
m
17
p
19
q
16
ʿh
FIG. 328
GROUP C.
1
ʾh.
4
d
12
l
FIG. 329
6
Y
a
v
b
FIG. 330
8
a
ch
b
FIG. 331
21
W
sch
a
b
c

Fig. 326–331.

of expression. It is ridiculous to suggest that on this background and with knowledge of these languages the Phoenicians went ahead and produced a system of writing that could express only 22 ideas, compared with the many (thousands?) combinations in cuneiform.

Such a 22-idea system would have been a retrogressive step, and could scarcely have formed the foundation for many of the written languages in use today—as indeed was the case with the Phoenician-Moabite script.

In *Fig. 327* (group B) we see the next seven symbols, which make use of the circle's arc. They are 9, 17, 2, 16, 14, 13 and 19.

Here again the symbols fit so closely into the lines of the diagram that there can be no second thoughts as to their origin: ancient geometry. Nos. 14 and 13 are particularly striking. Admittedly a circle with a stroke or a cross through it can be selected from the imagination as a phonetic symbol, but hardly so symbols 13 and 14.

The next group is C, and in *Fig. 328* we see the three symbols drawn from this diagram, which of course includes the acute-angled triangle, symbols 1, 4 and 12.

These also fit naturally into the diagram, and this accounts for 19 of the 22 Phoenician symbols. We are in no doubt as to the origin of these 19.

This leaves us with three symbols, nos. 6, 8 and 21. For these the diagram provides one or two alternatives—hence the inability to state categorically precisely where their origin lies.

In *Fig. 329* we find two possible formations for symbol no. 6. The first alternative (a) is perhaps more likely to be correct than the second, and appears several times in this form on the Moabite Stone. The second alternative follows only partly the lines of the diagram, but the resemblance is obvious. It is possible that the original sign was chosen as in (a) but developed with use into (b) or was written thus by the individual carver who chiselled out the Moabite inscription.

We see symbol no. 8 in *Fig. 330,* which illustrates yet another variation of the diagram, the lines of 4-part division having been introduced. We find this variation used in one or two other letters/symbols —and later, in our study of the Nordic runes, we shall see that it was the principal geometric source of the ancient Scandinavian alphabet.

Fig. 330 (a) shows the symbol with one slanted transverse stroke between two verticals, while (b) shows a horizontal between the verticals. Symbol no. 8 in its original form actually has *two* slanted transverse strokes, and the diagram therefore fails somewhat to meet fully the shape of the original.

The same 4-part dividing lines are also seen in *Fig. 331* in which we see the final symbol in the alphabet.

There are three possibilities, (a), (b) or (c). (a) employs the same two vertical lines as used in the preceding illustration, but in fact the shape of the letter does not coincide exactly with the lines of the diagram, althought geometrically the choice could be defended.

In practice—on the Moabite Stone—the symbol often appears as in (b), i.e. shorter in height than the other symbols but the same width. It is not unreasonable to surmise that the letter began as (b) but developed to (a) on the grounds that it is unpractical to have one letter smaller than the others. (c) is another possibility, but admittedly not so convincing.

Broadly speaking we have been able to trace the origin of 19 out of 22 of the Phoenician letters without the slightest trouble. The remaining three give rise to doubt only to the extent that one has to choose between alternatives—no great de-

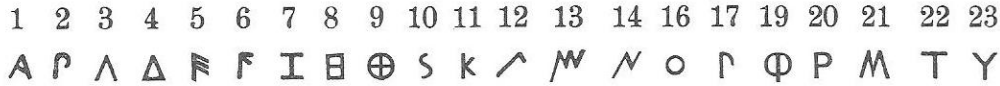


Fig. 332.

gree of imagination is required to associate the symbols with the geometric diagram.

Strictly statistically, approx. 86.5 % of the alphabet fit the diagram perfectly, while there is a little doubt as to 13.5 %. But it should furthermore be borne in mind that this was a handwritten script not (apparently) drawn on carefully prepared geometric diagrams. Despite this factor, we have achieved a very high degree of success.

The Phoenician alphabet and its variously formed letters were passed on to the Greeks. This is obvious when one compares the two scripts. Whether the legend is true that the alphabet arrived in Greece with a Phoenician prince by the name of Cadmas who was searching for his kidnapped sister I am afraid I cannot say! We are told that the sorrowing Cadmas stayed in Greece long enough to establish a township, and it was during this period that he passed on the secrets of the alphabet.

It may be the case that the alphabet arrived in Greece in this way, but at the same time we should recall that the Grecian Temple engaged in friendly relations with sister-temples throughout the cultural area of Central Europe and the Near East. Her priests would therefore no doubt have come in contact with the Phoenicians' "new" script.

Greek priests were perhaps told not only of the Phoenician alphabet but also of the background from which it was constructed. They then returned to Hellas and used their new knowledge and experience to produce their own rationalised alphabet.

The early Grecian alphabet was strongly and obviously inspired by Phoenician script, but not to the degree that one could call it a take-over of the latter. But naturally if the Greeks constructed the letters of their alphabet from the same source as the Phoenicians, it is only to be expected that points of recognition should creep in.

In *Fig. 332* we see a line reproduced from Arthur Mentz's book, and this he declares to have been the oldest known form of the Greek alphabet. The list of letters is numbered 1 to 23 but nos. 15 and 18 have inexplicably been omitted. Moreover, in relation to the Phoenician script the whole alphabet has been reversed, i.e. the Phoenician symbol no. 22 is no. 1 in the Greek alphabet, etc.

When we compare the two alphabets geometrically we find that only three letters are identical in each: nos. 9, 16 and 19. These contain the circle, a vertical stroke and/or the vertical cross.

Many of the other letters are inverted. This is explained by the fact that the Phoenician alphabet was written from right to left, the Greek from left to right.

In taking as their inspiration the letters of the Phoenicians, the Greeks may not have had access to the original diagram. They may have fitted the Phoenician symbols into a related though slightly differing diagram, or they may have used the same diagram but selected different lines from within it. Only with symbols that contained the circle was there neither doubt nor opportunity for alteration.

The following geometric analysis of the ancient Greek alphabet will be conducted in the same way as our examination of the Phoenician alphabet. The letters will be split into groups.

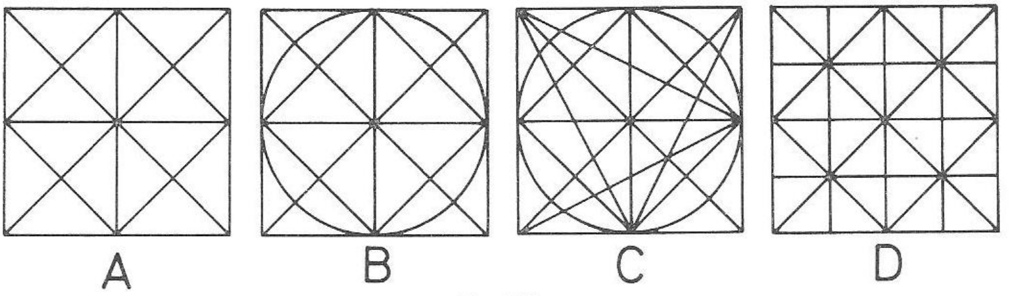


Fig. 333.

Group A will be placed in the diagram with the two squares and two crosses, *Fig. 333 A*.

Group B letters will include the use of the circle or parts of it, from *Fig. 333 B*.

Group C will include the above plus the acute-angled triangle, horizontally and vertically, *Fig. 333 C*.

In addition to the above groups, the Greek alphabet also made use of another geometric variation for one or two letters. This was the triangulation of the two above-mentioned squares. This departure (or addition) was also used by the Phoenicians for their letters nos. 8 and 21. We shall term these letters group D, and we see the diagram in *Fig. 333 D*.

In group A we have eight letter, nos. 7, 8, 11, 12, 22, 23, 6 and 13. See *Fig. 334*.

The letters and their geometric birthplace are plainly and certainly linked. The two fit together perfectly. Note particularly nos. 6 and 12.

In *Fig. 335* we find the five letters that concern the circle, i.e. nos. 9, 19, 16, 10 and 17.

The third group, C, is found in *Fig. 336* which makes use of the acute-angled triangle. These are letters 3, 4 and 1.

From this diagram the Phoenicians appeared to have employed both the vertical and the horizontal triangles, while here we see that the Greeks seemingly used only the vertical.

Only letter no. 1 is doubtful. There are two possibilities, the difference between them being the position or angle of the sloping stroke inside the letter. (a) follows the horizontal acute-angled triangle, while (b) follows the diagonal cross in the main square. Both alternatives have equal claims to being the source of no. 1.

The next group is D, which as we saw is in reality a continution of diagram 333 A. Here we have three symbols, nos. 5, 14 and 21, seen in *Fig. 337*, and we observe how no. 5 fits into the general diagram except that the middle of the three small strokes requires further subdivision of the diagram.

No. 14 shows a lean to the right, which is unusual when we examine the old Greek alphabet as a whole. The other letters are erect.

I believe the lean is the result of a poorly shaped script on the part of the writer, or of an idiosyncracy which may have crept in either at a particular period or in a particular temple area.

In *Fig. 338* we see two samples of Greek lettering. 338 b shows letter no. 14 with the mentioned lean towards the right (an extract from an ancient political manifesto?!), while 338 a illustrates no. 14 as an erect letter "N". This proves that both forms were in use. Two alternative theories concerning their origin are shown in Fig. 337.

In *Fig. 339* we see two alternatives for

FIG. 334

GROUP A.

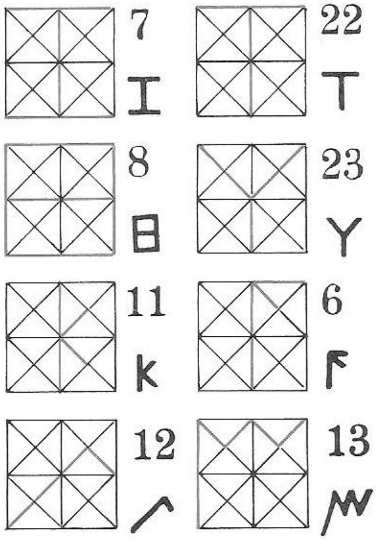


FIG. 335

GROUP B.

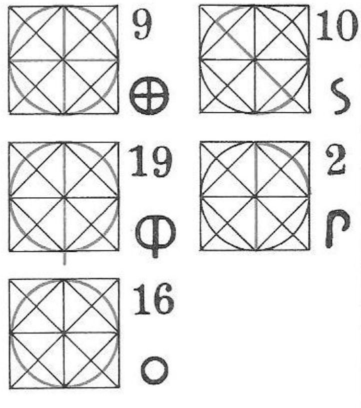


FIG. 336

GROUP C.

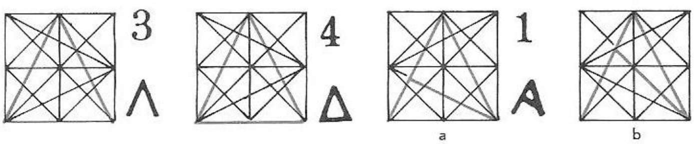


FIG. 337

GROUP D.

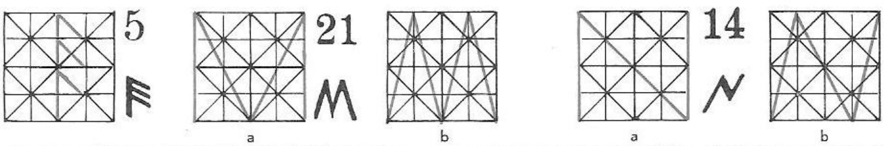


FIG. 339

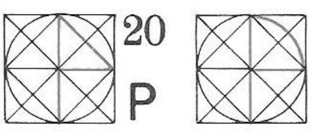
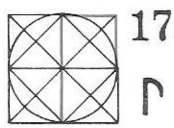


FIG. 341



Figs. 334-337, 339 and 341.

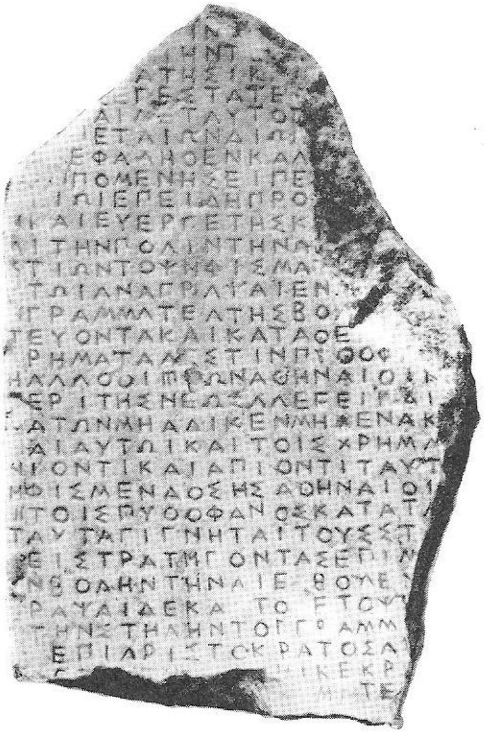


Fig. 338 a.

the formation of letter no. 20, but none matches exactly the pictured symbol.

One theory (a) follows the straight lines of the diagram, the other (b) follows the arc of the circle part of the way, but neither gives full explanation for the letter, which we of course know today as "P".

It is highly probable that the letter underwent a slight alteration as the alphabet developed, finally ending as illustrated in Mentz's book. It is not only probable, it is certain that at one time the letter resembled the alternative shown in Fig. 339 (a), i.e. with a sharp, angled loop instead of a rounded one.

We see a sample of this in *Fig. 340* which shows two inscriptions from the 6th century B.C. The sharply angled "P" is obvious.

The final letter in the list produced by Arthur Mentz is no. 2. We find its explanatory background shown in *Fig. 341.*

This letter closely resembles no. 20, and also shares a link with no. 17. It is not however really related to either of these since it follows neither the straight nor the rounded lines of the diagram.

As with no. 20 we should bear in mind the style in which the letter was reproduced. Perhaps this particular example was the work of an individualist.

Fig. 338 b.

Fig. 340.

In *Fig. 342* we see an inscription in marble dating from about 490 B.C. Several places in this inscription we find the letter as illustrated in diagram 341. In effect no. 2 ought to belong to group A, as should no. 20, since samples of inscriptions have proved that they were at some stage made up of straight lines.

Writing gained a rapid rise in popularity in Greece, bound up no doubt with the fact that paper production in Egypt and China eventually reached the stage where it could cope with the most pressing (Temple?) demands.

But development blossomed at such a rate that it became impossible to lay down hard and fast rules regarding the authorised formation of the individual letter symbols. They differed from temple to temple. This accounts for the remarkable fact that different areas of Greece had their own provincial adaptions.

Dr. Wilhelm Lerfeld, in his *Handbuch der Klassischen Altertums Wissenschaft, griechisch Epigrafik,* Munich, 1914, reproduces 93 different variations of the Old Greek alphabet in which the letters correspond roughly—but only roughly. There is still sufficient difference for Dr. Lerfeld (and the Greeks) to enumerate 93 individual *local alphabets.*

It was not until printing was developed that the letters were given a standard appearance and order was introduced into the chaotic situation.

It would be difficult to put a "date" on the invention of printing. We know that the German, Johann Gutenberg, invented in the early 15th century the system of movable type which has dominated to the present day (although photographic processes have been gaining considerable ground over the past decade). Gutenberg's method was to cast molten metal from existing model types—thus cutting down the need for so many finely etched printing plates.

We are aware, too, that the Italian brothers, Johan and Wendelin da Spira, set up a book-printing house in Venice in 1464 and made use of the Gutenberg system. This was a tremendous boost to the art of book-printing.

As Italy was at this time a dominating nation in the world of commerce, with considerable traffic through the principal north Italian cities of Florence, Genoa, Milan and Venice, the language of the Italians had become well-known and distributed throughout Europe. Italian script was of the so-called Latin variety, and this therefore gained a position of power in Europe.

But Latin lettering was not invented by the Romans and native Italians. No doubt with their stylishness the Italians improved the looks of the type, and were masters in cutting type-faces, but the actual formation and structure were a legacy from the Etruscan inhabitants of west central

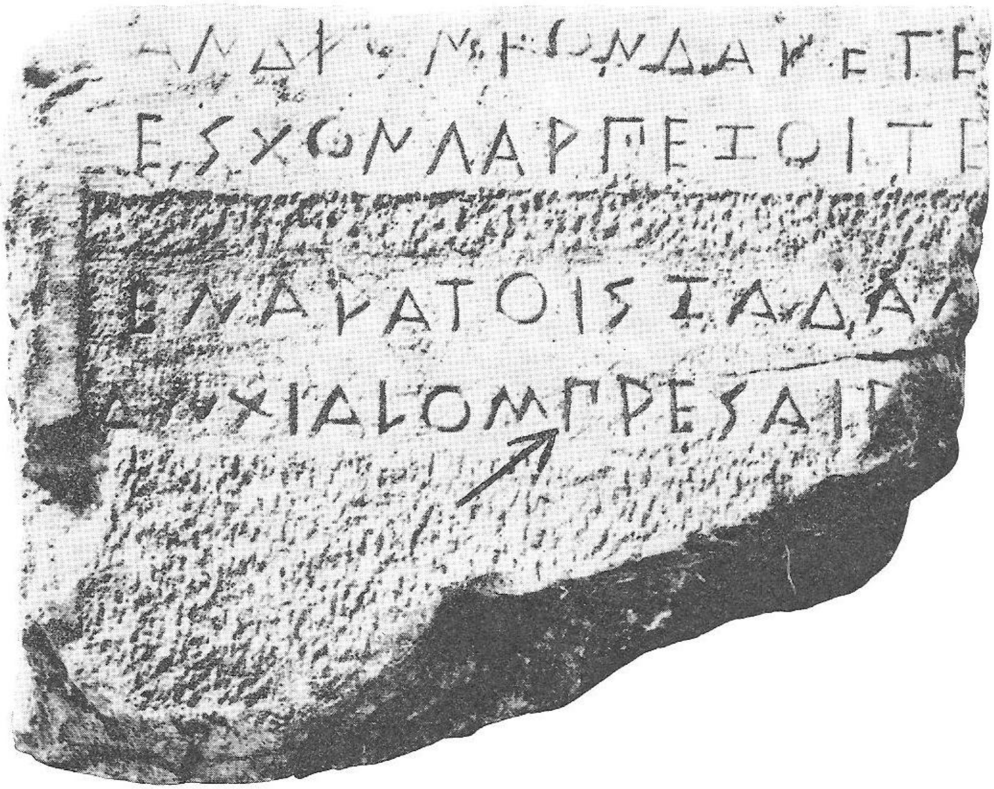


Fig. 342.

Italy, who in turn obtained them from the Greeks.

And the inventor of *italic* type, the Italian printer Aldus Manutius, who set up business in Venice in 1490, was a staunch Hellenist. With the assistance of Greek advisers and compositors hired to work in his Italian print-shop, Manutius had ancient Greek works reprinted in Latin. They included writings of Aristotle, Aristophanes, Sophocles, Herodotus, Plutarch, etc.

The point we should note here, in addition to the fact that Greece was ahead of Italy in this field, is that Greece was so advanced that her Italian neighbour actually had to borrow type-setters to work in a Venetian printing house.

This infers that Greece had an advanced process of printing even at that date (and earlier) which corresponded in some way to the European system. Printing moreover must have been an ancient, well-established trade in Greece for she would not otherwise have been able to afford to export her printing workers.

Printing processes have been widespread all over the world for thousands of years. For example, in her look into the interior of Tibet, *Parmi les mystiques et les magiciens du Thibet,* Alexandra David-Neel reports how the isolated Tibetans have operated their own printing system longer than history records, and the system is still in use. Or at any rate exists side by side with more modern processes.

Mrs. David-Neel describes a visit to the largest printing house in Tibet, situated

Α Β Γ Δ Є Ι Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ C Τ Υ Φ Χ Ψ Ω

Fig. 343.

in the town of Na-thang. She watched printing employees at work with *hand-carved wooden plates,* one for each page. The printing house is owned by the Temple and the work done by teams of specialist monks.

The wooden surfaces are coated with ink, a suitable sheet of paper placed on the plate, and an impression taken. The carved plates are passed first to monks who do the inking, then to others who take the proofs, and finally to the librarian.

This was perhaps one of the most fascinating parts of Mrs. David-Neel's story. The plates are then stored in a gigantic building, housing row upon row of shelves —all piled high with old printing plates. There is presumably a history of several thousand years in that library.

This would indicate that the Temple in Tibet operated a printing process (long before European and Greek brethren) for reproduction of sacred literature.

Fig. 344.

It is impossible to calculate or discover how long handwritten script existed in Greece before the advent of printing. The two presumably ran side by side until printing took over completely.

But to set up a printing technique demands that a stand be taken on the formation of lettering. The printer must decide how the symbols are to appear in print, and must select his ideal.

In *Fig. 343* we see Arthur Mentz's idea of the perfect script as he thinks it existed originally. There are few changes between this script and that which we find around 500 B.C.

In *Fig. 344* is an inscription from about 398 B.C. in which we find agreement with the ideal deduced by Mentz.

There is however one variation. Arthur Mentz shows letter no. 7 (the one we recognise as "H") with a slight lean to the right. The stone inscription on the other hand shows this letter perfectly vertical. The following analysis will therefore regard the letter as vertical.

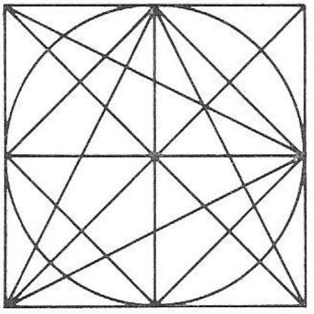
We have a line this time of 24 symbols, an increase of two.

As we did earlier, we divide the alphabet into groups. Here however we combine two groups, A and C. Thus we shall find that the main group of letters will be found in the diagram containing two squares, vertical and diagonal crosses, and the acute-angled triangle.

The completed diagram, containing the circle, is seen in *Fig. 345.*

In *Fig. 346* group A we see the 15 letters of the first group, i.e. nos. 1, 3, 4, 6, 7, 9, 10, 11, 13, 14, 16, 19, 20, 22 and 23.

All 15 follow closely the lines of the diagram and require no comment, except



C

Fig. 345.

to remind the reader of the decided lean of no. 7.

The next group, B, is seen in *Fig. 347* where we discover that the principal geometric shape required is the circle. The letters are nos. 15, 8, 5 and 18. Again, not the least doubt as to the letters' origin. They fit perfectly into the diagram.

We have succeeded in tracing 19 out of the 24, and all fit exactly.

Letter no. 17 (the "P") now appears on the scene with an attractively curved head instead of the sharp angle we saw earlier. We see three possible alternatives in *Fig. 348,* and recall that (a) was suggested as the most probable shape.

It is much the same with letter no. 2 (the "B"), two possible varieties of which we see in *Fig. 349*. The basic symbol was probably that seen in (a) and with usage developed later into the more recognisable "B" in diagram (b).

Letter no. 21, however, is one we have not met before. We see its most likely source of construction in *Fig. 350,* which would mean that it is a completely new arrival on the scene. It is not impossible however that it evolved out of letter 19 in Fig. 335.

No. 24, like the "P" and "B", has advanced somewhat from its original geometric structure. We see the two stages in *Fig. 351*.

Finally, we see letter no. 12 (the "M") in *Fig. 352* and note that it is the same as in Fig. 337.

The reason for departure from the strict lines of the diagram in the case of one or two of the letters may have been that the later versions were regarded more beautiful in form. In any event it involves only a minimum number of letters.

The strong resemblance borne by all three alphabets and their obvious geometric background can lead only to the conclusion that this diagram was in fact the origin of Phoenician and Greek lettering. Most letters fit exactly, and one or two here and there deviate minutely from the diagram's lines.

*

We next find the diagram in use in ancient Rome where its lines were used to construct Latin numerals. The system introduced by the Romans was simple and logical. They started with a couple of symbols (I and V). I = 1, II = 2, III = 3. Since V = 5, the Romans represented 4 as follows: they placed I *before* the V to indicate subtraction thus IV = 5 − 1 = 4. In the same way for the numeral 6 they placed I *after* V to indicate addition thus VI = 5 + 1 = 6. VII = 7, and VIII = 8.

So far so good. For the numeral 10 they selected the Greek letter X. And by the above system IX = 10 − 1 = 9.

For 50 they made a symbol L, for 100 a symbol C, for 500 symbol D, and for 1000 symbol M. A stroke above a numeral indicated that it should be multiplied by 1000, thus $\overline{\text{IX}}$ signified 9 × 1000 = 9000.

Compared with the ancient civilisations of Persia, Babylon, Egypt, Greece and others, the Roman empire was an infant in cultural development. But in the eyes of many parts of Europe (perhaps on account of proximity) Rome was a pioneer

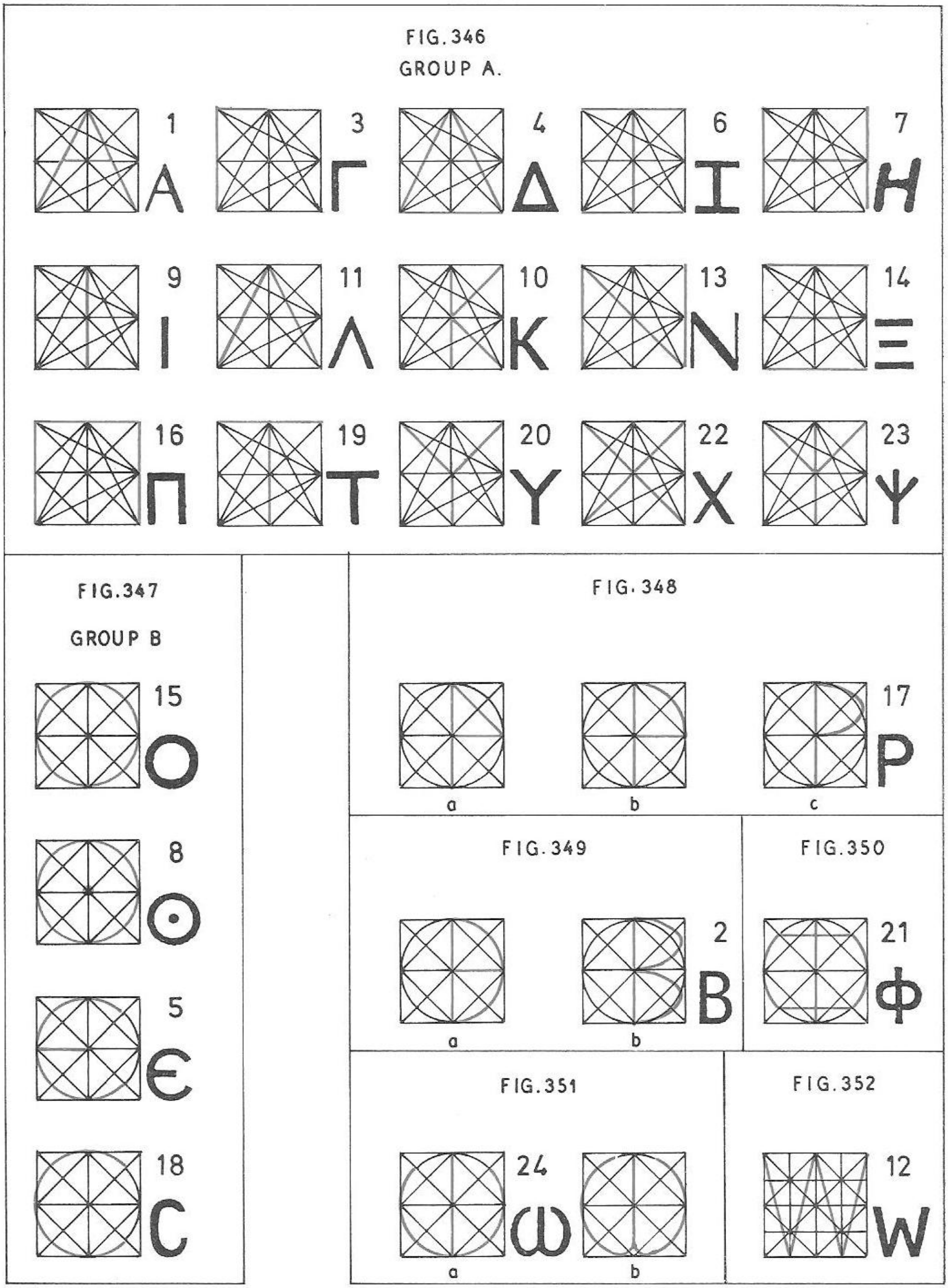


FIG. 346
GROUP A.
1 Α
3 Γ
4 Δ
6 Ι
7 Η
9 Ι
11 Λ
10 Κ
13 Ν
14 Ξ
16 Π
19 Τ
20 Υ
22 Χ
23 Ψ
FIG. 347
GROUP B
15 Ο
8 ⊙
5 Є
18 C
FIG. 348
a
b
17 Ρ
c
FIG. 349
a
2 Β
b
FIG. 350
21 Φ
FIG. 351
24 ω
a
b
FIG. 352
12 W

Figs. 346–352.

state where inspiration was there for the picking.

It is difficult to work out why the Romans chose this arrangement of symbols to represent their numerals. Perhaps they had their own numerical system before the influence of Greece spread to Italy.

Roman priests were familiar with ancient geometry and chose to design their lettering along the same lines as the old-established nations of the Near East had done. Their first need was a set of numerical symbols.

Four of the Roman symbols (I, X, C and M) already existed in the Greek alphabet, whereas the remaining three (V, L and D) were newcomers and unknown to the Greeks.

If two people (or groups of priests) were faced with exactly the same geometric diagram/system and were intent on constructing a set of numeral from the lines of the diagram, it is highly probable that some of the symbols in each camp would be identical. True, the more symbols required, the more complicated the diagram would become. But the vertical and diagonal crosses are two obvious sources of inspiration.

The Roman system of numerals possibly existed in its "present" form before they took over the alphabet from the Greeks. Had the Romans waited until the Greek alphabet arrived, they would have been able to select all seven numerals from the new alphabet, which contained 24 letters. The evidence suggests that the Romans had developed their numerals before Greek and Etruscan influence arrived with a new system of lettering.

Roman temples were acquaint with geometry. Mystery schools were constituted in southern Italy after Pythagoreans were banished from Greece. It is very possible that Pythagorean refugees brought with them a version of Greek lettering and numbers, and equally possible that travelling priests (from and to Italy) introduced the geometric diagram in Rome from which Greek and Phoenician scripts had been developed.

Whatever the precise order of events, Roman numerals (and language) were used throughout Italy and Europe for centuries, thanks to the lasting influence of Rome's empire-building legions and to religion. In many places and spheres Roman numerals lived side by side with the incoming Arabic numerals for centuries. It was not until about the 10th century A.D. that the latter finally broke through the barrier of established tradition into general and popular usage.

We see the Roman numerals in *Fig. 353* where their origin in the geometric diagram is obvious.

The persistence of Roman numerals is surprising. Even today, when Hindu-Arabic numerals dominate everywhere, the Latin I, II, III, etcetera, still survive, in books, on clocks, etc.

Roman numerals are used only to indicate complete numbers today, they are no longer employed for calculation purposes. They have long given way to the more rational Arabic symbols. (Adding three relatively simple Roman numerals together is such a complicated process that one wonders whether the method can ever have been used successfully!)

When the Romans adopted the art of writing (whether from the Greeks or Etruscans) it was presumably as handwriting and not in a printed form, and remained a system of handwriting until printing was introduced in northern Italy in the early 15th century.

Although printing was late in arriving in Italy, it is feasible that a technique of printing had existed elsewhere along the Mediterranean coast or in Europe somewhat resembling the method we know today (with type on paper as opposed to

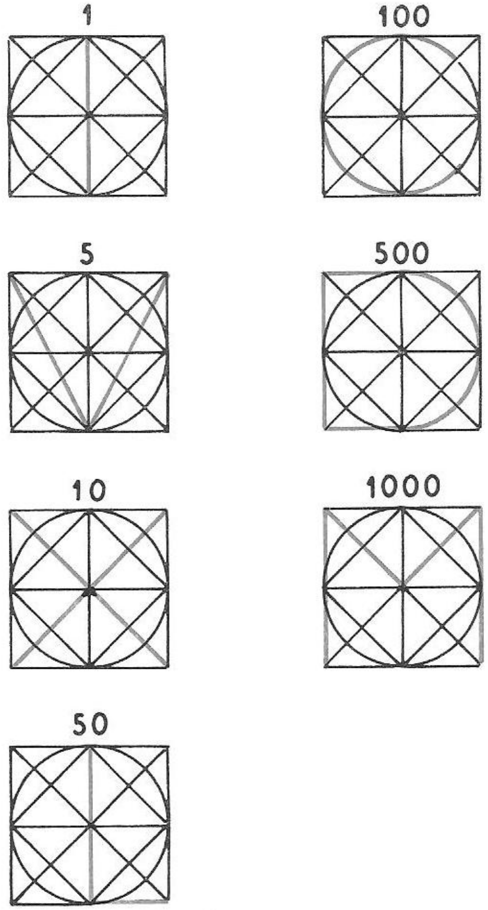


Fig. 353.

impressions in clay or other plastic materials).

Such a technique would not necessarily be transferred from one country to another, nor need it be in wider use than handwriting. Only a few centuries back, around 1400, a printing technique was developed but it by no means ousted handwriting. The latter was more prolific than printed matter for a long time afterwards—partly because it is not every thought, fact or report that requires to be printed and reproduced in numbers. (Only now, with the wide distribution of typewriters and all models of printing equipment, is handwriting beginning to suffer. Think back 20—30 years to the beautifully formed handwriting of those days!)

The Romans received a system of writing from outside sources about the year 500 B.C. But the existence of a writing system does not necessarily mean that a nation's advancement takes place overnight. Just how slowly Europe developed in this respect was emphasised by Keith Gordon Irvine in his book, *The Romance of Writing.*

In the year 1200 (A.D.), he wrote, even the richest monasteries in western Europe had libraries of less than 150 books, many of which could more aptly be termed booklets. And 200 years later the position had scarcely altered. This, said Irvine, was due not so much to the fact that people had no desire to write, nor that they lacked important or interesting ideas. It was simply that they lacked sufficient parchment to write on . . . .!

From the date the Romans acquired the art of writing to the time printing was really well and truly established is nearly 2000 years. And it is significant to observe that development did not get fully under way until Rome (or Italy) became a seafaring nation and able to acquire as much parchment and paper as she desired. Previously the nations surrounding Egypt had requisitioned all available supplies of writing materials, and very little had reached the northern shores of the Mediterranean.

Development proceeded and the symbols the Romans adopted from the Etruscans and Greeks were amended to suit Latin taste. New symbols were added to account for variations in pronunciation, and the latinised alphabet as we know it today gradually took shape.

This alphabet, too, fitted into the geometric background of the Greek and Phoenician symbols. The Roman Temple certainly was familiar with ancient geo-

FIG.354

GROUP A.

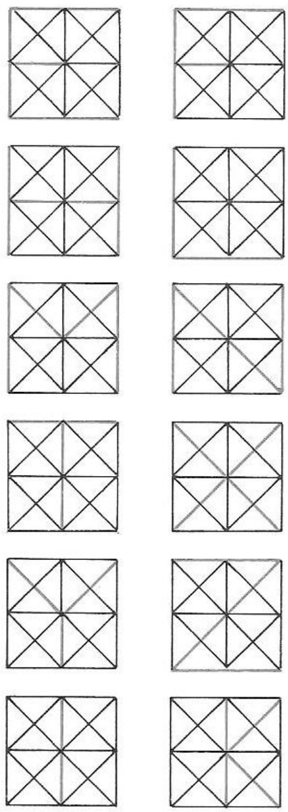


FIG.355

GROUP B.

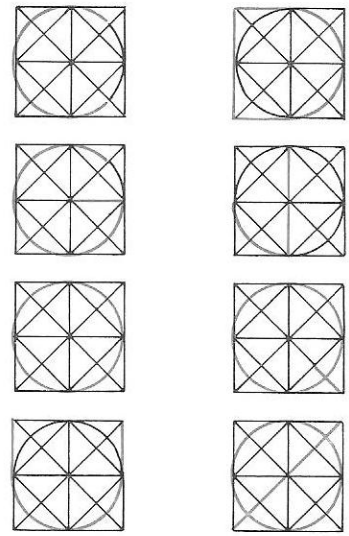


FIG.356

GROUP C.

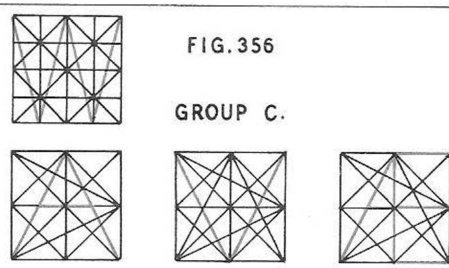
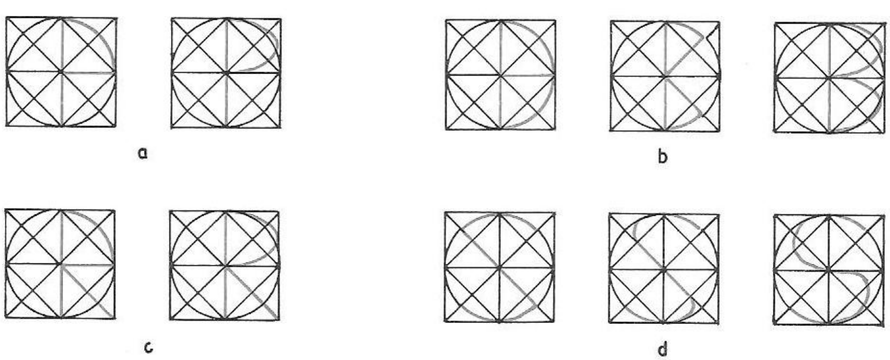


FIG.357



Figs. 354–357.

metric principles, and acquaint with the origin of the symbols it had adopted.

Let us examine the Latin alphabet as we know it in Europe today.

In *Fig. 354* we have 12 symbols, E, F, H, L, M, N, T, X, Y, Z, I and K.

For these the Romans applied only the two squares, and two crosses of the diagram. There is no doubt about the origin of these 12.

In *Fig. 355* we find eight letters which make use of the circle within the basic square: C, D, G, J, O, Q, U and the Danish Ø. Again the diagram is followed faithfully.

Fig. 356 shows four symbols which make use of the acute-angled triangle: A, V, W and the Danish Æ.

This brings us to 24. The English alphabet has 26 letters, the Danish an extra two. We shall now look at the remaining four letters: B, P, R and S.

In their case there appears to have been either a degeneration or development of the original symbol since the sharpened corners have been worn off with time.

We see the development in *Fig. 357* and can trace the change from angled shapes to rounded ones.

In fact, therefore, only four out of the range of 26 (or 28) Latin letters have undergone any alterations from the originial symbol .... an infinitely small degree of deterioration when one considers that the letters were mainly handwritten for a period of nearly 2000 years.

*

Latin lettering gradually spread throughout most of Europe and, among other places, in Germany the symbols were subjected to a process of ornamentation with squiggles, banners and other flourishes which collectively has been associated with central Europe. Such Gothic script came very close to being unrecognisable except by trained experts, and was in danger of being labelled the result of imagination with no basis whatever in any established system. But this flowery script, too, had a limited lifetime. The mass of people moved back to the simplicity of the original symbols.

On its slow journey across Europe Latin script encountered another form of writing that resembled the written language of the Romans to a certain degree—without being a copy in any way. This script was the runic alphabet of northern Europe, practised by the inhabitants of the Teutonic-Scandinavian region.

The runic alphabet (or runes, as the individual characters are called) flourished in Scandinavia, and parts of Scotland and Germany about two thousand years ago. It gradually gave way to Roman lettering and the spread of Christian writing, but was still being used occasionally (e.g. isolated parts of Sweden) as late as the 18th century.

The two alphabets, Roman and runic, had certain points in common: individual symbols represented sounds which could be placed together to make words; the number of letters was similar (26 Roman, 24 runic), etc. But the appearance, and order of phonetic values differed.

Whereas the Roman alphabet began with the sounds and symbols A, B, C ...., the runic alphabet started with symbols representing the sounds F, U, Þ .... The phonetic value of this latter symbol does not exist in the Latinised alphabet. Although entirely gone elsewhere, the sign ð still exists in the Icelandic language. It lives on in harmony with Roman lettering but cannot be dispensed with since there is no Roman symbol to cover exactly the sound in question.

In *Fig. 358* we see the complete runic alphabet with the approximate Roman equivalent. The sample has been photographed from *Norges Indskriftor med de ældre Runor* by Sophus Bugge, Christiania

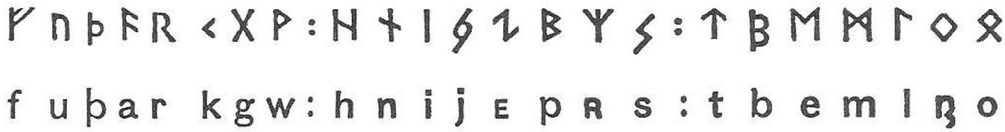


Fig. 358.

1905—1913, a historical account of runes, their possible origin and interpretation.

Bugge devoted an immense amount of time and study to the research of runic history, and consequently I regard his lettering as the most correct in this field.

In addition to the main area of use of the runic alphabet, Bugge states that runes have also been discovered in Poland and deep into the Rhine and Danube valleys, as well as on Iceland. The most probable centre of runic origin was Scandinavia.

Bugge's theory assumes that runes originated around 600—800 A.D. with an alphabet of 24 characters. About 100 years later another version of runes developed, with 16 characters, based on the existing alphabet.

We see the second alphabet in *Fig. 359*.

Thus we find that a system of writing originated or existed in northern Europe as a series of phonetic symbols, i.e. the same type of system employed by late Greeks and early Romans. The runes apparently came into being about 1300 years later than the period during which the Greeks received their alphabet from the Phoenicians.

Call to mind the development that preceded the birth of an alphabet in the cultural civilisations of the Near East and Mediterranean: there were the 600 hieroglyphs of the Egyptians, then came the approx. 350 characters of the cuneiform language, later reduced by the Persians to 100. Along came the Phoenician traders to further rationalise writing to a system of 22 symbols with clear-cut phonetic meaning. These were passed on to the Greeks, who gave the message to the Etruscans, who in turn passed the material to the Romans. The latter are honoured by the world as inventors of Roman letters—when in fact it took thousands of years of evolution, development, experience and experiment to arrive at the Latin script.

And then it is suggested that a new alphabet was invented in the North *with no apparent background, no proof of prior development, out of thin air.*

Was is possible?

The Nordic people of those early days were warriors. They were an up-and-coming civilisation. Their reputation was not based on philosophy. They were a material, tangible people—and surviving material portrays them to be more a nation of barbarous warmongers than meditating wise men. Although there had naturally been a sprinkling of philosophers among them, they were ruled by brute strength and savagery.

How could people of this calibre, who loved the whistle of the sword more dearly than the scratch of a pen, how could

Fig. 359.

they create a perfect alphabet without outside help; without assistance from some party familiar with the business of writing?

How could these warriors, few of whom could read or write, warriors without writing materials, create a system of writing which in its development and sophistication was equal to the experience of much more ancient civilisations?

It is scarcely possible to believe that they did. A system of writing cannot be drummed up out of the blue. It takes (and needs) time to develop. I believe there is only one explanation to the existence of the runic alphabet in northern Europe: it was learned from an outside source who already possessed a system of writing. The Scandinavians either adopted a complete system, or developed a system that had already been started.

How then, if the runic alphabet was adapted from or based on an existing system, can it differ so completely in appearance from other systems that no link can be traced? So completely that the world has attributed runes solely to the Scandinavians?

I think the explanation may be something on these lines.

We have seen throughout the whole development of planning, building, art, language, etc., that the Temple was the centre of all knowledge. We saw several alphabets evolve on this basis, and how Temple leaders took the initiative in most civilisations.

But we also recall the strict sense of secrecy that surrounded everything developed by the Temple and shared by its initiates.

Let us imagine for a moment that one or more Temple brethren from a part of the world where writing was already established arrived in the North, i.e. northern Europe, presumably Scandinavia. He may either have come of his own free will or have been captured during a Scandinavian campaign and dragged off to the wilds of the North.

He discovered that (apart from lacking manners!) his hosts had no knowledge of writing. They had no means of written communication. He had a strong desire to share his knowledge in this direction. At this point the theory has two possible developments.

The Teutonic language perhaps gave him no chance to transfer directly the phonetic symbols of his native tongue; the Nordic coarseness was guttural in pronunciation, much harder than his own language.

Or perhaps he had to take into account the oath of secrecy sworn at his Temple initiation.

He may have been able to adapt his own language's symbols to the tongue of his hosts—but to instruct others in the make-up of the symbols would have required to reveal the secret diagram in order to illustrate how the symbols could be remembered.

Since he had learned the use of the diagram in the Temple and at the same time swore never to reveal its secrets to outsiders, he could not explain the original symbol to his new pupils.

Thus it may be a combination of these possibilities that caused him to produce a complete set of new symbols: he would in this way be able to construct new symbols to suit the Teutonic tongue, and at the same time be able to comply with his oath of secrecy——for he would draw a different symbol from that used in his homeland.

If this theory is correct, then we should discover that the runic alphabet was constructed according to a geometric diagram —as were the earlier languages examined. And this apparently was the case.

In *Fig. 360* we see two geometric diagrams. *A* is the one on which the Phoeni-

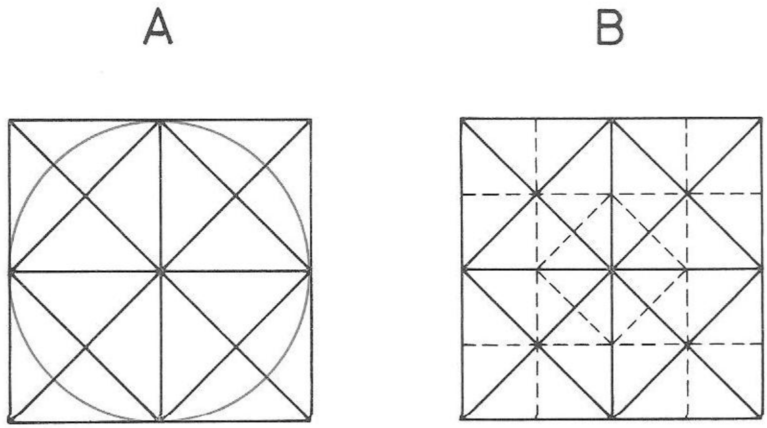


Fig. 360.

cian, Greek and Roman alphabets selected individual letters, and which is identifiable as symbol "E" (see Chapter Five). *B* is identified as symbol "G". This diagram was also used for one or two Phoenician and Greek symbols.

We see in *Fig. 361* the 24 symbols of the runic alphabet placed in the ancient geometric diagram from which they originated. The symbols are the same as those illustrated in Fig. 358.

We observe, as previously, an astounding similarity between the lines of the diagram and the shape of the individual symbols. At the same time we ought to bear in mind that the symbols are a reconstruction produced by Sophus Bugge—yet no compensation is required in order to fit the symbols into the Fig.

We see that rune no. 3 is not exactly the same as the original geometric shape, being rounded instead of angled. This may be because it was amended slightly to distinguish it from rune no. 8 which resembles it and which still retains its angled "pennant".

No. 18 has also been rounded off, so has no. 5.

But these variations do not disturb the general picture, and are in fact natural developments.

Many of the runes have remarkable shapes, which strongly supports the theory that this diagram was the basis of their formation. Otherwise their strange shapes would never have fitted the geometric pattern.

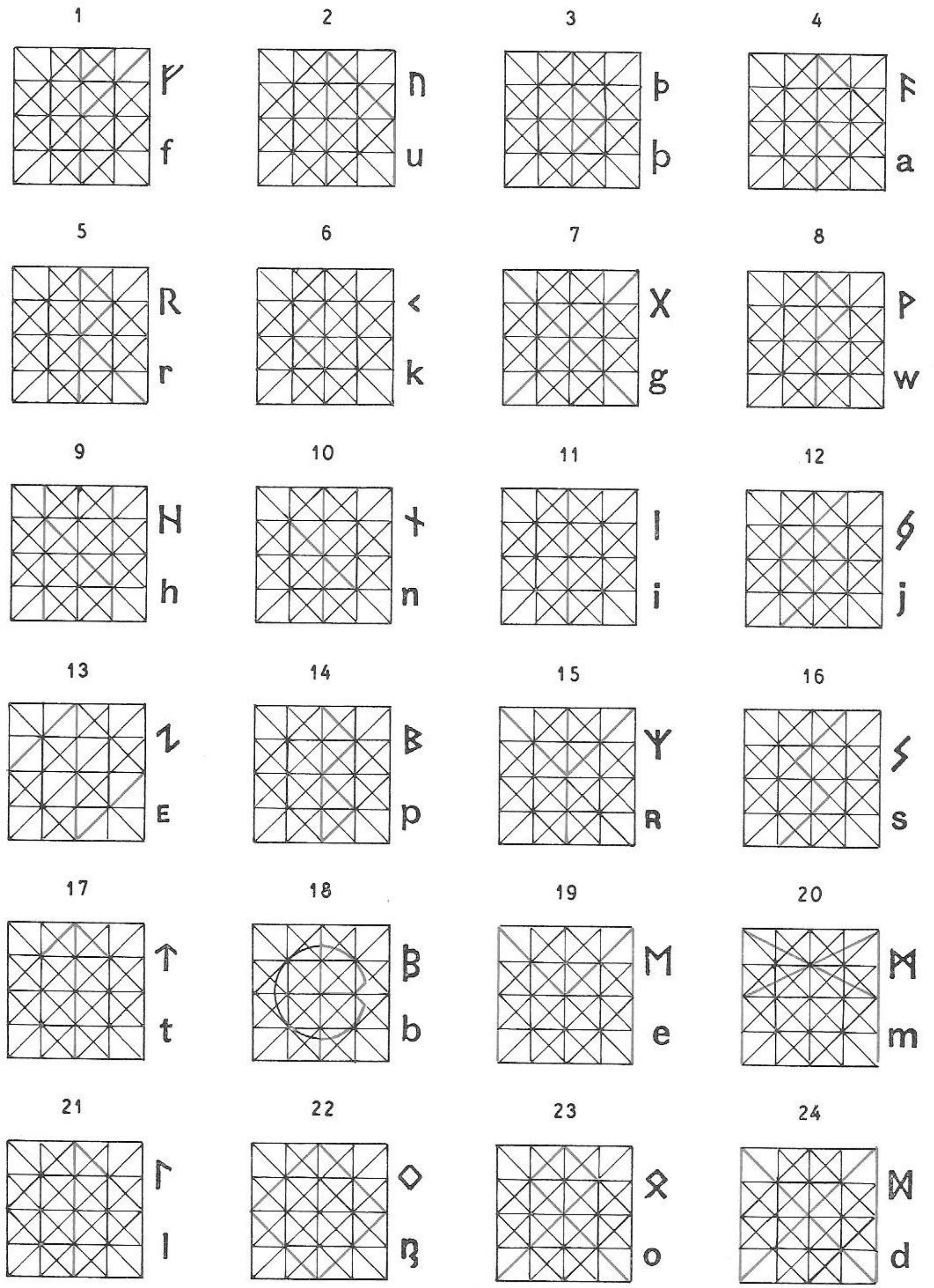
The diagram is based on geometric knowledge, and was used by Eastern civilisations to construct phonetic symbols to allow them to write. It is thus more than probable that the people of northern Europe who are responsible for the runic alphabet received their inspiration from outside.

The later version of the alphabet, with 16 symbols, is seen analysed in *Fig. 362*. These also fit the lines of the diagram.

This latter alphabet of 16 characters is eight fewer than the preceding, but the 16 are not identical with the earlier set of symbols. There are both variations on the old letters and new symbols altogether. But adaptations and newcomers all meet the requirements of the diagram.

This was the same development encountered in Greece where nearly 100 local alphabets evolved from just such variations, amendments and local individualism.

Let us compare the two runic alphabets. We shall term the first (24-letter)


1 ᚠ f
2 ᚢ u
3 þ þ
4 ᚨ a
5 ᚱ r
6 ᚲ k
7 ᚷ g
8 ᚹ w
9 ᚺ h
10 ᚾ n
11 ᛁ i
12 ᛃ j
13 ᛇ E
14 ᛈ p
15 ᛉ R
16 ᛊ s
17 ᛏ t
18 ᛒ b
19 ᛖ e
20 ᛗ m
21 ᛚ l
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23 ᛟ o
24 ᛞ d

Fig. 361.

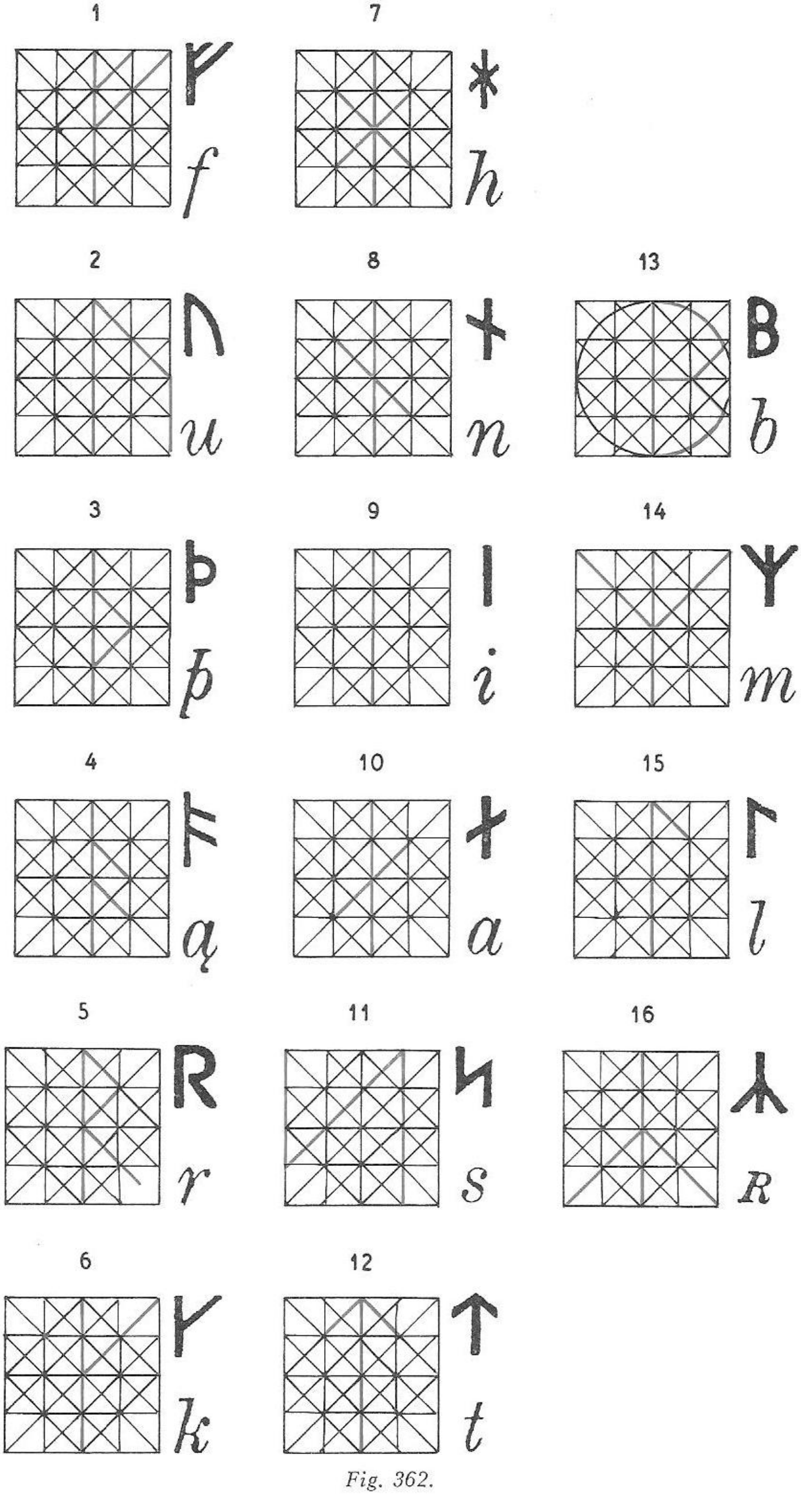


Fig. 362.

Fig. 363.

version "A", and the later version "B". We see that runes 1, 2 and 3 in both A and B are identical. A-4 and B-4 are almost identical except that the upper stroke in the latter has been lowered slightly. No. 5 in both alphabets are the same. Symbols A-6 and B-6 bear no resemblance to each other, the latter is a new letter but resembles no. 1.

A-7 has a distinct relationship to B-7, yet the difference is sufficient to allow us to record a new letter.

We omit A-8 and A-9 since these do not exist in B, and note A-10 and B-8 are identical, A-11 and B-9 are identical, while A-12 and A-13 have been dropped from B. A-14 and B-13 are the same, which would indicate that the phonetic value has altered from p to b.

B-10 and B-11 are also new symbols, not found in A. A-15 and B-14 are the same, but here again there has been a change of phonetic value. A-16 has been dropped from the later alphabet. A-17 and B-12 are identical. A-18, A-19 and A-20 are not to be found in B. A-21 and B-15 are identical. A-22, A-23 and A-24 are not included in B. And B-16 is a new symbol.

Thus of the 16 symbols in the new alphabet only 10 are the same as the old symbols, while the remaining six have varying degrees of resemblance to the old alphabet.

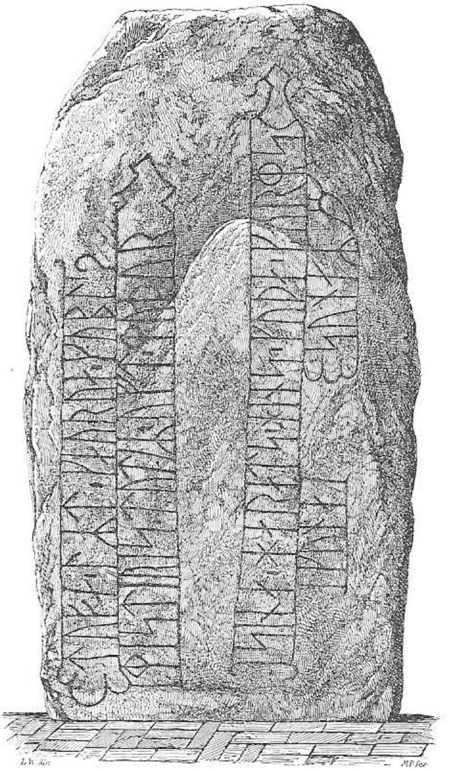


Fig. 364.

Fig. 365.

This makes a total of 30 runic symbols, all of which were apparently constructed on the basis of the geometric diagram.

It is difficult to stipulate why the number of letters should be reduced from 24 to 16. Perhaps 24 symbols were too many for the warring Vikings. They needed a simpler version. In rearranging the alphabet, the cultural leaders discovered that certain sounds were poorly represented in writing. Hence a reassessment of existing symbols, a general tidying-up, and the introduction of six new characters for the sounds that were missing.

It is not the purpose of this book to discover why the Vikings should alter their written language all of a sudden. We are intent here purely upon illustrating that, in company with other early scripts, the runic alphabet was based on a solid philosophical foundation: the diagrams of ancient geometry.

It may be a coincidence that the new alphabet contained 16 symbols, and the diagram contained 16 small squares. On the other hand, it may not . . . .

We have several fine examples of runes in the next few illustrations. *Fig. 363* shows the great Jelling Stone, dating from about 940 A.D. In *Fig. 364* we find the Sønder Vissing Stone, discovered in the Aarhus district of Jutland. Both these illustrations were borrowed from Ludvig F. A. Wimmer's book *De Danske Runemindesmærker,* 1895.

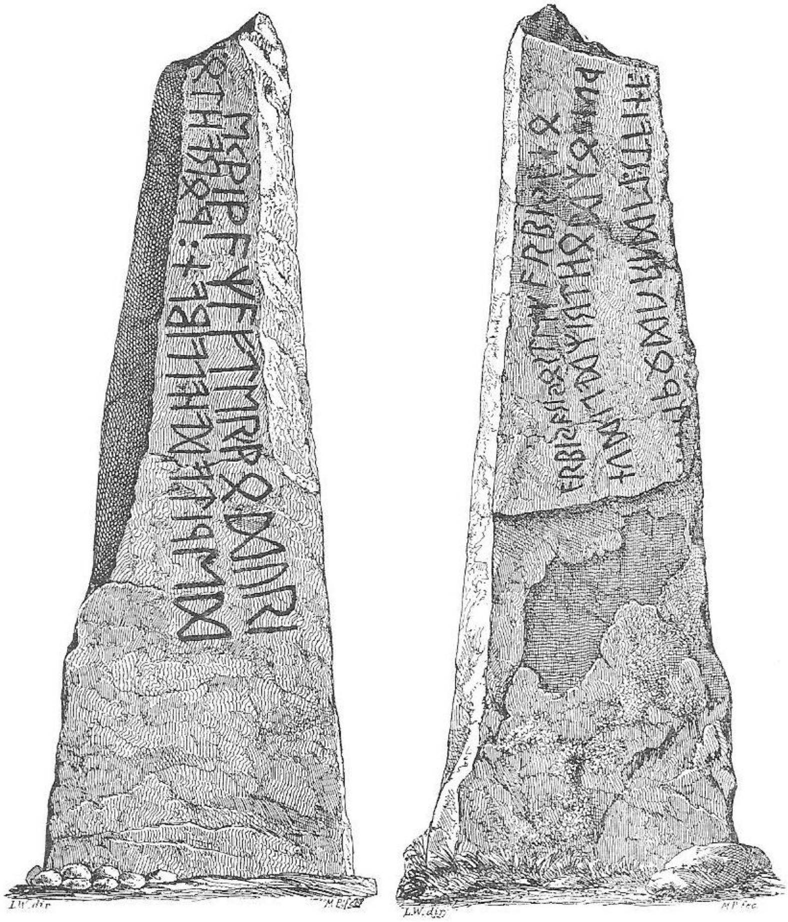


Fig. 366.

The two remaining illustrations were also taken from the same book but stem not from Denmark but from Sweden and Norway respectively.

Fig. 365 is a picture of the Åsum Stone from Skaane in southern Sweden, and it illustrates how in this case the text should be read from the top downwards, then continuing from the bottom upwards—precisely as a farmer ploughs his fields.

The Tune Stone in *Fig. 366* is from the Smaalenenes district of Norway.

The runic alphabet did not enjoy the same growth (or conditions for growth) as the script of the Romans. It was placed in the possession of a people who were not sufficiently developed to practise and hone this, one of culture's finest tools. Nor were the Scandinavians concerned with examining the possibilities of writing.

They were more often on the warpath than not, and trade and commerce did not attract them. If they fancied a herd of cattle, a group of blonde damsels, (not to forget an extensive area of England!), they did not trade—they took. The sword in their case was superior to the pen.

Nevertheless runes survived long enough to spread over a wide area. Samples have been found in Denmark, Norway, Sweden

and also, as mentioned earlier, Germany and north Britain.

But although samples have come to light throughout this area, their number has been few: less than a couple of hundred.

The gods of the marauding Vikings were principally Odin and Thor, and the warriors supported a priesthood to look after the interests of and appease the deities.

The priests had of course assembled, partly through their own labour and experiment, partly from inheritance, a certain ability and knowledge over and above the art of daily living. They were familiar with calender observations, a primitive system of mathematics, and other similar philosophic accomplishments.

It was not necessarily from the mysterious lands of southern climes that the Nordic priests learned and practised the principle of secrecy; the concealment to one's advantage of knowledge of this type is just as much an instinct as a conscious ability.

There were no mystery religious groups in the North resembling those of the ancient cultural nations of the Near and Far East, apart perhaps from the priesthood. There was no organisation or society with centuries of tradition behind it to compare with, for example, the Egyptian or Babylonian Temples.

In relation to the old-established nations of the South and East, the north of Europe was culturally at a decidedly primitive stage, little above the barbarian.

A select handful in the North were aware that southern colleagues were part of a development which was centuries ahead of northern Europe.

When runes were first brought into being (possibly in the manner described earlier in the chapter) few in Scandinavia were familiar with the material for which the alphabet was intended: papyrus or parchment. It is thus probable that the script was written mainly in wood, carved with a knife edge, or chipped into stone.

It is equally probable that a minority of priests wrote the alphabet on the materials for which it was created, but if this was indeed the case, the moist climate of the North has since destroyed all traces of paper, parchment or papyrus from that era. At any rate no runes have been discovered in this form.

*

Although the presentation of the runic alphabet to the warring Vikings was like tossing pearls before a herd of swine (some might say pigs!), the gift of writing to the Romans was not accepted in the same way. The Romans fully appreciated the value and use of the cultural tool.

The Empire had previously developed a system of numbers, and had built up a comprehensive trade with nations outside and inside the Empire area—hence the widespread use of the Latin language and script, more than was the case with any language earlier in history.

From whom did the Romans get their script? There are a number of possible alternatives, two being the most likely.

Either the inspiration came direct from Greece, or—which I believe to be more probable—the Latin script was based on the rational system of writing operated in northern Italy by the Etruscans.

Who were the Etruscans? Again theories have been put forward by the score, without any one proving conclusive.

At the time when the Latin alphabet arose, the Etruscans were apparently settled in Italy. They had arrived from the outside and conquered large parts of the country between Florence and Rome.

There is general agreement that the natives of the area of Italy known at that time as Etruria were seafaring people, who had arrived on the shores of Italy

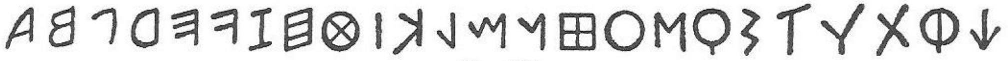


Fig. 367.

by boat. The most popular and commonly held theory is that they stemmed originally from Asia Minor, but their stepping-off point cannot be traced, nor can the reason for their emigration. Why should an apparently advanced people uproot and sail bag and baggage to Italy? The question is so far unanswered.

They mastered both warfare and peace. As well as being a nation of conquerors, they were adept at the production of items of art—and writing.

The script which the Etruscans introduced to Italy was neither Phoenician nor Greek, but a collection of 24 symbols unmistakeably of the same family as the other two languages.

The family link derived from the fact that the individual letters were planned and executed from the same geometric symbol employed by the Greeks and Phoenicians.

We see the Etruscan alphabet in *Fig. 367,* and one notices immediately the vague likeness to, for example, Old Greek. But if the individual letters are examined closely, it is discovered that none of those in the Etruscan alphabet is in fact identical to any in the other (Greek and Phoenician) alphabets.

The geometric diagram used by the Etruscans in the construction of their alphabet was the square, the vertical and diagonal crosses, the inner circle and the 4×4 lines of division. The Etruscans omitted the acute-angled triangle from their diagram: a factor also dropped by the planners of runes.

We see the geometric symbol in *Fig. 368,* and in *Fig. 369* we find the complete alphabet set in its geometric background. I thought it unnecessary here to group the letters according to structure.

As in the case of the alphabets examined earlier, the individual letters have been numbered for identification purposes, and only two of the 24 have required an addition to the diagram: nos. 18 and 23. For these we have entered the circle's half-size version as the basic form. Apart from these two, however, the other symbols follow unswervingly the lines of the standard diagram.

Thus we see that, in the same way as the Phoenicians and Greeks, the Etruscan cultural and/or religious leaders produced a script containing 24 symbols which differed from other existing scripts.

In the opinion of the author this illustrates that the Etruscan people were an independent group, uninfluenced by the Greek cultural life that affected the affairs of so many other contemporary civilisations. They were in a position to enjoy the same traditional knowledge as Greece, and to drink at the same cultural spring, but in choice of alphabet had selected their own form.

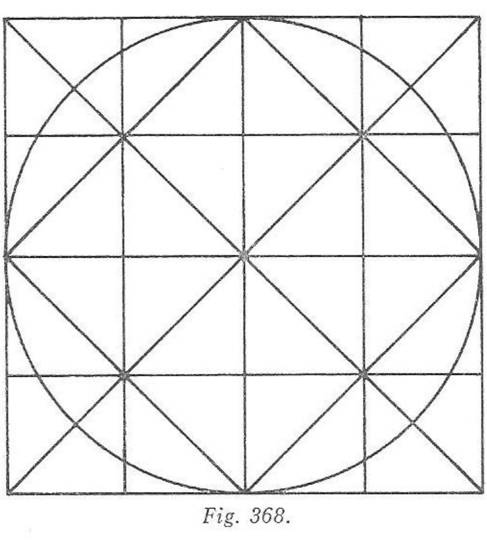


Fig. 368.

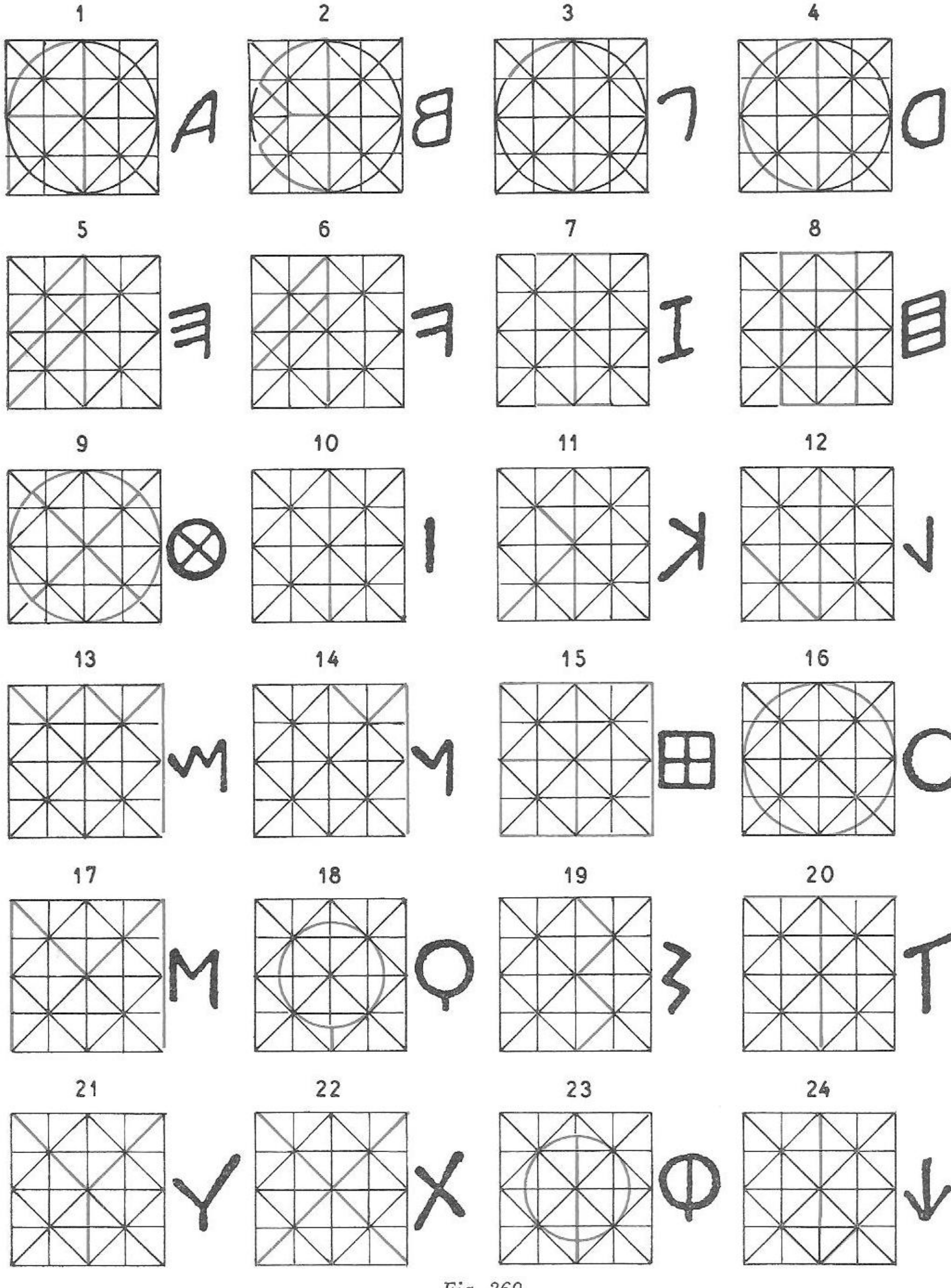


Fig. 369.

Apart from the alphabets and cultures examined in this chapter, a number of other civilisations have had a crucial effect on the written languages and culture of the present day. They were the giants, the supreme powers, the Sixes and the Sevens, the NATOs of the ancient world.

But it would be much too lengthy an epic for this book to study each in turn; we have space and time only for a representative cross-section. But I think I have shown the constructive path taken by the alphabet most used by nations of the Western world: the Latin script.

In the course of my research I have naturally subjected other written languages to the ancient geometric test, and I may report that the great majority of them fit the picture when one is able to track down a sufficiently old version of the script.

My theory is therefore that the Temple (in the respective civilisations) was the kiln in which alphabets and written languages originally formed, developed and hardened. It was from the Temple that new scripts emerged, and additions were made to extant alphabets.

The Temple saw to it that the script was kept up to standard, and that the letters closely resembled their geometric ghosts. And the local temples decided whether the people should write and read from left to right or vice versa, or in the case of the runes a combination of both directions.

The theory fits, I think, historical events as they have been recorded. On the subject David Diringer wrote in his book, *Writing,* that it was true both of the Slav alphabet and the Arabic alphabet that they followed religion.

Russians, White Russians, Ukranians, Bulgars and Serbs accepted the Cyrillic alphabet together with the Greek Orthodox Church. The Roman Catholic religion brought the use of the Roman alphabet to the Slovenes, Croats, Czechs, Slovaks, Poles, Wends and Lusatians.

Thus we find Diringer's information matching my theory: that the appearance of a complete alphabet hand-in-hand with religion is not something that happens gradually. It is introduced as a finished product which tolerates no personal amendment if it is to survive.

CHAPTER NINETEEN

Trelleborg — a Viking Stronghold

WE HAVE become familiar with the application of ancient geometry, and have seen that a system of this type could be fitted into many spheres of society.

The order in which geometry penetrated society was probably, first, that the various Temples procured its secrets on a strictly philosophical and geometric basis, and secondly, that it gradually filtered down "through the ranks" from temple to temple as a practical tool to be used in design and planning. Eventually it was spread throughout a wide area, the bounds of which are almost impossible to set.

If one specific geographical area is selected and a search mounted for signs of ancient geometric planning, it may be next to impossible in most cases to pinpoint the order of application of the geometric symbols.

As far as buildings are concerned we have various tests, dating methods, and styles which can be applied in an attempt to discover their age. When it is ascertained that they were built according to the rules of dimension detailed in earlier chapters of this book, then we can state with a fair amount of certainty the date at which ancient geometry was used in the particular building. But this reveals nothing regarding other buildings, temples, memorials, etc., which may not yet have been excavated from oblivion or which may have been completely destroyed. Nor does the use of geometry in the plan of a particular building reveal anything about geometry's employment in other spheres of culture.

In the case of my own country, Denmark, the position is somewhat different from that in the ancient cultural societies. We are able roughly to trace the order of use of ancient geometry as its practical aspect was so late in arriving in the North that it is possible to date its appearance reasonably accurately in at least two spheres.

One is language. We saw in the preceding chapter how the diagrams of ancient geometry formed the source of origin for the runic alphabet around 800 A.D. This was several thousand years after the same (or similar) diagrams were used to construct the ancient Phoenician script.

The other stage on which geometry performed was a familiar one: the art of planning in the building industry. One of Denmark's best-known works of excavation is the old Viking fortress, Trelleborg, at the west side of the island of Zealand, near the town of Slagelse. The fortress lies near the Great Belt sea passage, within easy sailing distance of the Kattegat and the Baltic.

As we shall presently see, the whole of this immaculate fortress structure fits into the symbols of ancient geometry—but not any old symbol. The diagram we shall

examine, and which we shall discover formed the basis of planning for Trelleborg, is no less than *the same geometric symbol used to construct the runic alphabet,* sub-divided to a finer degree.

Trelleborg dates from around 1000 A.D., and we see therefore an interval of 200 years between the system's application as the basis of a writing alphabet and its use in building.

The historical background to Trelleborg is vague. There are many theories regarding its origin and purpose, ranging over most of the spectrum of imagination.

The two principal beliefs, however, are that it was built either by a group of outside conquerors during a stay in Denmark, or that it was the work of the Danish king, Svend Tveskæg, (Swein Forkbeard), and his Vikings. The theory is that Svend had Trelleborg constructed as a gathering centre for international Viking "conferences". The bearded, skin-clad, helmeted Vikings—claim the theorists—assembled at Trelleborg from all over Denmark to prepare for their raids. It was during this period that Viking longships sailed up the Baltic and attacked the eastern Russian seaboard; and Svend Tveskæg about the same time vanquished large parts of England.

The theory that Trelleborg was associated with Svend and his Vikings is one of the most likely. It is supported by the fact that fortresses have been discovered in England—of almost the same construction.

One of these lies at Worham Camp in Norfolk. The fort's local name is the *Danish Camp.* The area in front of the fortress is called *Sweyn's Field,* and at the other side of the bordering river is *Sweyn's Meadow.*

Thus we see a fortress in England, similar to Trelleborg in Denmark, connected with Denmark through King Svend. Are we justified in believing that King Svend and his followers did indeed construct Trelleborg?

The reason to a certain extent that theories on Trelleborg persist in encompassing foreigners is that the plan and method of construction have an alien appearance. It seems to have had outside inspiration. But I do not think it came from a band of conquerors. Planning Trelleborg was the work of experienced practitioners from southern climes, who visited Denmark and instructed the people (or more likely the priests) in the practical side of ancient geometry.

Archaeologists have done a lot of work on Trelleborg. Thanks to careful excavation and inspection of every detail concerned with the fortress. The combined archaeological report was published in a book by the Royal Nordic Society for the Preservation of Ancient Texts (Det Kongelige Nordiske Oldskriftselskab) under the title *Trelleborg.* The volume was compiled by Poul Nordund and a number of assistants.

The once-proud fortress is today in ruins—and scattered ruins at that. But the archaeologists and historical experts have been able to trace all the main dimensions, and have pieced together a most complete (virtually perfect) reconstruction of the stronghold.

Trelleborg, we find, consisted of a main fort, surrounded by an encircling wall. The fort was not built of stone, as was the case with most of the traditional strongholds of the Middle Ages. The outer wall was a circular embankment of earth, covered with stones. Inside the wall, in a symmetrical pattern, were 16 wooden buildings each 30 meters long.

Outside the circular earthworks was a kind of advance line of defence in the form of a moat and 15 timber buildings of almost the same dimensions as those within the main fort.

The whole scene has been reconstructed

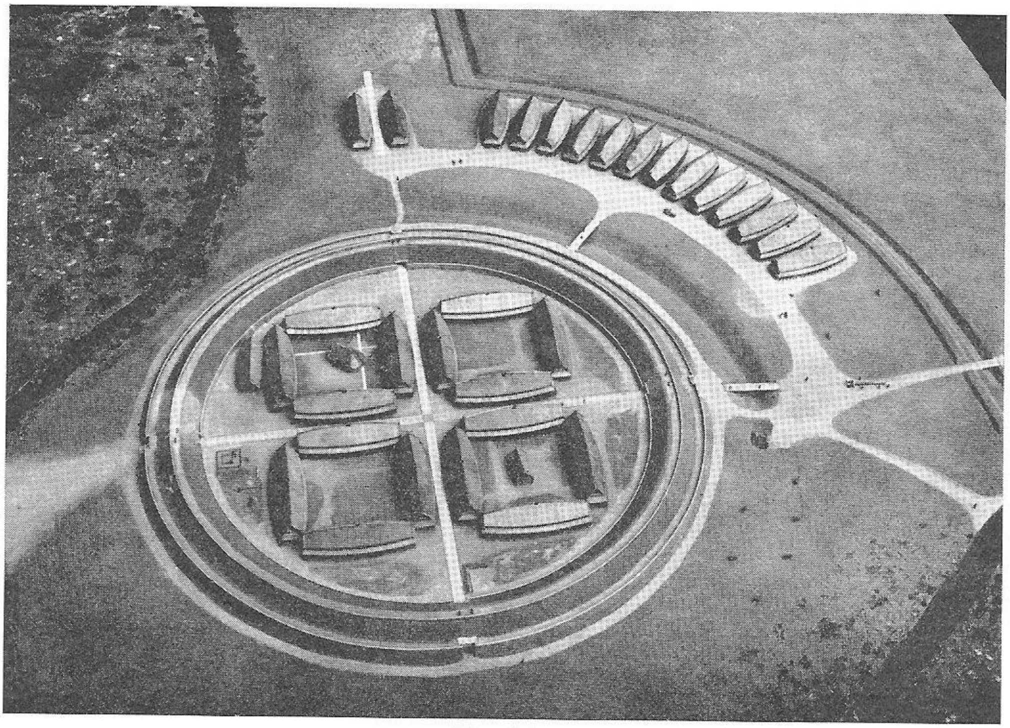


Fig. 370.

by the National Museum of Denmark, and we see a photograph of Trelleborg reconstructed in *Fig. 370.*

The mass of theories regarding the fort's history and origin has certainly been matched by a similar number of theories about its plan of construction.

The visitor to Trelleborg (or to several of Denmark's museums) can obtain a neat brochure, giving in Poul Norlund's words an account of the most commonly accepted theory on the system by which Trelleborg was planned.

Here is a translation of the summarised text:

"The amazing regularity of structure, both in form and measurement, indicates that a well-prepared plan was made prior to the actual construction process, and in marking out the plan on site the builders displayed tremendous accuracy. Some extremely experienced engineers must have been responsible for the construction of Trelleborg.

"The Roman foot was used as the unit of measurement; it is illustrated in all the main dimensions, most clearly in the timber houses within the circular fort. These are 100 Roman feet long. The small buildings in the quadrangles are 30 × 15 Roman feet, the houses in the advance fort are 90 feet in length, and the thickness of the circular rampart is 60 Roman feet. The large moat has the same width, and between the rampart and the moat is an area 200 feet wide. The average length of the foot used in the construction of Trelleborg was equivalent to 29.33 cm, or 14 mm shorter than the standard Roman foot of 29.57 cm.

"To start the projection work the planners selected a central point from which,

among other things, they marked off the arcs which bound all ramparts and ditches. Moreover, the same point forms the intersection of the two main axes which, at right-angles to each other, divide the area within the circular rampart into four equal parts and which are continued out through the four gateways.

"To set out the right-angles the constructors must have made use of a cross with arms exactly at right-angles: a simplified version of the Roman 'grand' or 'stella', on which plumb lines were hung on the tips of the cross and sights taken from the centre of the cross through the plumbs. The two principal axes were laid out with the utmost accuracy and care—it has proved impossible by modern means to discover even the slightest deviation from true. The four groups of houses within the rampart are also positioned with a great respect to the right-angle, although slight misplacements have been recorded by modern instruments.

"The curved sides of these 'long houses' are invariably symmetrical about a central axis, and there is no doubt that they were constructed on an ellipse. Two houses at an angle to each other had the same point of focus, and for the four ellipses of each block of houses it required only four focus points, forming the corners of a square the sides of which measured approx. 36.4 meters or 124 Roman feet. The whole structure within the circular rampart was constructed in squares.

"Of the 16 block-houses 100 feet in length, the shortest was measured by the Danish Geodetic Institute to be 29.21 meters, the longest 29.59 m., while 11 of the houses measured between 29.33 and 29.49 m.: deviation of up to 16 cm or a mere ½ % of the planned length. The radius to the inner edge of the circular rampart is 68.4 m. or 234 Roman feet. This dimension is repeated in several places: the distance between the two moats of the same size is 234 R.ft., while the distance from the centre of the fort to the nearest gable of the advance blockhouses is twice this figure.

"The accuracy with which these gables were placed on a circle perimeter is totally amazing, apart from the two northernmost houses which have a special placing. Twenty-one of the 26 corner points are from 237.21 to 237.45 m. from the centre. The measurements were made by the Geodetic Institute."

This text by Norlund demonstrates a certain agreement between my theories and the discoveries recorded, as well as with the conclusions resulting from the research.

It is maintained that the fortress was built in strict accordance with a predetermined plan prepared by specialists, every detail of the construction being accurately measured and fixed upon axes.

It is also suggested that construction was based on squares, although Norlund and his team do not go into detail. This assumption was presumably held because of the placing (in four squares) of the 16 blockhouses inside the main citadel.

The comments fit excellently ancient geometry, with its intimate use of the square. It produced invariably a construction which, when faithfully executed, was both accurately measured and fixed upon axes.

As usual the modern researcher tries to fit ancient building into a fixed standard of measure, and I believe a mistake is made on this count.

The Roman foot, it is suggested, was adopted by the builders as the standard of measure—but at no stage do the theorists illustrate a direct connection between the Viking Trelleborg and Roman builders.

Norlund chose the Roman foot because it is the unit which best fits the measurements made at Trelleborg, but it is ad-

mitted that there is a deviation between the selected unit and the unit known today as the standard Roman foot, the "Trelleborg foot" being 14 mm shorter. Is an error in the basic unit of measurement likely to have been made by engineers so adept at surveying and setting out dimensions?

I believe that as with the other buildings examined earlier in this book Trelleborg was built without resort to any unit of measure.

The fortress, like the other buildings, was planned according to the principle of comparative measure, i.e. the drawn plan was reproduced on a given ratio. Requirement decided how large the basic square should be, and the latter with its lines of division provided all the fortress's dimensions.

Experienced builders were certainly able on their plan of construction to assess how much space was required for the various parts of the fortress, and when they decided on the dimensions of their smallest area, they were able to calculate how large

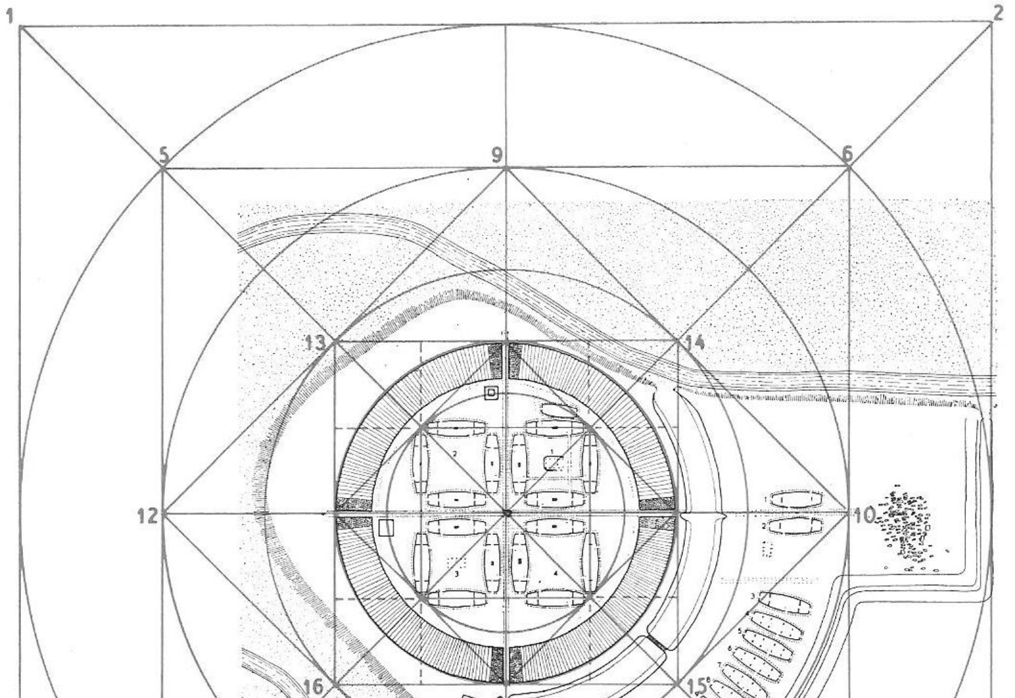


Fig. 371.

their constructive basic square should be in relation to the available building site.

They had by all means employed some means of measuring unit to estimate or calculate the areas required, for without such a standard measure in which to reproduce one's thoughts one cannot think in terms of length. But once preliminary estimates were made, they had no need for any standard measure. They could happily leave the rest to the rules of ancient geometry.

In *Fig. 371* we see the reconstructed ground-plan of Trelleborg placed in an ancient geometric diagram. The diagram consists primarily of four diminishing squares within each other, each with its inscribed circle.

The basic square is 1-2-3-4, line 2-3 following the straight part of the moat which has been extended out of alignment with the circular part.

This square's inscribed circle provides the basis for constructing the main square's half-size version, 5-6-7-8. The circle inscribed within this square marked the circular moat outside the advance fortress. Only $\frac{1}{4}$ of this moat remains today.

Within this construction we now draw the half-size version of square 2, turning it through 90°. The new square is 9-10-11-12. The circle inside this square marked the outside edge of the wide carriageway lying between the circular rampart and the 15 houses of the advance fort.

The diagram is supplemented by construction of the fourth square, 13-14-15-16. Its inscribed circle, we see, marked the outside of the great circular rampart surrounding the fortress proper.

The next (and almost the final) step in constructing our diagram is to divide square 13-14-15-16 4×4 in the same way as the diagram used for the runic alphabet, and we see immediately how this marked the 16 houses at the centre of the fort. These match the four inner squares.

If a circle is drawn outside the latter four squares, it indicates the inside of the rampart and thus its thickness.

I have not the slightest doubt that a closer examination on a more detailed reconstruction would reveal that the curved shape of the block-houses was produced in the same manner as demonstrated in the curves of the ceramic vases in an earlier chapter. And it is extremely interesting to observe that the inner square, 13-14-15-16, shares a definite link with the diagram used to form the runic alphabet.

From this one is entitled to assume that 200 years before Trelleborg was built Denmark was in possession of the geometric basis of the fortress's construction—only at that time the geometric diagram was reserved for writing only.

Whether Danish priests developed during the 200 years that passed sufficient expertise and manipulation of ancient geometric symbols that they were able without outside assistance to plan and construct together with their Viking people the fortress at Trelleborg must remained an unanswered question. But it is certainly possible that this was the case.

CHAPTER TWENTY

The Origin of Chess?

As with our alphabets, numeral systems and a list of other cultural components, the game of chess is one of the factors handed down to us from an ancient, shrouded past.

It is no exaggeration to say that chess is the best known of history's many games and pastimes, and is today a regular feature of international leisure at all levels. Its exponents grow every day, there are no signs of the game dying out of existence. Annually loudly proclaimed international tournaments are staged by national clubs, and these international 'meets' arouse considerable public interest.

The game itself is played on a square board divided into 64 squares: i.e. the sides of the board are divided 8×8.

Each player begins the game with 16 chessmen or pieces, placed on the board in a special order according to rank and order of importance of the pieces.

I do not intend going deeply into the rules of the game here since they are widely known and available to all, and an extensive variety of literature has already been written on the subject explaining the rules and relating details of well-known and classic chess games played between recognised masters.

Many attempts have been made to estimate, discover or calculate the age and place of origin of chess without any clear conclusion ever having been reached. The observer is told variously that the period of origin lies "somewhere" between 6000 B.C. and 600 A.D.

About 1785 the English researcher, Sir William Jones, wrote in *Asiatic Researches* (vol. 2) that evidence showed that the game in its earlier day had been called *chaturanga.* "That is, the four ongas or members of an army, which are said in the Amarakasha to be elephants, horses, chorials and foot soldiers."

Jones' book is principally based on translations of Bhawishya Purana in which a description is given of a four-handed game of chess played with bricks.

He relates that in a conversation with a Brahmin pundit, he was informed that the game was mentioned in India's oldest and most revered collection of laws and history, and that by this means the age of the game was reckoned to be four or five thousand years.

Sir William goes on to relate that chess was passed from Hindustan to Persia along with the beliefs and religion of Buddha around 400—500 A.D. under its Sanskrit name of *chaturanga,* but that the name was altered to the Persian *chatrang.*

When Persia subsequently fell to the Arabs, the game lived on in the hands of the conquerors but as the early Arabic alphabet and language had no expressions

precisely to cover the name chatrang, the name changed once more: this time to *shatranj*.

One of the earliest authorities to write on the wide subject of chess and its origin was the Arab, Masudi. He recorded in 950 A.D. that *shatranj* had existed at an "incalculably" early date in history.

Masudi's message can in any event be taken to mean several centuries prior to his own period, and from this statement alone we can with confidence assume that the game of chess is at any rate 1000 years old today.

Thus we see that the game is (a) extremely old, and (b) has existed in a number of countries under slightly different appellations.

The latter may present difficulties to later investigators who discover two nations discussing a game under two different names without delving into the rules and play. Yet there is every chance that the two widely spaced games (both from a time and geography point of view) are in fact our present-day game of chess.

Another consideration is that with such an ancient pastime as the pursuit of chess it is astonishing that the game has undergone so few and such superficial alterations as we can discover has been the case. The names and ranks of the individual pieces have perhaps changed occasionally, but it appears unlikely that their significance and direction of movement have altered through the centuries.

I venture to suggest that the stable form of the game has been due to the positioning and movement of the pieces according to certain, clear-cut rules of ancient geometry—and these geometric guide-lines as we have seen repeatedly in our study of the ancient, simple form of geometric diagram have withstood the changes of time.

An examination—even cursory—of the historical period around 1000 A.D. shows that chess was originally no game for the common citizen. We read generally that it was a pursuit of kings and wise men. We sense that chess was a game for the upper, leading classes.

Following this train of thought as best we can back through time, regarding chess as a special interest of the intelligentsia, we land as so often earlier back at the walls of the Temple. This was the perfect milieu for the origin of a brain-racking game like chess.

The Temple assembled a nation's creative, thinking leaders and employed them daily with tasks demanding speculation, meditation and intelligence. But all work and no play made even the keenest Temple inmate stale, and there is no doubt that there were a variety of games with which to while away the leisure hours.

We can imagine the immediate acceptance and success of chess, the new society game. The game that appealed to the brain, the imagination. Almost devoid of dependence on luck, its players pitted their mental skills against each other, encouraging other Temple brethren to join in.

If indeed these were the surroundings in which chess originated, it would probably have been kept secret initially, reserved for the top brains of the Temple. The fact that it was a pursuit engaged in by brethren in their leisure hours did not mean that it was something that could be passed openly to non-initiated. For the bond of secrecy that invisibly united the many ranks of initiated brethren together was also tied tightly around their free time.

If this was correct, chess could have lived many hundreds of years as a game known and played by initiated Temple inmates—and in the same way as other Temple knowledge could have been passed from Temple to Temple, country to country by visiting brethren without at

any time breaking through into public gaze.

In the later phase of the powerful epoch of mystery religions stretching over thousands of years, a change began to take place in the Temple's structure of power.

Whereas in the early years of Temple society it was the Temple and its inner brethren who actually ruled behind king and country, two groups of brethren now began to emerge.

The first, original, hard-core group spent their entire adult lives within the Temple's administration, emerging into the public scene only in the service and on instructions of superior brethren if their particular abilities were required "on the outside".

The second and larger group of reliable brethren were men admitted to the religious order for training for a specific numbers of years. At the close of this period of training they returned to civil life, fitted into existing society as part of the upper crust of leaders, and in this manner gave the Temple real contact with and control of the eminent branches of the social tree.

This latter group of Temple brethren were probably responsible for bringing the venerable game of chess outside Temple walls. They had perhaps learned it during their stay in the midst of regular Temple brethren, and on leaving the close circle had taken chess with them as a thoroughly invigorating pastime, although it had already been known within the Temple for thousands of years unknown to the outside.

The "part-time" brethren doubtless met each other in their capacities as public leaders, and socially they had inevitably played chess, a secret and forbidden pursuit of the Temple. But as time passed one or two, and then more, outsiders got to know about chess and its rules—and the big tournament began. Gradually the game filtered down from the ruling classes until it became widespread among the common people.

I am certain that the game of chess began within the framework of the Temple partly because it was a natural birthplace for such an exercise of intelligence and mental skill, partly because the picture fits the structural society in which it would later prove impossible to trace the game's origin on account of the veil of secrecy that covered the Temple and its activities.

The appearance of the game outside the Temple was not synonymous with its *invention* outside the Temple. And the passing on of its secret rules of play to the population at large may well have occurred at widely differing times in the individual societies depending on development within a society's Temple. Centuries may have separated the stages of development, for example, in India and Egypt.

A further factor confirming my theory that chess was a child of the Temple is that the form of the game follows remarkably closely the lines of well-known geometric figures. Its inventor(s) was evidently familiar with the mysteries of ancient geometry.

Since ancient geometry throughout its long underground life was one of the closely guarded secrets of the Temple, it must be admitted that if a link can be demonstrated between the game and its formation and ancient geometry and its symbols, it is extremely probable that the creator of the game was acquaint with these things, and therefore one of the Temple's initiates. This helps cement the game's rules and mystical origin into the Temple background.

The chessboard itself comprises a square board divided into $8 \times 8 = 64$ small squares.

When we begin our reflections on why this particular division should be selected

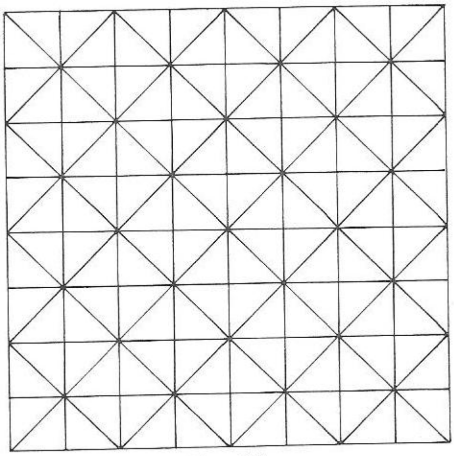


Fig. 372.

in preference to any one of a number of other suitable combinations, we may quickly observe a possibility: that is precisely one of the divisions which results when we divide a square by triangulation into smaller triangular units.

When we divide a square into 128 small triangles, we simultaneously produce a division of $8 \times 8 = 64$ small squares.

We can recall this division from our earlier look at Egyptian papyri with triangulation of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$. The next step, $\frac{1}{128}$, produces the 64 required squares. We saw this division to be a natural part of the Egyptian mathematical picture, e.g. in Fig. 120, under the section on measures of capacity.

A similar division, we remember, was taken as the basis, the geometric basis, of the origin of cuneiform symbols. In the case of cuneiform the triangular division of the square had nothing to do with numbers.

We saw this in Figs. 301 and 302, and now begin to recognise the chessboard to be representative of a very important constructive factor in ancient speculation.

This symbol of the square divided into a number of small triangles is almost an optical puzzle picture. If one concentrates on triangles, one sees the main square as a mass of triangles; on the other hand if one concentrates on squares, one finds that the main square is also composed of small squares.

In creating the symbols of the cuneiform script, the Temple brother responsible for the task was inspired by the triangle. The creator of chess however made exclusive use of the small component square, eliminating the guide-lines with which he produced the squares.

The division of the square is seen in *Fig. 372*.

Each of the game's two players has, as mentioned, 16 pieces, comprising

1 king
1 queen
2 bishops
2 knights
2 rooks (or castles)
8 pawns.

The names are not important. Indeed they alter from language to language, as we have already seen. The Danish name, for example, of the English bishop means "runner", and the knight is known as the "jumper". The pawns in Danish are "peasants".

In *Fig. 373* we see the pieces positioned at the outset of the game, prior to the opening player's gambit. The game consists of moving the individual pieces according to strictly governed rules from which no divergence is tolerated.

Apart from the design of the playing board, with its 8×8 small squares, the movement of the pieces brings immediately to mind a peculiar link with the lines of ancient geometry even though these cannot be seen on the board. This applies to the square's diagonals.

The king may move only one step (or square) at a time. *Fig. 374* shows that this piece may move backward and forward, parallel to line AB, or from side to side parallel to CD. But at the same time the

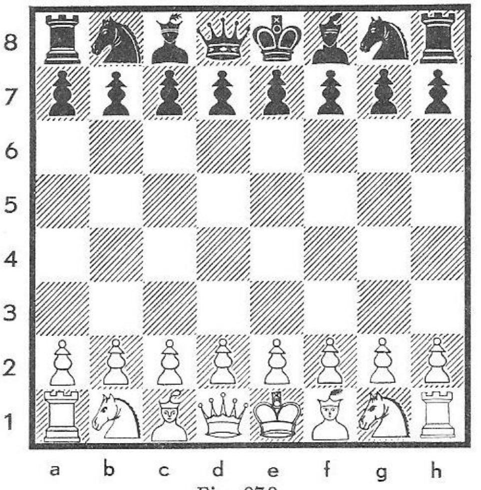


Fig. 373.

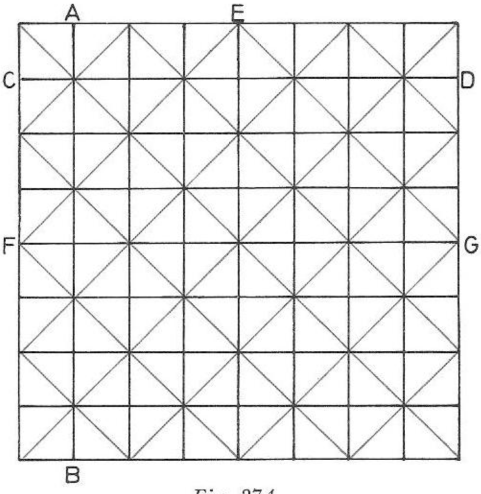


Fig. 374.

king may move from square to square parallel to diagonals FE or EG. It is restricted to these moves regardless of its whereabouts on the board. It is free however to move backward or forward on the lines indicated: either up, down or across the board parallel with the sides as governed by the small squares, or on the invisible lines of the square's diagonals.

The next major piece on the board is the queen, the most mobile of all. It may be moved any number of squares in any one direction: either parallel to AB (backward or forward) or to CD. Or parallel to the diagonals EF or EG. But like the king, the queen is still bound by the visible and invisible lines on the board.

The rook may move only on the visible lines of the board, i.e. parallel to CD/DC or to AB/BA, but never in the direction of the diagonals EF or EG. The piece can however be moved as many squares as required in one move in one direction as long as there is no hindrance in its path.

As opposed to the rook, the bishop is governed by the diagonal lines EF and EG (the invisible lines of the diagram, shown in red in Fig. 374) and must on no account move on the visible lines, i.e. parallel with the sides of the board.

The pawns in the game are confined in principle to slow movement forward (never backward as with the other pieces) in the direction of and parallel with line AB. When in a position to capture a piece belonging to the opposite side, however, the pawn moves diagonally forward to occupy the opponent's square.

Thus we see that of the five groups of pieces examined, only one (the rook) is confined irrevocably to movement on the visible lines while the other four groups either make exclusive use of the invisible lines or may use a combination of visible and invisible.

If the possibilities for combination were exhausted at this point, a sceptic might well with some justice claim that the combinations seen so far do not in themselves prove that the game of chess is tied up with ancient geometry. A chequered board encourages the user to move either parallel to the sides or along the diagonals, one might say. Although there are in fact other possibilities open.

And there is no denying that this doubt might be justified, if the sixth piece, the knight, did not so obviously place chess firmly and squarely in the framework of ancient geometry.

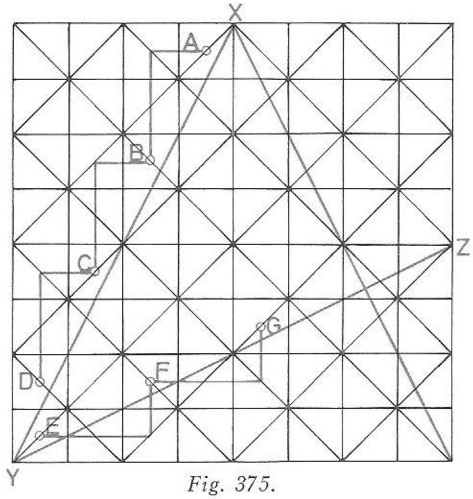


Fig. 375.

From the point of view of ancient geometry the movement of this piece is quite remarkable as it follows neither the visible nor the invisible lines we have hitherto discussed in connection with our chess investigation.

The movement of the knight is, from any given square, to move one to the side and two forward *or* two to the side and one forward *or* two forward and one to the side. And this rather interesting manoeuvre is based on the fact that the initial (given) square and the final square in which the knight comes to rest lie on the line or parallel lines *of the acute-angled triangle in the main square.*

We see this in *Fig. 375* in which we have the chessboard complete with triangulation and squares—and in red the acute-angled triangles.

If to illustrate the geometric connection we place the knight at point A (the square normally occupied by a queen or king), we see clearly that with a combination of one-side-two-forward the piece lands at point B. In other words in terms of direction from A to B it follows the side (xy) of the acute-angled triangle. And so on from B to C and C to D.

In moving back along the same line the knight makes use of another of its moves, two forward and one to the side. The third possibility, two-side-one-forward, follows the side (yz) of the transverse triangle: from point E to F and to G.

Thus the acute-angled triangle has covered all three possible combinations open to the knight.

When the complete set of 2 × 16 pieces is arranged in playing order on the board the knight is not directly connected with the acute-angled triangle. But two of its possible three moves bring it in direct contact with either xy or yz.

Although the knight is restricted in its movement to this 2-1 or 1-2 combination, it does not mean that it must at all times stay in immediate contact with the lines of the acute-angled triangle. The latter is not after all visible on the board. With the three possibilities at its disposal the knight is then "free" to roam the board.

The special characteristics of this chess-piece in my opinion place the game of chess securely within the speculative sphere of ancient geometry. With the other factors already discussed, it is extremely likely that chess originated in the ranks of initiated Temple brethren whence—like other cultural bricks used to build up society—it was passed through the veil of mystery, voluntarily or otherwise, into the hands of the people.

CHAPTER TWENTY-ONE

Ancient Geometry and Modern Times

WE HAVE followed a trail in the past two volumes, through history, on the track of ancient geometry and its use in a number of spheres from Early Egyptian art and architecture to a period that lies about 500 years before our own time.

The philosophical, mathematical and purely practical knowledge contained in the principles of ancient geometry, we saw, was kept a virtually unbroken secret by the Temple and later the Church as part of their esoteric wisdom. This secret was maintained despite the fact that the religious ideals of the respective civilisations underwent change, and that they presumably differed from nation to nation.

The application of the principles of this secret geometry left an unmistakable mark (for those able to interpret it) on the cultural development of early times up almost as far as our era. Having gathered some knowledge of the system's principles and their influence, one inevitably ponders on whether traces of the old science exist today.

Yes, certain aspects of ancient geometry are put to work even in our own time. At the time of writing I have been able to discern two spheres in which ancient geometry can be seen. One of these is in an essentially practical form, used every single day in drawing offices, typists' pools, engineering workshops, architects' premises, and in most homes in Europe.

It is the means of measuring and dividing paper in sizes known as the A-format: A-2, A-3, A-4, etc. These pages measure width to length in the ratio $1:\sqrt{2}$. And this means that they involve the sacred cut!

The other sphere in which we can find traces not of ancient geometry itself in practical use but of the ritual aspect similar to that which existed in the ancient orders of builders, with whom we have made a passing acquaintance throughout this book, is the worldwide movement of Freemasonry. As far as I have been able to trace it originated in the old orders or clans of builders.

The A-format

ONE OF the most precious concepts valued by the ancient geometer was, as we have seen, the sacred cut. Its significance was immense within both geometry and construction.

It is thus at the same time amusing and remarkable to record that the sacred cut is applied to dimensions of paper today without anyone previously having connected it with ancient times. On the contrary, the paper sizes we know as A-3, A-4, etc., are regarded very much as a modern innovation, a rational action on the part of 20th century experts.

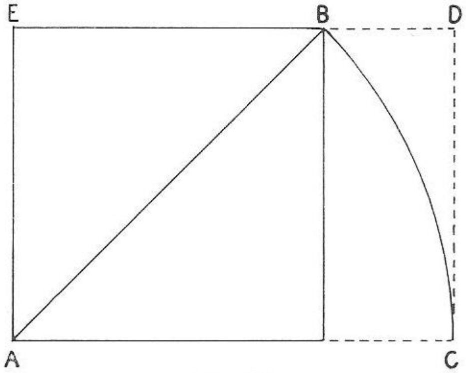


Fig. 376.

To say that the sides of the A-format page are in the ratio $1:\sqrt{2}$ tells the non-mathematician little about the shape of the page—if he is not already acquaint with it. (Although the A-format is in widespread use throughout the mainland of Europe, many British and American readers may not be familiar with it. It is only in the past year or two that there has been mention of introducing these standard sizes in Britain.)

To deepen our understanding of what is meant in effect by the A-format, we should perhaps translate the modern term into the language of ancient geometry.

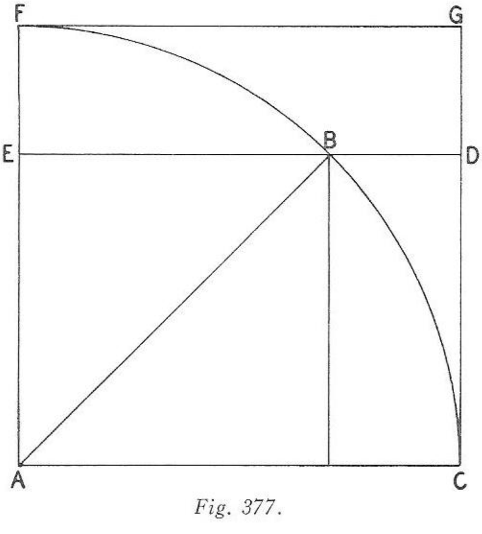


Fig. 377.

If we have a square, each of the sides of which measures 1 unit in length, the diagonal of that square measures $\sqrt{2}$.

In *Fig. 376* we see the said square. The diagonal is AB.

The diagonal is laid out along the base-line to AC, permitting us to construct rectangle ACDE. The long side is AC, the short side is AE. This rectangle has thus the proportions of the A-format.

But we do not see the sacred cut until we enlarge the diagram by constructing the small square's double-size version. We see this in *Fig. 377* in which the arc CB is extended to point F, after which the double-size version can be entered.

We see that line ED is the upper horizontal sacred cut in square FGCA.

The A-format is not one particular size. It is a ratio, represented by five basic sizes, each twice (or half) the size of its neighbour.

If we imagine that the construction in the preceding figure represents the smallest of the sizes, A-5, in rectangle ACDE, we move in the next illustration one size up to format A-4: *Fig. 378.*

Here, with radius AG, we have constructed another square, AKLJ, the upper horizontal sacred cut in which is FH. Thus format A-4 is the rectangle contained by the latter sacred cut, rectangle FHJA.

This is the same geometric construction as we discussed earlier in Fig. 40, with three squares inside each other. It is also the same figure as mentioned by Plato in his account of Timaeus's speech.

When we examine the diagram we find that the long side of the smallest format is identical in length to the short side of the next size. Thus we turn A-5 through 90° it fills exactly half of a sheet of A-4 format.

In *Fig. 379* we have the four most commonly used sizes of the A-format series. The largest is A-1, the next A-2 and so

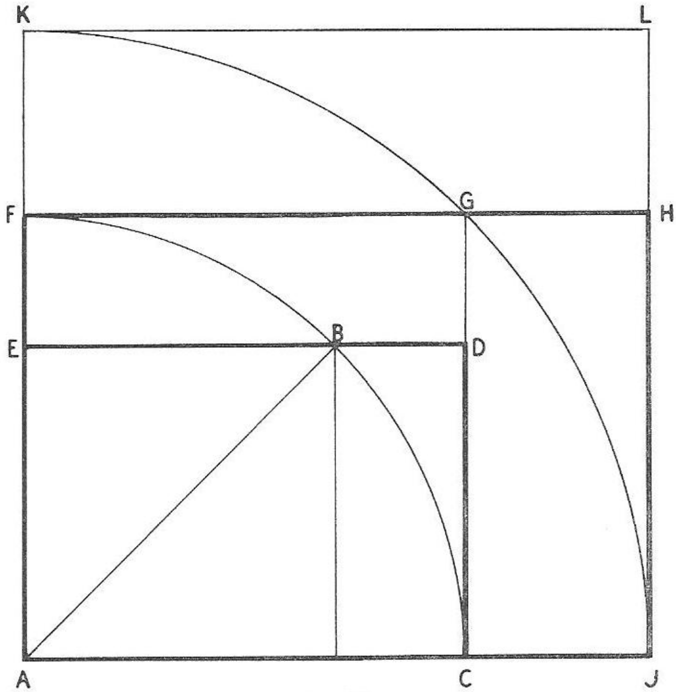


Fig. 378.

on. Thus we can have two sheets of A-4 from one of A-3, etc.

This series of page formats is extremely rational and economical in use. It has brought simplicity and good order to every stockist of paper who has practised the system. If he has a stock, for example of A-1 sheets, he is able quickly and without any waste whatsoever to cut a supply of any of the other sizes—an important factor in the busy printing industry of today.

The sizes allotted to the ratio must depend on whichever figures are accepted by a society.

This was, we saw, also true of the system of Egyptian capacity and area measure. It was a system of division and subdivision. We proved that we were able to work with the system without knowing the length or capacity of the approved standard unit.

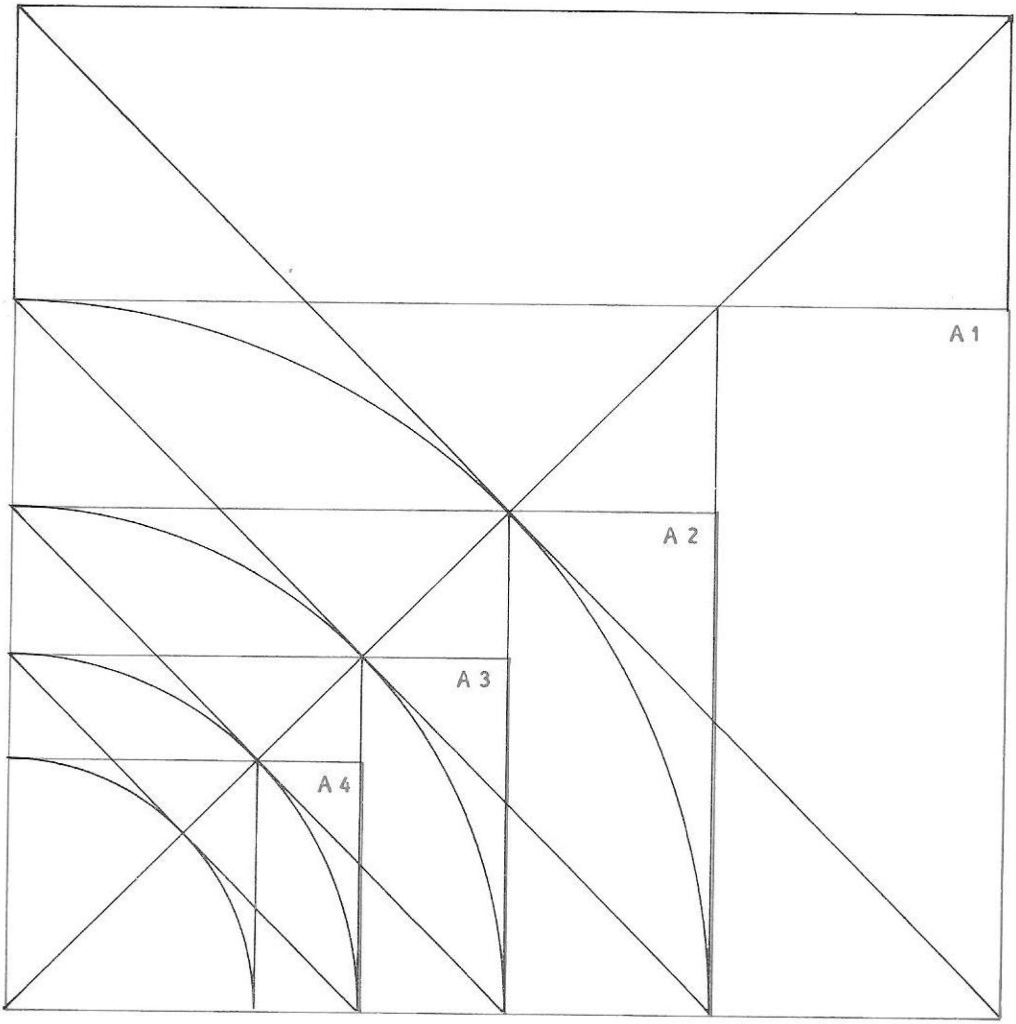
So with the A-format. We need not know the sizes in order to lay down the proportions, but for the record the respective sizes in use in Europe are as follows:

A-1 $=$ 840 $\times$ 594 mm
A-2 $=$ 594 $\times$ 420 mm
A-3 $=$ 420 $\times$ 297 mm
A-4 $=$ 297 $\times$ 210 mm
A-5 $=$ 210 $\times$ 148.5 mm

Research into the history of the A-format leads to a misty wilderness.

It is possible to discover when a number of European countries began popular use of the format, but it is difficult to find out just where the format originally started—or what unit of measure was used to plan the series.

Today the A-format is mentioned usually in the same breath as the metric system of measure. But if the assumption that the two are associated is examined intensively, it is impossible (or rather I have found



A4 – 297 x 210. A 3 – 420 x 297. A2 – 594 x 420. A1 – 840 x 594.

Fig. 379.

it impossible) to see what the connection really is.

In a special issue of the industrial publication *Industritidningen Norden,* dated 1948, one reads that as early as 1790 the A-format was in use in France, with pages being halved to provide the next smaller size. But it is not indicated whether the principle was an old one at that time.

The system is, as we have seen, based on the division of a series of squares placed within each other. If we assume that it existed in the paper or printing industry prior to the introduction of the metric system, one of the squares—perhaps the smallest—had to be selected as the basic square.

If this was made to measure a *Roman foot* along each side, we would discover that this fitted exactly the present size of

the A-4 format since one side of the A-4 measures 297 mm, which equals the old standard Roman foot.

According to Carl Herning's *Ready Reference Tables,* New York, 1914, the old standard French foot measured approx. 325 mm—considerably more than the Roman measure.

Prior to 1750 France, like many other European countries had a number of different lengths for the unit "foot", including the Roman.

The French Church, and its subsidiaries, was very much under the influence of the Catholic citadel in Rome, and it is highly probable that the Church printing houses, under the same influence, operated with the Roman foot instead of a local version.

Indeed it is also possible that the French Church printers actually took over the system of dividing paper into the formats discussed above *direct from Rome,* which of course boasted printing facilities at a much earlier stage than France.

If this (the Roman foot) was applied as the unit of measure from the beginning, we can better understand the irregular dimensions of which the system consists.

When the decimal system with its meters and centimeters was introduced in France it was highly inconvenient to alter the paper formats since printing machinery, paper guillotines, and the printer's other tools and machinery were based on the already accepted sizes. Instead therefore of altering the paper sizes to "round" figures in the metric system, it was decided to express the old Roman foot dimensions in the appropriate number of millimeters and to retain the sizes unaltered.

As the previous chapters have shown, the system of sub-division itself is really ancient.

It is thus, as stated, entirely within the limits of credibility that both the system of division and the unit of measure were imported at some early stage in France's printing history direct from Rome. Use of the format system and sizes spread from Church printing houses to other printers in France, until it became the established means of dividing and measuring paper.

From France the sizes and system gradually worked their way throughout the rest of Europe, each country accepting in turn the sizes in question on account of their rational proportions.

The rational part about the A-format of paper is not the actual size of the individual sheet but the system of division which produced the proportions.

The system was superior to all other sizes and shapes of paper, and—in any event in Europe—slowly ousted all competitors.

Thus a tiny aspect of ancient geometry found its way unobserved into our everyday life and filled a practical role—without anyone thinking of the concept's extremely significant history.

Freemasonry

FROM THE beginning of time, knowledge was assembled by Society's witch-doctors, medicine men, priests, Temple and Church. It was from these, Temple and Church, that learning and education stemmed.

There were no other centres of knowledge to which one could turn for learning. The Temple was the centre of all power.

Books have been written in our own time, based on historical and archaeological reports, of how the Temple, for example, in Egypt had various scientific branches the initiated brethren could frequent at the end of their obligatory seven-year training. We learn that the Temple built up an efficient system of medical research and team of doctors.

Medical instruments of excellent quality have been recovered from excavations in Egypt, surgical instruments, for instance, that had been employed for trepanation operations on the brain. These instruments have actually been tested by modern surgeons—and found perfect for the purpose.

From these literary sources therefore we can see that the Temple nourished, apart from ancient geometry, other scientific fields which developed and improved slowly with time, practice and application.

It is easy to visualise the picture of training in ancient Egypt. All brethren admitted to the Temple underwent a common training for a period of seven years. At the end of that period they split into different groups, each to receive a specialised further education.

Pythagoras was the first person history records as having broken with the ancient tradition. He opened a school in Greece around 600 B.C. at which he had only a form of preparatory school, followed by a more advanced study of statesmanship, philosophy and geometry.

Pythagoras educated ordinary men, with no Temple initiation, there being however a form of selection by the school of desirable and undesirable pupils at the end of a trial period.

The school was a tremendous success, but met resistance from the class of population who regarded it as a breach of etiquette and tradition.

A long time was to pass though before non-Temple schools became a common sight in civilisations of Europe and the East. The tendecy had however started. Perhaps it was further inspired by the followers of Pythagoras, who opened "underground" schools in countries surrounding Greece after the banishment of their old master.

Development proceeded until the stage when, in the early Middle Ages, the Church's powerful monastic orders assumed charge of education and professional training.

One or two monastaries distinguished themselves from others in their order by specialising in particular subjects or professions, a condition of admittance being that students had undergone the basic education available in the lower-rated monastary prep school, including reading and writing.

This was one of the first signs of the structure we recognise today, in which primary and junior schools are one part of education, and senior and more advanced schools a quite distinct and different part.

As a result of this developing tendency individual monastaries took on specialist arts and sciences and gradually excluded all others from their curriculum. In this way the subjects to which they devoted their attention were cultivated intensively—and benefited as a result. But the monastaries, educationally, drifted further and further from each other, as did the one-time linked sciences.

As regards the study of ancient geometry, it was irrevocably bound up with building and architecture, which as a subject had also been developed separately and been made the private province of one or two monastic orders, perhaps the science with the largest following of them all.

Historical literature informs us that as late as 1260 it was known that in Europe the massive building clans were amalgamations of specialist monastaries with headquarters in Cologne, Vienna, Berne and Magdeburg. These main centres established "branch offices" in other areas of Europe, and monastaries throughout the continent were linked (figuratively!) by a bridge of bricks. One of the leaders of this wieldy building union was Meister

Gerard, mentioned in an earlier chapter on the planning of Cologne Cathedral.

The principal centres named above were presumably the training quarters, while the various local monastaries were directly implicated either in the actual building process or in the production of materials such as bricks, etc.

From southern and central Europe the building clans spread their influence north and west where, for example, in England a strong branch of the organisation was founded, and erected many beautiful edifices.

Simultaneously with this evolution of the monastic building section, there was a second development within several of the other scientific spheres: a gradually increasing number of trainees were breaking away from the influence and hold of the Church in order to practise on their own account. Training was switching from the Church to private institutions, the breakaways spread the word that the Church no longer had a monopoly on the building site.

Maybe the development erupted when a collection of fresh theories and experiences burst upon the individual branches of science. The old, traditional centres of training (churches and monastaries) refused to acknowledge the innovations—because they contradicted tradition.

An atmosphere of tension and strife set in between the established and the radical schools of thought, to be released only when the "new thinkers" broke with convention and began independent work.

In form the development probably began on a modest scale, with only one or two rebels refusing to accept the established training and to conform to traditional rules. But it eventually reached the stage where the State or Society saw fit to take over the new range of schools that arose.

As discussed earlier, a development of this kind is more likely and easier to effect in a purely philosophic sphere, or with subjects in which the individual is complete within himself, such as medicine.

But if the training is part of a larger picture, in which the individual is a cog in a wheel, the breakaway is more difficult. To obtain sufficient teachers for a school in the latter subject, one person would have to break off simultaneously from each of the many phases of learning.

For this reason the Church's building industry enjoyed a longer lifetime than the other scientific subjects.

But time eventually caught up with the monastic building clans, too, for needs arose in Society which could not be met directly by the Church.

Harbours and ports were required, roads had to be built, more and more houses and homes were needed for ordinary people, and warehouses had to be constructed to store the increasing quantities of merchandise that changed hands and travelled by land and sea.

And the old Church and monastic orders of builders simply could not stand the pace. At some point in time the same happened within the building industry as in other branches of science—more and more initiated brethren broke off from the main tree and set up independent workshops to cater for the new requirements, while the old orders continued to devote themselves to the construction and adornment of church buildings.

Denmark's history provides ample illustration of the press to which the old building guilds were subjected.

The country has of course a large number of old parish churches, dating mostly from the period 1100—1200. Church records show that they were in many cases built by tradesmen who came from the south.

The custom was for a church to be

"ordered" by the local congregation—but 10—12 years might pass before a start was made to the work, so busy were the builders. In the intervening years the locals assembled loads of stones and builders to be used in the building so that the bulk of the material was ready when the master builder and his journeymen arrived to begin work—bringing the completed plans with them from their southern base.

A population can perhaps wait 10—12 years for a church, but if it urgently needs a harbour, a road or a warehouse, it can scarcely wait that length of time, which was why development created—naturally—a number of independent contractors to do these more mundane works. There is not the slightest doubt that these contractors had their original roots within the Church's old building organ.

The builders who broke away from the Church's official order took with them all or part of the experiences, knowledge and training offered by the Church. It was not always the best-trained and most able men who started up on their own. But irrespective what stage they had reached in their training, they were bound by an oath of secrecy which allowed them to use their knowledge themselves, but which forbade them to share structual secrets with uninitiated outsiders.

Another factor which perhaps played a part here was that the traditional knowledge, training and ability may not have suited the requirements of the new building projects, since the latter differed so completely from established church-building projects. New methods had to be introduced.

Dissolution of the Church's building orders presumably occured gradually—maybe even with the blessing of the old orders of brethren, who at first retained a form of control over their former craftsmen colleagues through the principle of secrecy, etc.

The old building orders may have been forced to acknowledge that they could not possibly honour every practical demand made by Society. They therefore stood aside as many of their members went into the "outside" world to take care of the emerging problems of construction—without anyone fully realising the consequences.

The private contractors became within a relatively short time a vital part of the community. In many ways they became superior to the old organisation, and round about the year 1700 had reached such proportions that they drew all the strength out of the Church's traditional building sects.

So many new ideas and theories regarding construction work came into being that special schools had to be set up to train and practise the new skills—just as outside schools had to be established in the other (once purely the domain of the Church) professions such as medicine, the sciences, etc.

The old methods became out of date in many ways, and it was eventually of no practical significance that one could not teach openly the skills and knowledge of the old organisation. They were not forgotten; they were simply left behind by the new skills.

At this time the old building organisation lived in the shadow of the Church. The once-powerful training centres became depopulated. There were no practical problems to be solved. The formerly vigorous organisation was dying a lingering death.

About this time fresh blood was injected in its veins by the brethren of the Scottish and English orders of Church builders. And that inspiration bore fruit which blossoms even in our time.

The impulse that came from Britain was the start of the Freemasonry fraternity which as an order of ethics now spans the

globe with a myriad of international and national branches.

Historical sources record that speculative Freemasonry began in England in 1717 when a number of nobles and other leading citizens combined to start a "grand lodge" as an ethical branch of a practically operating building lodge. Building lodge was the term used to cover "a building organisation made up of craftsmen from various trades".

But the branch grew so rapidly and strongly that it within a short time became the main trunk of the tree, the purely practical building work taking a back seat.

While the lodge existed as a branch of the operative building organisation, one has a picture of dukes and earls together with other leading citizenry mixing freely with craftsmen directly and indirectly since the latter from apprentice to master were of course lodge members. And if we have recorded the picture correctly, the operative members belonged to the main lodge, while the nobility were merely members of a branch organisation.

There is something here that does not ring true of the scene in Britain in 1717.

Britain at that time was split severely into classes and the difference between the upper and lower classes was so great that no personal contact whatever was possible between them. The only common centre or assembly point shared by the whole community was the Church.

The upper class was the nobility, with all its rights and privileges. Only the higher ranks of churchmen stood on the same level as the nobles.

Under the nobility came the officer class which directly stemmed from the nobles. It cost a considerable sum of money to purchase a commission, and it was generally only sons of the aristocracy who could afford the necessary cash.

The next rung of the ladder was occupied by the gentry and landowners who could not quite claim sufficient blue blood to call themselves nobility. The next group was made up of merchants and traders who had succeeded in assembling fortunes on the strength of their commercial ability. This group included the country's bankers, money-lenders, etc.

The richest of these were accepted—but only just—by the nobility on account of their cash. They were regarded as upstarts who might well take a tumble if the market changed form.

Lesser merchants, and the so-called middle class were a necessary evil to be tolerated and used but not to be encouraged. While purely working class people were allowed to exist—if they could.

Britain was perhaps the most class-conscious society the world has seen (some claim the position has changed little even today) and every class was looked down upon and held down by the one above. Each had its own points of assembly, clubs, houses, areas, etc., and it was simply inconceivable that a person from a lower class should frequent one's own private club or eating house.

With this set-up in mind it is unthinkable that the top buds of society should suddenly plunge down among the masses of the lower class and start a club, society or lodge in connection with something as vulgar as the building industry. It is equally unbelievable that the upper class should without warning begin frequenting the circles of ordinary craftsmen. It would be such a break with tradition as to be doomed from the start to failure.

What is much more likely is that the equals of the nobility, i.e. the Church's leaders, had their entire building organisation lying dormant with few practical projects left to be completed.

Within this organisation, dating from the infinitely distant past, was a ritual which started with the admission of the

apprentice, a ritual which altered in forms as the apprentice progressed upwards through the various degrees and became a master himself.

There were—as in the civil population—extreme differences in "class" between the apprentice and the supreme master, who was frequently the head of the Church. The difference approached that between the ordinary worker and the king. But despite the distance from bottom to top, the spirit within the building organisation was rather more democratic than it was outside.

In spite of the different degrees of importance and training everyone within the organisation, from uppermost leader to the youngest apprentice, worked toward the same end: creation of beautiful buildings. And the holders of the respective degrees taught those of lesser degrees in order to improve their position and status. In theory anyone could achieve the highest degree of learning.

In daily operations holders of the respective degrees had their own meeting premises, so that there would always be a physical difference between the upper and lower degrees. The organisation had probably erected special buildings for the purpose of meeting.

It would be natural for the organisation to have such buildings and to observe a certain amount of ritual for we must not forget their original purpose: to erect churches. When we regard the ornamentation of old church buildings, both from a structural and detail point of view, then we must assume that the masters responsible for the work also knew how to apply their ability and knowledge to ornamentation when it was required as part of their own private rites.

An inspired group of Church leaders, left with the relatively empty shell of the building brotherhood recognised the social need for an organisation which—without preference to any particular class—could try to wipe out some of Society's inequalities. At the same time they realised that such an organisation had to include representatives of the ruling classes since without the co-operation of these the plan might collapse.

Thus the entire order of building brethren was reorganised; large sections of the strictly practical side of the traditional ritual were nevertheless retained since the founders were aware that where a thing might once have been of practical significance it could now take on a symbolic and ethical meaning.

The principle of secrecy was directly transferred to the new organisation—which was to become known as Freemasonry—and is still an integral part of the movement today.

Action of this kind on the part of the Church's top brains could not be ignored, and a large part of the nobility joined in the plan, which gave them an opportunity to lend a strictly anonymous helping hand where required. The organisation gained considerably in size and popularity throughout the length and breadth of Britain.

Outside the island's shores, presumably with the assistance of religious colleagues abroad, the movement and its principles were acknowledged and introduced in the same manner as in Britain. Within half a century of the foundation of the Grand Lodge in London, Freemasonry had spread not only to the European continent but also to America and Asia. Freemasonry was able to step into the shoes of the established but empty building brotherhood, and thus spread amazingly quickly until 250 years later it forms a network all over the world.

It is difficult to say whether the inspiration that began in Britain was completely spontaneous, or whether it was modelled on or prompted by even older

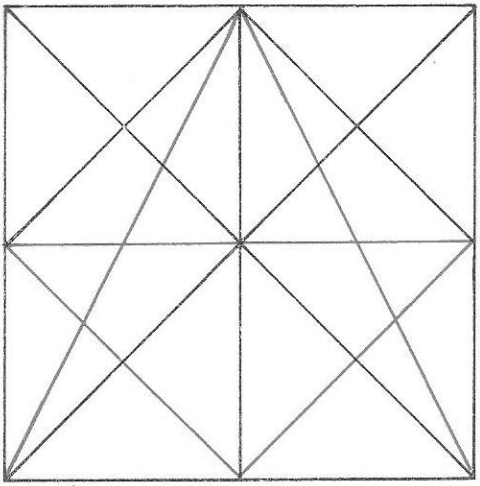


Fig. 380.

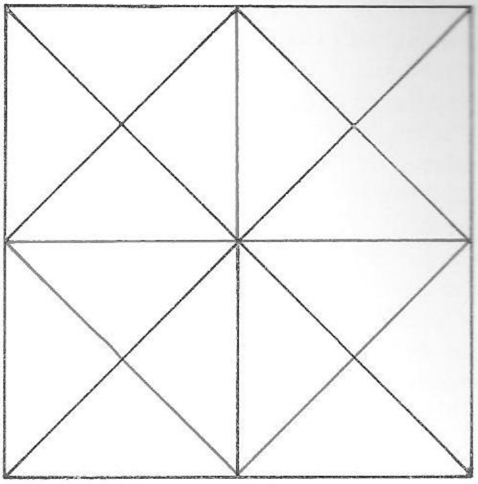


Fig. 381.

orders of knighthood, etc. It was a fact however that the form to which the building brotherhood was converted in Britain suited exactly the period, and is presumably why the new order has developed to a greater degree than other similar, though older, movements.

I think there can be little doubt that the founders of Freemasonry stemmed from within the Church, since they were the only group able to set up such a mixed organisation, cutting successfully through the ties and entanglement of class distinction.

Many things within the Freemasons' lodge demonstrate that the founders were familiar with the rules of ancient geometry. Of the outer signs of the movement, I can mention the square and compasses. This symbol has been selected from one of the well-known geometric diagrams appearing throughout the past chapters: the square, its half-size version and the acute-angled triangle.

The diagram and symbol are shown in *Fig. 380.*

Another typical symbol of Freemasonry is the tiny bricklayer's trowel. It too is taken from the same symbol, but without use of the acute-angled triangle: *Fig. 381.*

Thus we see that the outer symbols of Freemasonry fit the same diagrams as we have earlier discovered in many cultural domains throughout these two volumes.

I personally have no doubt that the origin of the movement lay in the roots of the old building fraternity, which can be traced directly back to a period of history and civilisation many thousands of years before our time.

Within the Freemasons' lodge and its work there are innumerable signs and examples of this link.

But this is not a matter into which I may go very deeply; the oath of secrecy that bound my predecessors applies also to me.

*

This book has followed a development which to our minds stretched over an infinitely long period, and we have examined such widely differing subjects as the pyramids of Gizeh and the beginning of a written language centuries later in Scandinavia.

We have also looked at the attempts of primitive Man to keep a check on the

days of the year, and have seen the impressive art of Greece. Our study has included the shape of vases, the design of temples, and the origin of Freemasonry.

It may surprise the reader that so many spheres can be covered and discussed on the basis of what I have termed ancient geometry, but it is not really surprising when one fully appreciates the connecting thread.

Ancient geometry and its many uses are the keys to very many fields in the research of our past.

When one holds the keys it is not difficult to open the different doors and discover what lies behind, but if one has no key, or if one does not understand how it should be used, the doors remain securely sealed, and the curious are obliged to guess what they possibly screen from sight.

This book is intended only as the introduction to an unlimited field of study, an aspect of our history which we are now able to probe from an entirely new point of view.